Environmental options and technological innovation: an evolutionary game model

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Abstract

This paper analyzes the effects on economic agents' behaviour of an innovative environmental protection mechanism that the Public Administration of a tourist region may adopt to attract visitors while protecting the environment. On the one hand, the Public Administration sells to the tourists an environmental call option that gives them the possibility of being (partially or totally) reimbursed if the environmental quality in the region turns out to be unsatisfactory (i.e. below a given threshold level). On the other hand, it offers the firms that adopt an innovative, non-polluting technology an environmental put option that allows them to get a reimbursement for the additional costs imposed by the new technology if the environmental quality is sufficiently good (i.e. above the threshold level).

The aim of the paper is to study the dynamics that arises with this financial mechanism from the interaction between the economic agents and the Public Administration in an evolutionary game context. The evolution of visitors and firms' behaviour is modeled in the paper using the so-called replicator dynamics, according to which a given choice spreads across the population as long as its expected payoff is greater than the average payoff. From the model it emerges that such a dynamics may lead either to a welfare-improving attractive Nash equilibrium in which all firms adopt the environment-friendly technology or to a Pareto-dominated equilibrium with no technological innovation and no tourism. As shown in the paper, the attraction basin of the virtuous equilibrium will be maximum if total reimbursement is offered by the Public Administration to the visitors and will be minimum if a simple entrance ticket is imposed on the tourists with no chance of reimbursement.

Keywords: environmental bonds; call and put options; technological innovation; evolutionary dynamics

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1 Introduction

Environmental problems arising from economic activity have become a well established research area in economics. Among the many proposals set forth to reduce pollution or protect the environment, much attention has been devoted in the literature to the introduction of specific financial assets that can integrate the traditional operating of the public sector by providing market incentives to achieve environmental objectives.

One of the most relevant examples of financial assets that can be issued in accordance with environmental purposes is constituted by the so-called *Environmental Bond (EB)*, introduced by Perrings (1987 and 1989).[1] The EB is a mandatory deposit paid to the public administration by any agent whose activity may damage the environment. The deposit is (totally or partially) refundable if the holder of the bond can prove to the regulation authority that he/she avoided the expected environmental damage of his/her activity. The EB represents, therefore, an incentive-based instrument of environmental risk control (Costanza and Perrings, 1990) and can be conceived as a generalization of the deposit-refund systems that have been applied in different contexts characterized by environmental risk, like compulsory deposits on waste lubricant oil, junked cars, beverage containers, dangerous substances contained in materials or products and so on (cf. Bohm, 1981; Huppes, 1988).[2]

The *EB* shares some common features with other policy instruments, like marketable permits, environmental taxes and subsidies. For instance, as some authors have pointed out (Torsello and Vercelli, 1998), the *EB* can be considered symmetrical to tradeable permits. In the latter case, the regulatory authority establishes the total quantity of the permits letting their price be determined by decentralized market decisions; while in the case of an *EB* system, the authority fixes the price of the *EB*, or risk premium for the possible damages caused to the environment, leaving the market free to determine the quantity of *EBs*.

Moreover, the *EB* can be regarded as the joint implementation of an environmental tax (the price of the *EB*) and a potential subsidy (the refund), but it is often considered politically more attractive than these two alternative fiscal measures taken separately. In an *EB* system, in fact, subsidies (refunds) are self-financed by taxes (deposits), therefore -differently from environmental subsidies- the *EB* does not imply any worsening of the public budget. Moreover, the prospective of a refund often makes the *EB* more acceptable to public opinion than the environmental taxes, since in the *EB* the punishment is proportional to the damage effectively produced and the refund is received only by the agents who can prove to deserve them.

The idea originally set forth by Perrings has been further developed by Horesh (2000, 2002a and 2002b), who has proposed a slightly different kind of

[1] Although Perrings was the first to use this term, a similar policy instrument had been previously suggested by Solow (1971) and Mills (1972) who had proposed the introduction of a material disposal tax.

[2] See also Gerard and Wilson (2009) for a possible application of *EB* to the nascent carbon sequestration projects.

EB, that is auctioned by the Public Administration (PA) on the open market, but, unlike ordinary bonds, can be redeemed at the face value only if a specified environmental objective has been achieved. They do not bear any interest, and the yield investors can gain depends on the difference between the auctioned price and the face value in the case of redemption. Economic agents involved in the environmental objective (either polluters or not), once in possession of the bonds, have a strong interest to operate in such a way that the objective itself is quickly achieved, so to cash in the expected gains as soon as possible.

In our paper we follow a rather different path, proposing two financial activities, issued by the PA of a tourist region *(R)*, which work like contracts between the PA and, respectively, visitors and firms operating in R - and can be regarded as (cash-or-nothing) *environmental call (EC)* and *environmental put (EP)* options. More specifically, the context we analyze has the following features.

An individual who desires to spend a period of time in the region R has to purchase the environmental call (*EC)* sold by the PA at a given price $\widetilde{p}$. This implies a cost for the visitor in the case of a satisfactory environmental quality, that is, when a properly defined environmental quality index Q is above a given threshold level $\overline{Q}$ fixed by the PA (the value of Q being evaluated by an independent authority); but offers the visitor the possibility of a reimbursement in the case of low environmental quality (namely, when $Q < \overline{Q}$). Consequently, buying the EC represents a self-insurance device that allows the visitor to *buy protection* against environmental degradation. Thus, potential visitors have to choose between the following strategies:

(V_1) visit the region R (and consequently buy the EC);

(V_2) do not visit the region.

