Corporate Debt Value
with Switching Tax Benefits and Payouts

Flavia Barsotti
Dept. Math. for Decisions
University of Firenze, Italy
flavia.barsotti@unifi.it

Maria Elvira Mancino
Dept. Math. for Decisions
University of Firenze, Italy
mariaelvira.mancino@dmd.unifi.it

Monique Pontier
Inst. Math. de Toulouse (IMT)
University of Toulouse, France
monique.pontier@math.univ-toulouse.fr

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Abstract

This paper analyzes a structural model of corporate debt in the spirit of Leland's model [7] within a more realistic general context where payouts and asymmetric tax-code provisions are introduced. We analytically derive the value of the tax benefit claim in this context and study the joint effect of tax asymmetry and payouts on optimal corporate financing decisions. Results show a quantitatively significant impact on both optimal debt issuance and leverage ratios, thus providing a way to explain differences in observed leverage across firms.

JEL Classification: G32, G33

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1 Introduction

A firm's capital structure decision is a complex issue due to many variables entering in the determinacy of corporate financing policy, i.e. riskiness of the firm, bankruptcy costs, payouts, interest rates and taxes. In particular corporate tax rates are a fundamental factor in corporate financing decisions, as early recognized by [12] and observed in more recent empirical studies [4, 13].

In this paper we consider a firm subject to default risk in a framework with bankruptcy costs and taxes, whose owners can optimally choose the capital structure by maximizing the total value of the firm. We extend the model proposed by [7] by means of a switching (even debt dependent) in tax savings and the introduction of a company's assets payout ratio. We perform a quantitative study of the effects of both factors on the optimal capital structure of the firm, obtaining analytical results in most cases. As a matter of fact, tax code provisions can vary across nations, across industries, across activity sector in which the firm is operating in, and also across time (e.g. when the tax code is modified to encourage investments, [14], [13] footnote 1). The economic insight we want to analyze is how asymmetry in tax-code provisions is incorporated in firm's financing decisions and moreover its quantitative impact on optimal debt issuance and leverage ratios. Our findings show that the combined effect of tax asymmetry and payouts produces predicted optimal leverage ratios which are more in line with historical norms (significantly reduced w.r.t. the ones in [7]) and empirical evidence.

Following [7] we consider an infinite horizon assuming that the firm issues debt and debt is perpetual. Debt pays a constant coupon per instant of time and this determines tax benefits proportional to coupon payments. A payout rate is also introduced as in [2] and [15]. Structural models assume constant corporate tax rates, thus tax benefits of debt are constant through time. The original assumption in [7] is that the firm has deductibility of interest payments for all firm's assets values above the failure level, producing a constant tax-sheltering value of interest payments. Leland argues that default and leverage decisions might be affected by non constant corporate tax rates, because a loss of tax advantages is possible for low firm values. Thus in [7] section VI.A the author suggests that, when assets value decreases, it is more likely that profits will be lower than coupon payments and the firm will not be able to fully benefit tax savings. The empirical analysis of [4] confirms that the corporate tax schedule is asymmetric, in most cases it is convex. The quantitative impact of this asymmetry on the optimal default boundary and leverage ratio is considered in [13] under the hypothesis of a piecewise linear tax function when the state variable is the operating income; the simulation study therein shows that the effect of tax asymmetry on the optimal leverage ratio is quantitatively significant, while it is lower on the optimal default boundary. Further [14] examines the relation between tax convexity and investment in the presence of a strictly convex (quadratic) tax function.

In this paper we extend Leland model by incorporating the possibility of two different corporate tax rates, namely τ_1, τ_2 and net cash outflows. We consider as state variable firm's current assets value. The switching from a corporate tax rate to the other is determined by the firm value crossing a critical level. We study two alternative frameworks: at first, the switching barrier is assumed to be a constant exogenous level; then we analyze a more realistic scenario in which this level depends upon the amount of debt issued by the firm. In fact, as pointed out in [7], under U.S. tax codes, a necessary condition required to fully benefit tax savings, is that the firm's EBIT (earnings before interest and taxes) must cover payments required for coupons. We obtain an explicit form for the tax benefit claim, which allows us to study monotonicity and convexity of equity function, to find the endogenous failure level analytically in the case with no payout and to prove its existence and uniqueness in the general case. Further, exploiting the linearity of the smooth pasting condition with respect to the coupon, we are able to study the optimal capital structure of the firm. Differently from [7], we solve the optimal control problem as an optimal problem in the set of passage times; the key tool is the Laplace transform of the stopping failure time [1, 3, 6].

As a measure of the degree of asymmetry, we refer to the ratio between the two corporate tax rates. Our study shows that tax asymmetry increases the optimal failure level and reduces the optimal leverage ratio, with a more pronounced effect on optimal leverage ratios, thus confirming results in [13]. Nevertheless, we find that, as far as the magnitude is concerned, introducing a payout produces an even more significant reduction in optimal leverage ratios. Thus the joint effect of tax asymmetries and payouts drops down optimal leverage to empirically representative values and seems to be a flexible way to capture differences among firms facing different tax-code provisions. For example observing firms belonging to different activity sectors, this could explain differences in observed capital structure decisions, mainly in leverage ratios. The analysis developed in [2] showed that introducing payouts in a structural model with a unique corporate tax rate has the effect of reducing both optimal leverage ratio and optimal failure level. In the present paper we find that this reduction in both optimal leverage and optimal failure level increases as the degree of asymmetry of the tax schedule rises, meaning as the difference between the two corporate tax rates is higher. We study both the impact of asymmetry in tax benefits on optimal capital structure in comparison with a flat tax schedule (i.e. a unique constant tax rate) as benchmark model, but also how these decisions change as the asymmetry varies showing two alternative approaches to measure the impact of asymmetry on corporate decisions. Finally, both optimal total value of the firm and optimal debt are decreasing functions of asymmetry and payouts: debt becomes less attractive (due to potential loss of tax benefits) and less assets remain in the firm because of payouts outflows. We then study optimal capital structure when the switching barrier is an increasing and linear function of the coupon level, in order to represent a more realistic framework in which EBIT is considered as a barrier determining a potential loss in coupon payments deductibility. In such a case a higher profit is needed in order to fully benefit from tax savings. Given a payout rate, as the optimal coupon decreases for higher degrees of asymmetry, then also the optimal switching barrier decreases: this trade-off concerning the potential tax benefits loss leads to empirically representative value in predicted leverage ratios.

We stress the point that the payout rate and the tax asymmetry parameter have a deeply different nature from an economic point of view: even if in our model the payout is exogenously given, we can also recognize that it can be partly modified or chosen by the firm, even when it is not a result of an endogenous decision (i.e. even if it does not depend on coupon payments, as in our case). Namely, opposite to this potential choice by the firm, the corporate tax schedule is imposed to the firm by an external authority. Moreover, the corporate tax schedule could be very different depending for example on the sector in which the firm is operating in. Thus, analyzing the joint effect of these two factors can be an interesting and flexible way to analyze and improve empirical findings inside a structural model of credit risk allowing to explain why a high dispersion in observed leverage ratios exists.

The paper is organized as follows. In Section 2 we introduce the model. We compute the tax benefit claim. In Section 3 the endogenous failure level is derived and the influence of tax asymmetry on it is analyzed. The optimal capital structure when the switching barrier is fixed or it is debt dependent, is achieved in Section 4. Section 5 concludes.

2 The Model

In this section we introduce a structural model of corporate debt in the spirit of [7]; nevertheless, our model exhibits two differences: the model for the firm's activities includes a parameter δ which represents a constant fraction of value paid to security holders (e.g. dividends, see also [2, 9, 15]), further, we consider a corporate tax schedule which is not flat, meaning we suppose the corporate tax rate being not unique and constant through time. We derive the value of the tax benefit claim in this framework, following Leland [7] in modeling tax benefits of debt: asymmetry in corporate tax code provisions becomes also asymmetry in tax benefits of debt.

We assume an infinite time horizon, as in [7]. This is a reasonable first approximation for long term corporate debt and enables us to have an analytic framework where all corporate securities depending on the underlying variable are time independent, thus obtaining closed form solutions. We consider a firm realizing its capital from both debt and equity. The firm has only one perpetual debt outstanding, which pays a constant coupon stream C per instant of time[1]. This assumption can be justified thinking about two different scenarios: a debt with very long maturity (in this case the return of principal has no value) or a debt which is continuously rolled over at a fixed interest rate (as in [9]). Bankruptcy is triggered endogenously by the inability of the firm to raise sufficient capital to meet its current obligations. On the failure time T, agents which hold debt claims will get the residual value of the firm, and those who hold equity will get nothing. Following Leland [7] we do not consider personal taxes, thus we model the tax benefits claim as a derivative depending directly on corporate tax rate provisions.

Suppose that firm's activities value is described by process $V_t = Ve^{X_t}$, where X_t evolves, under the risk neutral probability measure, as

$$dX_t = \left(r - \delta - \frac{1}{2}\sigma^2\right) dt + \sigma dW_t,\ X_0 = 0, \tag{1}$$

[1] Instantaneous coupon payments can be written as $C := cF$, where F is face value of debt, supposed to be constant through time, as in Leland [7].

where W is a standard Brownian motion, r the constant risk-free rate, r, δ and $\sigma > 0$. The term δV_t represents the firm's cash flow: we can think of it as an after-tax net cash flow before interest, since we only consider tax benefits of debt. When bankruptcy occurs at stopping time T, a fraction α $(0 \leq \alpha < 1)$ of firm value is lost (for instance payed to who takes care of the bankruptcy procedures), the debt holders receive the rest and the stockholders nothing, meaning that the strict priority rule holds. The failure passage time is determined when the firm value falls to some constant level V_B: define

$$T_{V_B} := \inf\{t \geq 0 : \ V_t = V_B\}.$$

The value of V_B is endogenously derived and will be determined with an optimal rule later.