Analogously, the PA offers to a potentially polluting firm operating in the region R the choice between subscribing or not to the environmental put option (EP) issued by the PA. This financial activity is a contract, which binds the firm to adopt a new environment-friendly technology, thus bearing a supplementary cost given by the difference between the cost of the new, non polluting technology (c_{NP}) and that of the old, polluting one (c_P), and implies a financial aid for the firm only if the environmental quality index Q results higher than the threshold level $\overline{Q}$.

Therefore, potentially polluting firms have to choose between the following strategies:

(F_1) adopting the new environment-friendly technology (and subscribing to the EP);

(F_2) carrying on its activity with the polluting technology in the region R.

The names of (environmental) *call* and *put options* for the financial activities we have described can be justified by the following interpretation of the above mechanism.

An actual (vs. potential) visitor can be said to have purchased the right of buying from the PA, at the end of a fixed period, paying a *strike price* equal to zero, a *security* whose value (immediately cashed in) is zero if the environmental goal is achieved and is equal to the reimbursement if it is not. Vice-versa, a firm adopting the environment-friendly technology can be seen as having acquired the right of cashing in, at the end of the same period, the difference between a *strike price* equal to the financial aid and the value of a *security*, which, again, is zero if the environmental goal is achieved and is equal to the financial aid itself in the opposite case. This way the *asset* underlying the two *options* can be interpreted as the cost, for the *PA*, of a *bad* environmental quality.

It is also clear that in a more detailed model reimbursement and financial aid can take several (even continuous) values.

In the following we will assume the value of Q to depend on the number of firms choosing the environment-preserving technology, i.e. subscribing to the *EP*.

Summing up, if $Q < \overline{Q}$, the visitors choosing V_1 receive a reimbursement for the low environmental quality experienced during the period spent in R, while the firms choosing F_1 do not receive any financial aid. If, on the contrary, $Q \geq \overline{Q}$, the visitors choosing V_1 bear a cost but can enjoy high environmental quality in the region R, while the firms choosing F_1 receive financial support for their investments aimed at protecting the environment. In this way, the *PA* can achieve the goal of improving the environmental quality at a relatively low cost. As a matter of fact, both visitors and firms have an incentive to protect the environment, the former in order to enjoy a better environmental quality in region R, the latter in order to get financial aid. Therefore, the costs born by the *PA* to finance the firms that subscribe to the *EP* can be compensated by the revenues the *PA* cashes in from selling the *EC* to the visitors.

The *PA* determines prices and reimbursements taking into account, among other things, the number of visitors and firms wishing to subscribe to the financial activities as well as the cost of the environment-preserving technological innovation.

The financial activities proposed here resemble, under certain aspects, the deposit-refund system implicit in the *EB*, but differs from it in other respects. In the *EB* the burden of the proof falls on the holder, which is often considered to be an attractive feature of the *EB*. However, this does not eliminate the monitoring costs for the regulatory authority that has to verify the evidence brought forward by the *EB* holders that their negative externalities were actually lower than expected. On the contrary, the regulatory authority may find it difficult and expensive to attribute the responsibility for a certain damage to a potential polluter (due to asymmetric information, scientific uncertainty, non-point sources and so on). In the present case, instead, the *PA* should only monitor the overall level of the chosen indicator Q (through an independent environmental authority, as proposed above), which might possibly reduce the monitoring costs of the system, while the agents do not have to suffer the burden of the proof that the environmental damage was lower than expected.

Moreover, the present proposal extends the application of the deposit-refund system typical of the *EB* from the set of potential polluters to the set of the visitors who would benefit from avoiding pollution. As a consequence, the mechanism described above generates a strong interdependency between firm and visitor payoffs. The aim of the paper is to study the dynamics that arises in this context from the interaction between economic agents (firms and visitors) and the *PA*.

To this purpose the choice process of firms and visitors is represented by a two-population evolutionary game, where the population of firms strategically interacts with that of visitors. The evolution of visitor and firm behaviour is modelled using the so-called replicator dynamics (e.g., see Weibull 1995), according to which a given choice spreads among the population as long as its expected payoff is greater than the average payoff. As it emerges from the model, such a dynamics may lead to a welfare-improving attractive Nash equilibrium, in which all firms adopt the environment-friendly technology and all potential visitors choose to visit region R. The attraction basin of this equilibrium expands as the reimbursement due to the visitors increases.

The paper has the following structure. Section 2 introduces the model and Section 3 provides the basic mathematical results. Section 4 concludes.

2 The model

Let us assume that at each period of time t potential visitors and firms play a one-shot population game (i.e. all agents play the game simultaneously). Each firm has to choose ex-ante whether to buy the *EP* and adopt the new environment-friendly technology (strategy F_1) or to keep on using the old polluting technology (strategy F_2). Similarly, each potential visitor has to choose ex-ante whether to buy the *EC* and visit the region R (strategy V_1) or go on holidays somewhere else (strategy V_2). Only the firms (potential visitors) who adopt the new technology (decide to visit the region) can buy the *EP* (*EC*). We assume that the potential visitors know ex-ante the criterion (specified below) that is used by the *PA* to fix the price of the *EC*. Therefore they also know in advance the maximum price they might have to pay to visit the region. At the end of the time period t, the *PA* decides whether to reimburse firms and visitors who bought the *EP* and the *EC*, respectively, on the basis of the data on the environmental quality in region R that are released by an independent environmental agency.