In the spirit of Leland we assume that from paying coupons the firm obtains tax deductions. In [7] the corporate tax rate τ is assumed to be constant; nevertheless, in Appendix A the author derives the endogenous failure level in the case when there are no tax benefits for the assets value going under an exogenously specified level. The empirical analysis of [4] confirms that the corporate tax schedule is asymmetric. Moreover [13] assumes the hypothesis of a piecewise linear tax function and reports a quite significant impact of this asymmetry on the optimal leverage ratio. These studies motivate our extension of Leland's setting in the direction of a structural model with endogenous default boundary presenting a more general (even debt dependent) corporate tax schedule.

Following [7, 11] tax benefits can be expressed as a defaultable claim written on the underlying asset represented by the unlevered value of the firm V. Let $\tau(\cdot)$ be the corporate tax function and $F(\cdot)$ the tax benefits function. Tax benefits of debt can be seen as the value of a claim written on V_t paying a continuous instantaneous coupon $\tau(V_t)C$ if there is no default and 0 in the event of bankruptcy. We now describe the general scheme to determine the value of this defaultable claim for a given corporate tax schedule.

Lemma 2.1 *For any stopping time T the value of the tax benefits of debt is equal to:*

$$F(V) = \mathbb{E}_V \left[e^{-rT} F(V_T) + \int_0^T e^{-rs} \tau(V_s) C ds \right], \tag{2}$$

where the expectation is taken with respect to the risk neutral probability and we denote $\mathbb{E}_V[\cdot] := \mathbb{E}[\cdot | V_0 = V]$.

In this section the asymmetric tax benefits schedule is specified through the introduction of an exogenously given level of firm's assets value at which the tax deduction changes. We modify Leland [7] assumption about a unique constant level τ, considering a piecewise linear model in which two different corporate tax rates τ_1, τ_2 are in force. We assume that the deductibility of coupon payment generates tax benefits for all value $V \geq V_B$, but these tax savings are reduced when V falls to a specified (exogenous) barrier V_S. As the firm approaches bankruptcy, it will lose tax benefits. Therefore the corporate tax function is equal to

$$\tau = \tau_1 \mathbf{1}_{(V_S,\infty)} + \tau_2 \mathbf{1}_{(V_B,V_S)} \tag{3}$$

depending on the firm's activities value V_t staying upon the prescribed level V_S. Obviously we assume $V_S > V_B$. The tax-sheltering value of interest payments will not be constant through time: it will be $\tau_1 C$ for $V \geq V_S$, and $\tau_2 C$ in case $V_B \leq V < V_S$. We assume $\tau_2 \leq \tau_1$, meaning loss of tax benefits below V_S.

The first passage time at V_S is defined

$$T_{V_S} = \inf\{t \geq 0 : V_t = V_S\}. \tag{4}$$

Note that $V_{T_{V_B}} = V_B$ and $V_{T_{V_S}} = V_S$, as the process V_t is continuous.

Using integral representation of tax benefits we can write (2) as:

$$F(V) = \mathbb{E}_V \left[e^{-rT_{V_S} \wedge T_{V_B}} F(V_{T_{V_S} \wedge T_{V_B}}) + \int_0^{T_{V_S} \wedge T_{V_B}} e^{-rs} C \tau(V_s) ds \right]. \tag{5}$$

where $\tau(V_s)$ is specified by (3).

It is easily seen that in order to compute (5), it is enough to have explicit formulas for the Laplace transform of a double boundaries passage time. As V_B is a failure level, we impose that tax benefits are completely lost at failure, then the required boundary condition is $F(V_B) = 0$. Finally we can state the following result.

Theorem 2.2 *Suppose that the deduction tax function $\tau(\cdot)$ is defined by (3), then the tax benefits claim $F(V)$ in (2) is equal to:*

$$F(V) = \left(A_0 + A_1 V^{-\lambda_1} + A_2 V^{-\lambda_2}\right) \mathbf{1}_{(V_B,V_S)}(V) + \left(B_0 + B_2 V^{-\lambda_2}\right) \mathbf{1}_{(V_S,\infty)}(V), \tag{6}$$

where

$$A_0 = \frac{\tau_2 C}{r} \tag{7}$$

$$A_1 = \frac{C\lambda_2 V_S^{\lambda_1}(\tau_2 - \tau_1)}{r(\lambda_1 - \lambda_2)}, \tag{8}$$

$$A_2 = -\left(A_1 V_B^{-\lambda_1} + \frac{\tau_2 C}{r}\right) V_B^{\lambda_2}, \tag{9}$$

$$B_0 = \frac{\tau_1 C}{r}, \tag{10}$$

$$B_2 = A_2 + A_1 \frac{\lambda_1 V_S^{\lambda_2 - \lambda_1}}{\lambda_2}. \tag{11}$$

and λ_1, λ_2 *are defined as*

$$\lambda_1 = \frac{\mu - \sqrt{\mu^2 + 2r\sigma^2}}{\sigma^2}, \qquad \lambda_2 = \frac{\mu + \sqrt{\mu^2 + 2r\sigma^2}}{\sigma^2} \tag{12}$$

with

$$\mu := r - \delta - \frac{1}{2}\sigma^2. \tag{13}$$

We discuss the effects of the tax asymmetry assumption on the value of the tax benefit claim $F(V)$ as follows. Under the hypothesis $\tau_2 \leq \tau_1$, it holds $A_1 > 0$, $A_2 < 0$, $B_2 < 0$. Therefore in both segments $V \geq V_S$ and $V_B \leq V < V_S$, the function $F(V)$ is strictly increasing w.r.t. firm's current assets value V. Further, we note that the tax benefits value in the segment $V \geq V_S$ is a strictly concave function of V, since B_2 is negative. If the tax rate were always τ_2, both above and below V_S, we would have

$$F_L(V, \tau_2) = \frac{\tau_2 C}{r} - \frac{\tau_2 C}{r} {V_B}^{\lambda_2} V^{-\lambda_2},$$

which coincides with the result obtained in [7]. We now compare this value with $F(V)$ in our framework in case $V_B \leq V < V_S$:

$$F(V) = A_0 + A_1 V^{-\lambda_1} + A_2 V^{-\lambda_2},$$

with A_0, A_1, A_2 given by Equations (7)-(9). Notice that now, for all assets values below the switching barrier (but obviously above the failure level V_B) the value of the claim $F(V)$ exhibits three terms instead of two: while the constant term $\frac{\tau_2 C}{r}$ appears in both models, in Leland framework the term depending on $V^{-\lambda_1}$ does not appear. The presence of $A_1 V^{-\lambda_1}$ captures the effect of: i) payouts, since in case $\delta = 0$ we have $\lambda_1 = -1$; ii) most important, it captures the possible switching from τ_2 to a higher level τ_1, thus representing the value of a possible gain in tax savings, through coefficient A_1. This is why it is positive and increasing w.r.t. V. Coefficient A_1 reflects exactly the asymmetry in the corporate tax schedule; it is increasing w.r.t. both $\tau_2 - \tau_1$ and the switching barrier V_S. Coefficient A_2 is instead negative and depends on both the asymmetry of the corporate tax schedule and the default event.

Remark 2.3 *In Appendix A [7] the author proposes a structural model in which the instantaneous tax benefit is zero, if the firm's value V falls under a prescribed level. We observe that in the particular case of* $\delta = 0$ *and* $\tau_2 = 0$*, we recover the same result as in [7].*

We are now ready to complete the description of the corporate capital structure model. Applying contingent claim analysis in a Black-Scholes setting, given the stopping (failure) time T_{V_B}, the expression for *debt value* is given by:

$$D(V, V_B, C) = \mathbb{E}_V \left[\int_0^{T_{V_B}} e^{-rs} C ds + (1-\alpha) e^{-r T_{V_B}} V_B \right]. \tag{14}$$

We stress that the corporate tax schedule does not affect directly debt value. Corporate tax provisions have an influence on capital structure decisions since issuing debt allows to have some tax savings, thus a potential increase in firm's value. But they do not affect directly debt value, which depends only on the coupon level and obviously, on the default event through bankruptcy costs α and the failure level V_B. The effect of asymmetry in the tax scheme will produce an impact on debt value only through the choice of the endogenous failure level and thus on the optimal coupon equity holders will choose.

The *total value* of the firm $v(V)$ consists of three terms: firm's assets value (unlevered), plus the value of the tax benefits claim $F(V)$ given in (6), less the value of the claim on bankruptcy costs:

$$v(V, V_B, C) = V + F(V) - \mathbb{E}_V[e^{-rT_{V_B}} \alpha V_B]. \tag{15}$$

Since the total value of the firm can be expressed as the sum of equity and debt values, finally it is possible to write *equity value* as:

$$E(V, V_B, C) = v(V, V_B, C) - D(V, V_B, C). \tag{16}$$

Equity holders have to define the capital structure of the firm. To this aim, they have to choose both the endogenous failure level and the optimal amount of debt to issue. As stressed in [2] these are interrelated decisions which can hardly be separated. Our approach to the problem is to conduct the analysis in two stages: i) at first we determine the endogenous failure level, ii) then we find the optimal coupon, given the endogenous default boundary.

3 Endogenous Failure Level

The aim of this section is to investigate the effects of a different asymmetric corporate tax schedule on the endogenous failure level chosen by equity holders. We conduct a detailed analytical study considering the influence of the corporate tax function in (3) on the firm's capital structure. Our analysis considers a given and fixed level of coupon payments, namely C.