We assume the two populations to be constant over the time and normalize to 1 the number of both potential visitors and firms. Let the variable $x(t)$ denote the share of firms choosing F_1 at time t, $0 \leq x(t) \leq 1$. Analogously, let $y(t)$ denote the share of potential visitors adopting choice V_1 at time t, $0 \leq y(t) \leq 1$, and let $E(x)$ be their expected benefit from the time spent in region R, that is assumed to be positively correlated with x: the higher the proportion of firms choosing the non-polluting technology, the higher the environmental quality that the tourists can enjoy during their visit to the region.

Let us indicate by $\widetilde{p}(x, y)$ the price (fixed by the *PA*) of the *EC* bought by visitors choosing V_1 (assumed to depend on the proportion of individuals choosing V_1 and of firms choosing F_1); and by $\widetilde{r}_V(x, y) = \alpha\widetilde{p}(x, y)$ the reimbursement due by the *PA* to these visitors when $Q < \overline{Q}$, where α is a parameter satisfying the condition $0 \leq \alpha \leq 1$ ($\alpha = 1$ means that the amount $\widetilde{p}$ is totally reimbursed, whereas if $\alpha = 0$ visitors are not reimbursed at all). Then, the payoff of a visitor buying the call option is $E(x) - \widetilde{p}$ if the environmental goal is attained ($Q \geq \overline{Q}$), whereas the payoff is $E(x) - \widetilde{p} + \alpha \cdot \widetilde{p} = E(x) - \widetilde{p}(1 - \alpha)$ in case it is not ($Q < \overline{Q}$).

Denoting by $\theta(x)$ the probability that $Q < \overline{Q}$ (assumed to depend negatively on the proportion of firms adopting the environment-friendly technology), the expected payoff of strategy V_1 is, therefore, given by:

$$EV_1(x, y) = E(x) - \widetilde{p}(x, y) + \alpha \cdot \widetilde{p}(x, y) \cdot \theta(x) = E(x) - \widetilde{p}(x, y)\,[1 - \alpha \cdot \theta(x)]$$

For the sake of simplicity, we assume:

$$E(x) = \beta x$$

$$\widetilde{p}(x, y) = \gamma + \delta y + \varepsilon x \tag{1}$$

where β, γ, $\varepsilon > 0$, $\delta \gtreqless 0$ and $\gamma + \delta > 0$.[3]

Notice that $\widetilde{p}(x, y)$ is positively correlated to the number x of non-polluting firms. As a matter of fact, the entries obtained by the PA from the visitors through the call options EC can contribute to finance the firms that adopt the innovative technology. Therefore, if x increases, the PA tends to increase the price of the EC to finance the larger amount of the potential reimbursements due to the non-polluting firms. In other words, the price paid by the visitors increases as technological progress spreads among the firms of the region, progressively improving its environmental quality Q.

The price of the EC, moreover, may be positively or negatively correlated to the number of visitors y, according to the sign of δ. On the one hand, an increase in the number of visitors raises the demand of call options, which induces the PA to increase their price ($\delta > 0$). On the other hand, an increase in the number of visitors tends to enhance the entries available to the PA, so that the latter may have an incentive to reduce the price of the call option in order to attract an even higher number of potential tourists ($\delta < 0$). The sign of δ, therefore, is a priori ambiguous and depends on which one of these two opposite mechanisms will tend to prevail.[4]

Finally, we assume:

$$\theta(x) = 1 - x$$

[3] The latter condition ensures that the price of the call option $\widetilde{p}$ is always strictly positive for any possible value of x and y.

[4] Notice that the upper bound of the call option price is $\gamma + \delta + \varepsilon$ if $\delta > 0$ (occurring when $x = y = 1$) and $\gamma + \varepsilon$ if $\delta < 0$ (occurring when $x = 1$, $y = 0$). One can imagine that the PA fixes the values of the parameters γ, δ and ε in such a way that the upper bound is relatively low, so that it does not discourage potential tourists from visiting the region. If so, the PA can attract tourism (through the possibility of getting a reimbursement in case of an "unsatisfactory" holiday) and uses the related entries as a fund raising mechanism to support the adoption of environment-friendly technologies in the region.

This is equivalent to saying that if all firms adopt strategy F_1 and invest in the non-polluting technology ($x = 1$), the environmental quality index Q will *almost surely* be above the threshold level $\overline{Q}$ (i.e., $\theta = 0$) and the visitors will not be entitled to any reimbursement; whereas such an index will *almost surely* be below $\overline{Q}$ (i.e., $\theta = 1$, and visitors have to be reimbursed) if all firms choose strategy F_2 ($x = 0$).

Under the assumptions above, the expected payoff of strategy V_1 becomes:

$$EV_1(x, y) = \beta x - (\gamma + \delta y + \varepsilon x)\,[1 - \alpha(1 - x)]$$

Without loss of generality, we can normalize to zero the payoff of individuals choosing V_2 (i.e. deciding not to visit the region):

$$EV_2(x, y) = 0$$

Turning now to the firm decision process, if the environmental goal is missed ($Q < \overline{Q}$), the profit of a firm subscribing to the put option is:

$$R(y) - c_{NP}$$

where:

$R(y)$ is the firm revenue, which is an increasing function of the number y of visitors (and is independent of the adopted technology, being assumed to affect only the production costs);

$c_{NP} > 0$ is a parameter representing the cost of the non-polluting technology plus the cost of the put option sold by the PA.[5]

Whereas, in case the goal is achieved ($Q \geq \overline{Q}$), the profit is given by:

$$R(y) - c_{NP} + \widetilde{r}_F(x, y)$$

where $\widetilde{r}_F(x, y)$ is the financial aid received by a firm choosing F_1 in case $Q \geq \overline{Q}$.