3.1 Failure level with exogenous switching barrier

Given the value of the tax benefits claim $F(V)$ in (6) we can write debt, equity and total value of the firm. First consider the debt function, which is not directly affected by (3), since for the moment we are considering V_B as a constant level and C is fixed. Debt value can be seen as the sum of a risk-free debt C/r plus a positive term depending on the risk of default, thus (14) becomes:

$$D(V, V_B, C) = \frac{C}{r} + \left((1-\alpha)V_B - \frac{C}{r}\right)\left(\frac{V_B}{V}\right)^{\lambda_2}. \tag{17}$$

Further using Theorem 2.2, the total value of the firm defined in (15) is equal to

$$v(V,V_B,C) = V + \left(\frac{\tau_2 C}{r} + A_1 V^{-\lambda_1} + A_2 V^{-\lambda_2}\right)\mathbf{1}_{(V_B,V_S)}(V) \tag{18}$$

$$+\left(\frac{\tau_1 C}{r} + B_2 V^{-\lambda_2}\right)\mathbf{1}_{(V_S,\infty)}(V) - \alpha V_B \left(\frac{V_B}{V}\right)^{\lambda_2}.$$

Finally from (16) we obtain

$$E(V,V_B,C) = V + \left(\frac{\tau_2 C}{r} + A_1 V^{-\lambda_1} + A_2 V^{-\lambda_2}\right)\mathbf{1}_{(V_B,V_S)}(V) \tag{19}$$

$$+\left(\frac{\tau_1 C}{r} + B_2 V^{-\lambda_2}\right)\mathbf{1}_{(V_S,\infty)}(V) - \frac{C}{r} - (V_B - \frac{C}{r})\left(\frac{V_B}{V}\right)^{\lambda_2}.$$

Equity function must reflect its nature of an option-like contract. For any C, we have $E(V_B, V_B, C) = 0$ meaning that when V falls to V_B there is no equity to cover the firm's debt obligations, thus equity holders will chose to default. We first analyze the equity value for $V_B \leq V < V_S$:

$$E(V,V_B,C) = V - (1-\tau_2)\frac{C}{r} + A_1 V^{-\lambda_1} + \left(-A_1 V_B^{-\lambda_1} + \frac{C}{r}(1-\tau_2) - V_B\right)\left(\frac{V}{V_B}\right)^{-\lambda_2}, \tag{20}$$

where A_1 defined in (8) is positive if $\tau_2 < \tau_1$. The first term $V - (1-\tau_2)\frac{C}{r}$ is the equity value considering a constant tax-sheltering value of interest payments $\tau_2 C$, unless limit of time (when there is no risk of default). Coefficient A_1 represents the opportunity-cost of V being in $[V_B, V_S)$ instead of $[V_S, \infty)$. In fact, considering Leland [7] framework, equity value is increasing with respect to the corporate tax rate τ: our assumption about the tax deductibility scheme modifies the unique constant τ introducing an asymmetry. Suppose V being in $[V_B, V_S]$: this asymmetry becomes an opportunity, since $\tau_2 < \tau_1$. Coefficient A_1 is then positive and increasing w.r.t. $\tau_2 - \tau_1$ and decreasing w.r.t. the switching barrier V_S. As the difference $\tau_2 - \tau_1$ increases, the possible gain in tax benefits in the event of $V = V_S$ is greater. As V_S becomes higher, coeteris paribus, the probability of V reaching V_S before reaching V_B is reduced, thus obtaining a gain in tax benefits becomes less likely and A_1 is lower. The last term is exactly the option to default which is embodied in equity. Observe that in this case, the option to default will compensate equity holders also for the tax deductibility asymmetry, through the term $-A_1 V_B^{-\lambda_1}$. Therefore it must hold

$$-A_1 V_B^{-\lambda_1} + \frac{C}{r}(1-\tau_2) - V_B > 0. \tag{21}$$

Analogously we consider the equity value for $V \geq V_S$:

$$E(V,V_B,C) = V - \frac{(1-\tau_1)C}{r} + A_1 \frac{\lambda_1}{\lambda_2} V_S^{\lambda_2-\lambda_1} V^{-\lambda_2} + \left(-A_1 V_B^{-\lambda_1} + \frac{C}{r}(1-\tau_2) - V_B\right)\left(\frac{V}{V_B}\right)^{-\lambda_2}. \tag{22}$$

The term $V - (1-\tau_1)\frac{C}{r}$ represents equity value when there is no risk of default, with a constant corporate tax rate τ_1. The term depending on $V^{-\lambda_2}$ captures two different effects: one due to the switching in tax benefits, the other related to default, both arising in the event of V falling to V_S. Observe that $A_1 \frac{\lambda_1}{\lambda_2} V_S^{\lambda_2-\lambda_1}$ is negative, reflecting the possible partial loss of tax benefits below V_S. Its negative effect on equity increases with an increase in the switching barrier V_S and also as a consequence of a higher difference $\tau_2 - \tau_1$. What remains is the option to default, which is activated only if V reaches V_S. As previously, the option to default must have positive value, meaning constraint (21) being satisfied.

In the following theorem we analyze the verification of constraint (21) to ensure convexity of equity function w.r.t. V.

Theorem 3.1 *Suppose that $\tau_2 < \tau_1$, then condition (21) is satisfied for $V_B < \overline{V}_B$ such that*

$$\frac{1}{1+A_1}\frac{C(1-\tau_2)}{r} < \overline{V}_B < \frac{C(1-\tau_2)}{r}.$$

Remark 3.2 *If $\delta = 0$, then condition (21) becomes $-A_1 V_B + \frac{C}{r}(1-\tau_2) - V_B > 0$, which is satisfied for*

$$V_B < \frac{1}{1+A_1}\frac{C(1-\tau_2)}{r}. \tag{23}$$

The convexity condition in [7] is $V_B < \frac{C(1-\tau_2)}{r}$; note that $\frac{1}{1+A_1}\frac{C(1-\tau_2)}{r} < \frac{C(1-\tau_2)}{r}$ as $A_1 > 0$. On the other side, under the hypothesis $\delta > 0$ and a unique constant corporate tax rate $\tau_1 = \tau_2$, then $A_1 = 0$ and the condition (21) becomes $V_B < \frac{C(1-\tau_2)}{r}$, as found in [2]. This emphasizes the fact that the difference between these two convexity constraints is due to asymmetry in tax savings: introducing asymmetry in tax benefits makes the convexity constraint on V_B more tight if compared to the case of a unique constant tax-sheltering value $\tau_2 C$. This result does not depend on the payout level.

Theorem 3.1 gives an upper bound for V_B; nevertheless, due to limited liability of equity, the failure level V_B cannot be chosen arbitrarily small, but $E(V, V_B, C)$ must be non negative for all values $V \geq V_B$. To this end we write equity function (20) as:

$$E(V, V_B, C) = f(V, C) + g(V, V_B, C),$$

where

$$f(V, C) := V - (1-\tau_2)\frac{C}{r} + A_1 V^{-\lambda_1}, \tag{24}$$

and

$$g(V, V_B, C) := \left(-A_1 V_B^{-\lambda_1} + \frac{C}{r}(1-\tau_2) - V_B\right)\left(\frac{V}{V_B}\right)^{-\lambda_2}. \tag{25}$$

Constraint (21) is needed in order to make the option embodied in equity having positive value and its value $g(V, V_B, C)$ in (25) being a convex and decreasing function of V. The value $g(V, V_B, C)$ increases as V approaches V_B, thus compensating equity holders for the reduction in equity due to a lower V. The function $f(V, C)$ in (24) represents equity value with no risk of default unless limit of time, when asymmetry in tax benefits is assumed. Under constraint (21) f is increasing and convex when $V \in [V_B, V_S)$. At point $V = V_B$, $g(V, V_B, C)$ and $f(V, C)$ have the same absolute value but opposite sign. Thus the following theorem provides a condition such that an increment in V must produce an impact on equity value without default risk $f(V, C)$ higher than its effect on the option to default $g(V, V_B, C)$. This allows to keep $E(V, V_B, C) \geq 0$ when the option to default approaches its exercise instant, i.e. $V \to V_B$.

Theorem 3.3 *The function $V \mapsto E(V, V_B, C)$ is increasing and strictly convex in $V_B \leq V < V_S$ if V_B satisfies constraints (21) and*

$$V_B(1 + \lambda_2) + A_1 V_B{}^{-\lambda_1}(\lambda_2 - \lambda_1) \geq \frac{C(1 - \tau_2)}{r}\lambda_2, \tag{26}$$

Moreover $E(V, V_B, C) \geq 0$ for $V \geq V_B$.

Remark 3.4 *If $\delta = 0$, constraint (26) becomes*

$$V_B \geq \frac{2(1 - \tau_2)V_S C}{V_S(\sigma^2 + 2r) + 2C(\tau_1 - \tau_2)}. \tag{27}$$

Recalling (23), in order to have equity increasing and convex w.r.t. V, the endogenous failure level has to satisfy:

$$\frac{2(1 - \tau_2)V_S C}{V_S(\sigma^2 + 2r) + 2C(\tau_1 - \tau_2)} \leq V_B \leq \left(2 + \frac{\sigma^2}{r}\right) \frac{(1 - \tau_2)V_S C}{V_S(\sigma^2 + 2r) + 2C(\tau_1 - \tau_2)}.$$