Therefore, the expected profit EF_1 of a firm choosing strategy F_1 is:

$$EF_1(x, y) = R(y) - c_{NP} + \widetilde{r}_F(x, y) \cdot (1 - \theta(x))$$

where $1 - \theta(x) = x$ is the probability that $Q \geq \overline{Q}$.

If, instead, the firm keeps on using the polluting technology (strategy F_2), its profit is given by:

$$EF_2(x, y) = R(y) - c_P$$

[5] Observe that, for the sake of simplicity and without any loss of generality, the cost of the EP can be set equal to zero. If so, the firms subscribing to the EP would have to face only a technological innovation cost. This would avoid one of the main criticisms moved to the use of the environmental bonds, concerning the potential liquidity problem that a firm purchasing an environmental bond may suffer as long as it is not proved that its activity did not cause any environmental damage (or, in the present case, as long as the overall level of Q is unknown).

where c_P is the cost of the traditional (polluting) technology and it is: $c_{NP} > c_P > 0$.

We assume:

$$\widetilde{r}_F(x, y) = \lambda + \mu y + \nu x$$

where λ, $\mu > 0$ and $\nu \gtreqless 0$ are parameters fixed by the *PA*.

Notice that the financial aid received by a firm ($\widetilde{r}_F(x, y)$) is positively related to the number y of visitors choosing strategy V_1. In other words, as pointed out above, the *PA* uses the entries deriving from the visitors' subscription of the *EC* to finance the firms' adoption of new, low-impact technologies. Moreover, the financial aid may be positively or negatively related to the share of *clean* firms x. In fact, on the one hand, an increase in x improves the environmental quality of R; this tends to lower the likelihood that the *PA* will have to reimburse the visitors, thus setting free more financial resources that the *PA* can use to subsidy the non-polluting firms. On the other hand, an increase in x implies that more firms will be entitled to the financial aid, thus reducing the reimbursement level at disposal for each single firm.

The process of adopting strategies is modelled by the so called *replicator dynamics* (see, e.g., Weibull, 1995), according to which the strategies whose expected payoffs are greater than the average payoff spread within the populations at the expense of the alternative strategies:

$$\begin{aligned} \dot{x} &= x\left(EF_1 - \overline{EF}\right) \\ \dot{y} &= y\left(EV_1 - \overline{EV}\right) \end{aligned} \tag{2}$$

where

$$\begin{aligned} \overline{EF} &= x \cdot EF_1 + (1 - x) \cdot EF_2 \\ \overline{EV} &= y \cdot EV_1 + (1 - y) \cdot EV_2 \end{aligned}$$

are the average payoffs of the populations of firms and visitors, respectively.

The replication system (2) can be written as follows:

$$\begin{aligned} \dot{x} &= x(1 - x)\left(EF_1 - EF_2\right) = x(1 - x)F(x, y) \\ \dot{y} &= y(1 - y)\left(EV_1 - EV_2\right) = y(1 - y)G(x, y) \end{aligned} \tag{3}$$

where:

$$F(x,y) = -(c_{NP} - c_P) + \lambda x + \mu xy + \nu x^2$$

$$G(x,y) = -\gamma(1-\alpha) + [\beta - \alpha\gamma - \varepsilon(1-\alpha)]\, x - \delta(1-\alpha)y - \alpha\delta xy - \alpha\varepsilon x^2 \tag{4}$$

We assume the parameters to satisfy the following conditions:

$$\begin{array}{l} C_1)\ \ 0 \le \alpha \le 1 \\ C_2)\ \ c_{NP} > c_P > 0 \\ C_3)\ \ \beta, \gamma, \varepsilon > 0;\ \delta \lesseqqgtr 0;\ \gamma + \delta > 0 \\ C_4)\ \ \lambda, \mu > 0;\ \nu \lesseqqgtr 0 \\ C_5)\ \lambda + \mu y + \nu > c_{NP} - c_P\ \forall\ y \in [0,1] \\ C_6)\ \beta > \gamma + \delta y + \varepsilon\ \ \forall\ y \in [0,1] \end{array} \tag{5}$$

We have already discussed the conditions $C_1) - C_4)$. As to conditions $C_5)$ and $C_6)$, they imply that, no matter what the number of visitors y is, strategies F_1 and V_1 are more remunerative, respectively for firms and visitors, than F_2 and V_2 (i.e. $EF_1 > EF_2$ and $EV_1 > EV_2$), if the share of non-polluting industries is sufficiently high ($x \sim 1$). In fact in that case $\theta \sim 0$ and consequently is quite high the probability that the environmental goal be achieved.