We consider the coupon rate C being fixed and maximize equity value in order to find the endogenous failure level. To this end we impose the smooth-pasting condition (see [7] footnote 20 and [10] footnote 60):

$$\frac{\partial E}{\partial V}|_{V=V_B} = 0. \tag{28}$$

Theorem 3.5 *Suppose constraint (21) holds, then the endogenous failure level $V_B(C; \tau_1, \tau_2; \delta)$ which satisfies (28) exists and is unique, under the condition*

$$V_S \geq \frac{(1 - \tau_1)C}{r} \frac{\lambda_2}{1 + \lambda_2}. \tag{29}$$

We note that condition (28) is equivalent to

$$\frac{\partial f}{\partial V}|_{V=V_B} = -\frac{\partial g}{\partial V}|_{V=V_B},$$

where f and g are defined in (24) and (25). Thus a solution of (28) is an implicit solution of [2]

$$(1+\lambda_2) = \frac{\lambda_2 C}{r} V_B^{-1} \left\{ \left(\frac{V_S}{V_B}\right)^{\lambda_1} (\tau_2 - \tau_1) + (1-\tau_2) \right\}. \tag{31}$$

Remark 3.6 *The choice of bankruptcy level by (31) also optimizes* $V_B \mapsto E(V, V_B, C)$*. Under constraints (21) and (29), equity is increasing (and convex) w.r.t. assets value* V *if* $V_B \geq V_B(C; \tau_1, \tau_2; \delta)$*, where* $V_B(C; \tau_1, \tau_2; \delta)$ *satisfies the smooth pasting condition, being the minimum failure level that equity holders can choose due to limited liability of equity. Consider the function* g *in (25) which is the option to default embodied in equity. Differentiating equity value w.r.t.* V_B*, we have* $\partial_{V_B} E = \partial_{V_B} g$ *which has the same sign of the decreasing function*

$$V_B \mapsto A_1 V_B^{-\lambda_1}(\lambda_1 - \lambda_2) + \lambda_2 \frac{C}{r}(1-\tau_2) - V_B(1+\lambda_2).$$

This function is positive then negative and vanishes at point V_B *exactly solution to Equation (31). It gives the optimal exercise time of the option to default embodied in equity.*

If $\delta = 0$, explicit solution can be obtained by solving equation (31) with respect to V_B

$$V_B(C; \tau_1, \tau_2; 0) = \frac{2CV_S(1-\tau_2)}{V_S(\sigma^2 + 2r) + 2C(\tau_1 - \tau_2)}, \tag{32}$$

thus extending [7] Appendix A (case $\tau_2 = 0$). Finally in case $\tau_2 = \tau_1 =: \tau$ we obtain:

$$V_B(C; \tau, \tau; \delta) = \frac{\lambda_2 C(1-\tau)}{r(1+\lambda_2)}, \tag{33}$$

which is the endogenous failure level in the case of a unique constant tax-sheltering value of interest payment τC with a payout rate δ. This result extends Equation (14) in [7] to the case of a non-zero payout rate for firm's assets value. See [2] for a detailed analysis of this aspect.

[2] In particular if we assume a switch to zero tax level, i.e. $\tau_2 = 0$, we obtain the optimal failure $V_B(C; \tau_1, 0; \delta)$ as implicit solution of

$$(1+\lambda_2) = \frac{\lambda_2 C}{r} V_B^{-1} \left(1 - \tau_1 \left(\frac{V_S}{V_B}\right)^{\lambda_1}\right). \tag{30}$$

This result completes the analysis in [9] Appendix B, in the case where we let the maturity $T \to \infty$ since the authors analyze the switch to zero tax level only in the no-dividend case.

Remark 3.7 *Observe that under constraint (29), the endogenous failure level* $V_B(C;\tau_1,\tau_2;0)$ *is lower than the switching barrier* V_S*. The following inequality*

$$\frac{2(1-\tau_2)V_S C}{V_S(\sigma^2+2r)+2C(\tau_1-\tau_2)} \leq V_S \tag{34}$$

holds under

$$V_S \geq \frac{2C(1-\tau_1)}{(\sigma^2+2r)} \tag{35}$$

which is exactly constraint (29) in the case $\delta = 0$.

3.1.1 Effect of corporate tax rate asymmetry on the endogenous failure level

In this paragraph we analyze the impact of the corporate tax function $\tau(\cdot)$ defined in (3) on the endogenous failure level. We fix τ_1 and V_S and then study the influence of τ_2 on the endogenous failure level, given the coupon C. Introducing asymmetry makes debt more or less attractive hence it should increase or decrease the optimal leverage ratio. Asymmetry also makes more or less attractive to keep a loss-making firm alive, hence it should raise or decrease the endogenous failure level and bring default closer or farer.

In order to work with explicit formulas we consider the no-payout case $\delta = 0$: therefore $\lambda_1 = -1$ and $\lambda_2 = \frac{2r}{\sigma^2}$. In the general case we will resort to numerical comparisons.