3 Analysis of the model

Let us consider the dynamic system (3) whose parameters satisfy (5). System (3) is defined in the unit square

$$[0,1]^2 = \{(x,y) : 0 \le x \le 1, 0 \le y \le 1\}\,.$$

All sides of this square are invariant, namely, if the pair (x,y) initially lies on one side, then the whole correspondent trajectory also lies on that side. The following Proposition holds $\forall \alpha \neq 1$.[6]

Proposition 1 *The four vertices of* $[0,1]^2$ *are equilibria of* (3)*. In particular,* $(0,0)$ *and* $(1,1)$ *are attractors, while* $(1,0)$ *and* $(0,1)$ *are saddles.*

Proof. Writing the Jacobian matrix J at the vertices of $[0,1]^2$, it easily follows from conditions (5) that: $\det J(0,0) > 0$, $\det J(1,1) > 0$, $\det J(1,0) < 0$, $\det J(0,1) < 0$; moreover, *trace* $J(0,0) < 0$, *trace* $J(1,1) < 0$. This proves the proposition. ■

Notice that at the four vertices of the square only one strategy is played by firms and potential visitors. In particular, at the attractor $(1,1)$ all firms adopt the non-polluting technology and all potential tourists choose to visit the region, as they are attracted by high environmental quality deriving from the widespread adoption in the region of new, environment-friendly technologies.

[6] See at the end of the section the case $\alpha = 1$.

The opposite holds at the attractor $(0,0)$: all firms keep on using the traditional technology causing high pollution in the region, therefore none of the potential tourists decides to come to visit R. At $(0,1)$ all firms are polluting, nevertheless all potential visitors choose to spend their holidays in the region R. In this case, therefore, the visitors are attracted by the reimbursement received rather than by the environmental quality of R. This fixed point might describe the case of some popular tourist destinations where - despite the low environmental quality (e.g. polluted sea and crowded beaches) - tourists are mainly attracted by the low costs of the area (which is equivalent to getting a reimbursement that lowers the holiday costs). Notice, however, that this fixed point is non attractive, therefore it is not a Nash equilibrium of the model. Mutatis mutandis, the same reasoning applies to the saddle point $(1,0)$: although the quality of the environment of region R is extremely high (all firms being non-polluting), potential visitors care more for the holiday costs than for the environmental quality of R. Hence in this case they do not come to the region because there is no chance of reimbursement.

Beyond the vertices of the unit square $[0,1]^2$, system (3) will be shown to have two more equilibria on the boundary and up to three equilibria in the interior of the square (see Propositions 5 and 6 in the Appendix). All these additional (boundary or internal) equilibria are either sources or saddles (i.e. unstable equilibria). Moreover, it will be shown (see Proposition 7 in the Appendix) that there is no limit cycle in $(0,1)^2$ (i.e. inside the square). It follows that *almost every* trajectory of system (3) approaches one of the attracting vertices.[7] Therefore, no matter what the initial conditions are, the dynamics of the system will *almost always* lead to one of the two attractors of the square.

Figure 1 describes the dynamic regime that emerges in the model when the highest possible number of internal equilibria occurs. Attractors, repellors and saddle points are represented in the figure by full dots, empty dots and squares, respectively. The attraction basins of $(0,0)$ and $(1,1)$ are separated in the figure by the bold line that connects the two boundary equilibria $(x_1,1)$ and $(x_2,0)$. This separatrix is constituted by the union of the stable manifolds of the boundary saddle points $(x_1,1)$ and $(x_2,0)$ and of the internal saddle point Q_2. As the arrows in the figure show, the dynamics is path dependent. In fact, if the pair (x,y) of the initial levels of firms and tourists buying the options lies above the separatrix, then all the other agents will tend to imitate their behaviour and the system will eventually converge towards $(1,1)$. If, on the contrary, the initial (x,y) lies below the threshold level given by the separatrix, then the opposite strategies F_2 and V_2 will tend to spread among the populations of firms and potential visitors and the system will converge towards $(0,0)$. Although the morphology of the attraction basins may differ from one case to the other, similar "threshold effects" emerge also in the other cases, regardless of the number (from zero to three) of the internal equilibria. Figure 2, for instance, shows the case in which there exists only one (saddle point) equilibrium in the interior of the unit

[7] The system does not converge to one of the attracting vertices only when it lies in one of the other equilibria or along one of the stable manifolds of the saddle points.

square and its stable manifold (the bold line) separates the basins of attraction of $(0,0)$ and $(1,1)$.

Actually the expected payoffs of the agents in the attracting vertices are easily compared.

Proposition 2 *Under the assumptions $C_1 - C_6$, the equilibrium $(1,1)$ Pareto-dominates the other attracting equilibrium $(0,0)$ of system (3); i.e. $EV_1(1,1) > EV_2(0,0)$ and $EF_1(1,1) > EF_2(0,0)$.*

Proof. Notice that the expected payoffs evaluated in $(1,1)$ and $(0,0)$ are, respectively:

$$\begin{aligned} EV_1(1,1) &= \beta - (\gamma + \delta + \varepsilon) \\ EF_1(1,1) &= R(1) - c_{NP} + (\lambda + \mu + \nu) \end{aligned}$$

and

$$\begin{aligned} EV_2(0,0) &= 0 \\ EF_2(0,0) &= R(0) - c_P, \end{aligned}$$

where it is always $EF_1(1,1) > EF_2(0,0)$ under assumption C_5 and $EV_1(1,1) > EV_2(0,0)$ under assumption C_6. ■

From the Proposition above it follows that $(1,1)$ is a "virtuous equilibrium", where the region achieves the highest possible level of environmental quality and tourism, and all agents (visitors and firms) are better-off than in the alternative sink $(0,0)$. The latter, on the contrary, may be interpreted as a "poverty trap"[8] to which the system may converge, leading to a "vicious equilibrium" in which the region R is extremely polluted and unable to attract any tourist. To minimize this risk, therefore, the PA will try to fix the parameter values so as to maximize the attraction basin of $(1,1)$, thus increasing as much as possible the set of initial values of x and y that make the system converge to the virtuous equilibrium. The following Proposition describes one possible way by which the PA may achieve this goal.

Proposition 3 *The basin of attraction of $(1,1)$ expands as α increases.*

Proof. As it can be easily verified (see the Mathematical Appendix), when $\alpha < 1$ the basins of attraction of the equilibria $(1,1)$ and $(0,0)$ are separated by a curve formed by the union of the stable manifolds of the saddle points (see, for instance, Figures 1 and 2). This separatrix is the graph of a decreasing function of x, $\widetilde{Y}(x)$, with slope $\frac{d\widetilde{Y}(x)}{dx} = \frac{\dot{y}}{\dot{x}} < 0$, if $\dot{x} \neq 0$.

[8] By this term we mean a situation in which private rational decisions lead to outcomes that are not optimal from a social viewpoint.

Let us indicate by $\widetilde{Y}_{\alpha_1}(x)$ the separatrix corresponding to $\alpha = \alpha_1$. Notice that if α increases (coeteris paribus), the value of $\dot{y}$ increases while that of $\dot{x}$ remains constant. It follows that setting $\alpha = \alpha_2 > \alpha_1$ the locus $\widetilde{Y}_{\alpha_1}(x)$ is crossed from the left to the right by the trajectories of system (3) with $\alpha = \alpha_2$. This implies that the basin of attraction of $(1,1)$ for $\alpha = \alpha_2$ is greater than for $\alpha = \alpha_1$. ■

Therefore, by increasing the reimbursement share α the *PA* can enhance the attraction basin of the first best outcome. In other words, the higher the reimbursement share α, the lower the initial values of x, given y, and y, given x, that are needed to converge to $(1,1)$. If this is the case, it is sufficient that a lower initial number of firms (visitors) choose the financial instruments proposed by the *PA* (*EP* and *EC*, respectively) to convince all other firms (potential visitors) to imitate their choice and behave the same way.

The attraction basin of $(1,1)$ will obviously be maximum when $\alpha = 1$ (i.e. the price of the call-option is totally reimbursed if the environmental goal is missed) and minimum when $\alpha = 0$ (i.e. no reimbursement occurs). Notice that in the latter case the price paid by the tourists becomes simply a tourist tax, that is, an entrance ticket that tourists pay to have access to the region.

We can conclude that the financial mechanism proposed here (allowing visitors to be refunded in case of an unsatisfactory environmental quality) is more likely to lead the system towards the virtuous equilibrium $(1,1)$ than the traditional entrance ticket without any refund possibility. Moreover, by proposing total reimbursement in case of low environmental quality, the *PA* actually minimizes the probability of refunding the tourists since this maximizes the attraction basin of the non-polluted equilibrium $(1,1)$.

Interestingly enough, as it can be easily verified, when $\alpha = 1$ (total reimbursement) there exists a continuum of equilibria along the side $x = 0$ and a unique trajectory leading to each of them (see Figure 3). This implies that, if the initial pair (x, y) lies below the separatrix, we can have any number of visitors at the *final* equilibrium. In this case we end up by having minimum environmental quality (all firms being polluting) and maximum reimbursement. Therefore it is not possible to predict a priori whether the tourists will be more attracted by the possibility of being totally reimbursed or more discouraged by the degradation of the environmental quality in region *R*.

4 Conclusions

The present paper suggests an innovative financial mechanism that the *PA* of a tourist region may adopt to attract visitors while protecting the environment. On the one hand, the *PA* sells to the tourists an environmental call option that gives them the possibility of getting a reimbursement if the environmental quality in the region turns out to be unsatisfactory (i.e. below a given threshold level). On the other hand, the *PA* offers the firms that adopt an innovative, non-polluting technology the possibility of getting a reimbursement to cover the additional costs imposed by the new technology if the environmental quality turns out to be sufficiently good (i.e. above the threshold level).

Since the two kinds of reimbursements (to visitors and firms) are linked to the same environmental index, they will tend to compensate each other. More precisely, if the environmental quality target is achieved, the entries that the *PA* gets from selling the call options to the visitors contribute to finance the financial aid given to the non-polluting firms. If, on the contrary, the environmental quality target is missed, the same entries plus those possibly earned by the *PA* from selling the put options finance the reimbursements due to the visitors. The fund-raising mechanism proposed here, therefore, could be a useful instrument to promote and spread across the firms a technological shift from a polluting technology towards a more environment-friendly one.

The mechanism described above extends to a two-population game the deposit-refund system, that provides the basic idea underlying the environmental bonds proposed in the literature. Differently from these instruments, however, in the present case the burden of the proof does not fall on the holder of the financial instrument, since the reimbursement is linked to the observed performance of an overall environmental quality index. This has a twofold effect: on the one hand, it removes the costs that a firm might encounter to prove ex-post that its activity did not actually damage the environment and, on the other hand, it generates a strong interdependency between the choices of the two populations (firms and visitors). The present mechanism, moreover, can reduce the risk of moral hazard behaviour that may arise with the environmental bonds. The latter instrument, in fact, may induce the *PA* to overestimate the environmental degradation provoked by a single firm to avoid refunding it, whereas in the present case the reimbursement depends on the observed values of an environmental quality index measured by an independent external agency.

As shown in the paper, the system is characterized by a multiplicity of possible equilibria (six fixed points along the boundary and up to three in the interior of the unit square). From the dynamics that emerges in the model it turns out that only two of these possible equilibria are attractors (namely, the fixed points $(1,1)$ and $(0,0)$) and that almost all trajectories will converge to them, since no limit cycle occurs in the interior of the unit square. At both attractors all the agents of each population choose the same strategy. Both firms and tourists would be better-off at the "virtuous equilibrium" $(1,1)$ in which all firms adopt the non-polluting technology and all potential visitors come to visit the region. However, the trajectories deriving from the interaction between the two populations may also lead to an attracting poverty trap where all firms are polluting and no tourist comes to the region.

Whether the system will converge to the first-best equilibrium or to the alternative attractor will depend on the initial share of firms (x) and potential tourists (y) that buy the environmental call and put options offered by the *PA*. The final outcome towards which the system will eventually converge is, therefore, strongly path-dependent for the existence of threshold effects and imitative behaviors that spread the most remunerative strategy across the agents within each population. The *PA*, however, can affect these threshold effects by modifying the reimbursement share due to the visitors in case of a low environmental quality in the region. If the *PA* aims at simultaneously achieving the maximum environmental quality and the maximum number of tourists, it should offer total reimbursement to the visitors as this maximizes the attraction basin of the virtuous equilibrium $(1,1)$. If, on the contrary, the *PA* levies a simple entrance ticket on the tourists with no chance of being reimbursed, this minimizes the attraction basin of $(1,1)$, increasing the critical mass of x or/and y that are needed to escape the poverty trap $(0,0)$. Increasing the reimbursement share, therefore, might paradoxically lower the costs of the financial mechanism for the *PA*: if the system converges to $(1,1)$ no reimbursement will be paid by the *PA* to the tourists and the entries obtained from the call options can be used by the *PA* to finance the firms for their virtuous (non-polluting) behaviour.

In our opinion, the present analysis could be extended into several directions in the future. In particular, using an optimal control model in which the *PA* aims at maximizing its own objective function, it would be interesting to compare the costs for the *PA* of the two alternative regimes described above (with and without reimbursement), taking its budget constraint explicitly into account.

5 Mathematical appendix

This section provides a complete characterization of the possible dynamics of system (3) in the unit square $[0,1]^2$.

Recall the expressions of $F(x,y)$ and $G(x,y)$ in (4). The possible equilibria of system (3), in addition to the vertices of $[0,1]^2$, are given by:

- the possible intersections of the loci $F(x,y) = 0$ and $G(x,y) = 0$ with, respectively, the *horizontal* and the *vertical* edges of $[0,1]^2$;
- the intersections between the loci $F(x,y) = 0$ and $G(x,y) = 0$ in the interior of $[0,1]^2$.

In order to study the existence and stability of equilibria of system (3), we prove the following Proposition.

Proposition 4 *The intersection of* $F(x,y) = 0$ *with the square* $[0,1]^2$ *is the graph of a decreasing function* $y = f(x)$ *defined in an interval* $[x_1, x_2]$, $0 < x_1 < x_2 < 1$, *with* $f(x_1) = 1$ *and* $f(x_2) = 0$. *Analogously, the intersection of* $G(x,y) = 0$ *with the square* $[0,1]^2$ *is the graph of a function* $y = g(x)$ *such that:*

- *if* $\delta > 0$, $y = g(x)$ *is an increasing function defined in an interval* $[x_3, x_4]$, $0 < x_3 < x_4 < 1$, *with* $g(x_3) = 0$ *and* $g(x_4) = 1$;
- *if* $\delta < 0$ *and* $\gamma + \delta > 0$, $y = g(x)$ *is a decreasing function defined in an interval* $[x_5, x_6]$, $0 < x_5 < x_6 < 1$, *with* $g(x_5) = 1$ *and* $g(x_6) = 0$.

Proof. The intersections of $F(x,y) = 0$ and $G(x,y) = 0$ with $[0,1]^2$ are, respectively, the graphs of the functions

$$y = f(x) = \frac{1}{\mu}\left(\frac{c_{NP} - c_P}{x} - \lambda - \nu x\right) \tag{6}$$

and

$$y = g(x) = \frac{1}{\delta}\left(\frac{-\gamma(1-\alpha) + (\beta - \alpha\gamma - \varepsilon(1-\alpha))\,x - \alpha\varepsilon x^2}{1-\alpha+\alpha x}\right) \tag{7}$$

It follows from conditions (5) that

$$\lim_{x\to 0+} f(x) = +\infty, \quad f(1) < 0$$

and that $f(x)$ either has no extreme (if $\nu > 0$) or has a negative maximum and a positive minimum point (if $\nu < 0$). Hence the intersection of $y = f(x)$ with $[0,1]^2$ is the graph of a decreasing function defined in $[x_1, x_2]$, $0 < x_1 < x_2 < 1$, with $f(x_1) = 1$ and $f(x_2) = 0$.

Analogously one can check that

$$\begin{array}{ll} g(0) < 0,\ g(1) > 1 & \text{if } \delta > 0 \\ g(0) > 1,\ g(1) < 0 & \text{if } \delta < 0,\ \gamma + \delta > 0 \end{array}$$

and that $g(x)$ has a negative minimum (maximum) and a positive maximum (minimum) point if $\delta > 0$ ($\delta < 0$). This proves the statements of the Proposition. ■

Let us now classify all the boundary equilibria.

Proposition 5 *System* (3) *has six equilibria on the boundary of* $[0,1]^2$, *i.e. the four vertices plus* $P_1 = (x_1, 1)$ *and* $P_2 = (x_2, 0)$. *The two vertices* $(0,0)$ *and* $(1,1)$ *are attractors, while* $(0,1)$ *and* $(1,0)$ *are saddles. Moreover,* P_1 *is a saddle or a repellor if, respectively,* $G(x_1, 1)$ *is* > 0 *or* < 0*; whereas* P_2 *is a saddle or a repellor if, respectively,* $G(x_2, 0)$ *is* < 0 *or* > 0.

Proof. All the statements of the proposition are easily proved by writing the expression of the Jacobian matrix and verifying the sign of its trace and determinant at each boundary equilibrium. ■

Proposition 6 *The internal equilibria of system* (3) *can be 0, 1, 2 or 3. More precisely:*

- *if* $\delta > 0$, *there is at most one internal equilibrium, which, in case it exists, is a saddle;*
- *if* $\delta < 0$, *the number of internal equilibria (counted by their multiplicity) is even if* $G(x_1, 1) \cdot G(x_2, 0) > 0$, *odd if* $G(x_1, 1) \cdot G(x_2, 0) < 0$*;*
- *no internal equilibrium is attractive: in particular, there exist at most one internal saddle and at most two internal repellors.*

Proof. Obviously $y = f(x)$ and $y = g(x)$ have at most three intersections in $(0,1)^2$. So, let $Q = (x^*, y^*)$ be an internal equilibrium and denote by $J(Q)$ its Jacobian matrix. Then

$$sign(\det J(Q)) = sign(\tfrac{\partial F}{\partial x}\tfrac{\partial G}{\partial y} - \tfrac{\partial F}{\partial y}\tfrac{\partial G}{\partial x})$$

$$trace(J(Q)) = x^*(1-x^*)\tfrac{\partial F}{\partial x}(x^*, y^*) + y^*(1-y^*)\tfrac{\partial G}{\partial y}(x^*, y^*)$$

Recalling conditions (5), it is easily checked that $\det J(Q) < 0$ if $\delta > 0$; while, if $\delta < 0$

$$\det J(Q) \gtreqless 0 \ \text{ iff } \ |f'(x^*)| \gtreqless |g'(x^*)|$$

Moreover, when $\delta < 0$, being $y = f(x)$ and $y = g(x)$ both decreasing in $[0,1]^2$, it follows that $\frac{\partial F}{\partial x}$, $\frac{\partial G}{\partial y}$ and thus $trace(J(Q))$ are positive.

Finally, suppose that $\delta < 0$ and three internal equilibria exist, say $Q_1 = (x_1^*, y_1^*)$, $Q_2 = (x_2^*, y_2^*)$, $Q_3 = (x_3^*, y_3^*)$, $x_1^* < x_2^* < x_3^*$. Then it is easily observed that $|f'(x_i^*)| > |g'(x_i^*)|$ when $i = 1, 3$, whereas $|f'(x_2^*)| < |g'(x_2^*)|$.

Clearly the previous considerations imply all the statements of the Proposition. ■

The phase portrait of system (3) can be fully described by combining the results of the previous Propositions with the following one.

Proposition 7 *System* (3) *admits no limit cycle in* $(0,1)^2$.

Proof. Due to the index Theorem (see, for example, Guckenheimer and Holmes, 1983) and the results of Proposition 6, a possible limit cycle in $(0,1)^2$ must surround some repellor (precisely, either one repellor or two repellors and one saddle). Because of Proposition 6, this implies $\delta < 0$. Hence, let $Q = (x^*, y^*)$ be an internal repellor. It is easily checked that either $P_1 = (x_1, 1)$ or $P_2 = (x_2, 0)$ is such that no other equilibrium exists in the strip $[x_1, x^*] \times [0,1]$ (or $[x^*, x_2] \times [0,1]$) and, correspondingly, P_1, or P_2, is a saddle.

Assume this is true for P_1 (mutatis mutandis the same applies to P_2) and consider the *triangoloid* $T = \{x_1 \le x \le x^*,\ g(x) \le y \le f(x)\}$, with *sides* $L_1 = \{x = x_1,\ g(x_1) \le y \le 1\}$, $L_2 = \{x_1 \le x \le x^*,\ y = g(x)\}$ and $L_3 = \{x_1 \le x \le x^*,\ y = f(x)\}$.

Then it is easily observed that the vector field points *out of* T along $L_1 \cup L_2 \cup L_3$ and there must exist a separatrix in T between the trajectories crossing $L_1 \cup L_2$ and those crossing L_3. It follows that such a separatrix must be a trajectory *joining* P_1 and Q, which can be represented by the graph of some decreasing function $y = h(x)$, $x_1 \le x \le x^*$. Thus Q cannot be surrounded by a limit cycle. ■

Being excluded the existence of a limit cycle, then *almost every* trajectory of system (3) approaches one of the two attracting vertices of the square $[0,1]^2$.

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