Let us consider: the endogenous failure level obtained with the constant instantaneous tax benefits in [7]

$$V_{B_L}(C;\tau_1;0) = \frac{2C(1-\tau_1)}{\sigma^2+2r}, \tag{36}$$

the level (32) obtained in the case of switching between two tax levels and, as a particular case of (32) with $\tau_2 = 0$, the switch to zero tax benefits (as in [7] Appendix A)

$$V_B(C;\tau_1,0;0) = \frac{2CV_S}{V_S(\sigma^2+2r)+2\tau_1 C}. \tag{37}$$

Theorem 3.8 *The function*

$$\tau_2 \mapsto V_B(C;\tau_1,\tau_2;0)$$

defined by (32) is decreasing. In particular $V_B(C;\tau_1,0;0)$ *in (37) is greater than* $V_B(C;\tau_1,\tau_2;0)$ *in (32). Further for any* $\tau_1 > \tau_2$

$$V_B(C;\tau_1,\tau_2;0) > V_{B_L}(C;\tau_1;0), \qquad V_{B_L}(C;\tau_2;0) > V_B(C;\tau_1,\tau_2;0).$$

We can observe that under our asymmetric corporate tax schedule, a higher τ_2 increases equity value (for each coupon level C), and reduces the endogenous failure level $V_B(C;\tau_1,\tau_2;0)$, thus, increasing tax deductions could be a way to support firms. Finally we conclude that in the model where $\delta = 0$ a higher asymmetry in the tax deductibility increases the failure level endogenously chosen.

We can introduce $\theta := \frac{\tau_2}{\tau_1}$ as a measure of the degree of asymmetry of the corporate tax schedule: $\theta = 1$ represents Leland framework (no asymmetry), $\theta = 0$ is the maximum asymmetry case, meaning full loss of tax shelter below V_S. Any other case $0 \leq \theta \leq 1$ represents nothing but an intermediate asymmetric scenario. As asymmetry increases, meaning θ closer to 0, the endogenous failure level $V_B(C; \tau_1, \tau_2; 0)$ increases for any value of the exogenous switching barrier. In such a case, in fact, below V_S the firm will have less tax benefits, due to the lower τ_2, bringing the endogenous failure level higher.

Remark 3.9 *Suppose for a moment τ_2 being fixed. Note that the application $\tau_1 \mapsto V_B(C; \tau_1, \tau_2; 0)$ is decreasing. This is in line with a reduction in the degree of asymmetry of the corporate tax schedule (i.e. θ closer to 1). A higher τ_1 allows the firm to have greater tax savings above V_S, bringing equity value higher both above and below (coefficient A_1 will be higher) the switching barrier V_S, thus bringing down the endogenous failure level.*

The impact of the deductibility asymmetry affects the endogenous failure level also through the exogenous switching barrier V_S. In the no-payout case it is easily seen the following.

Corollary 3.10 *The following result holds: the failure level $V_B(C; \tau_1, \tau_2; 0)$ in (32) is increasing (resp. decreasing) with respect to the exogenous barrier V_S if $\tau_2 < \tau_1$ (resp. $\tau_2 > \tau_1$).*

Assume that $\tau_2 < \tau_1$. Given a certain degree of asymmetry, i.e. θ being fixed, an higher V_S will increase the endogenous default boundary. Starting from $V \geq V_S$, the switching from τ_1 to τ_2 will be more likely, thus it will be more likely losing some tax benefits. As V_S increases indefinitely, the model approaches Leland framework with a unique constant tax-sheltering value of interest payments $\tau_2 C$. Equity holders will lose the opportunity to switch from τ_2 to a higher tax savings region, thus equity will be lower and the endogenous failure level higher, as shown in Theorem 3.8. This result depends on the hypothesis about the coupon C being fixed. Our findings are in line with those obtained in a framework with a flat corporate tax schedule τ_2: when the coupon is fixed, the endogenous failure level is decreasing w.r.t. the corporate tax rate (see [7]).

Observe that a change in θ or a change in V_S produce a different effect on the asymmetry of the tax schedule: we propose to interpret θ as a *vertical* measure of asymmetry, V_S as *horizontal* measure. What we mean is that θ modifies the degree of asymmetry, by acting on the distance between the two corporate tax rates, thus measuring the potential instantaneous loss of tax benefits at point $V = V_S$. A change in V_S represents an *horizontal* measure of asymmetry since it modifies the range of firm's values for which the firm faces a higher (lower) deductibility. When $\theta \to 1$, or $V_S \to V_B$, the limit-model is a framework with a flat corporate tax schedule with a constant corporate tax rate τ_1, but the economic intuition behind is completely different. Consider a coeteris paribus analysis in which all variables except θ are constant: as θ moves, what is changing is only the measure of the potential loss in tax benefits, meaning the distance between the two levels. As opposite case, when only V_S varies, the potential loss in tax benefits is still the same, what changes is the probability of reaching the barrier, thus the likelihood of the potential loss.

3.2 Failure level with debt dependent switching barrier

In subsection 3.1 the switching barrier V_S was exogenously given. Nevertheless, this hypothesis is not completely realistic, and we expect that V_S will depend on the amount of debt issued by the firm (see [7] section VI.A). If assets value falls, it is more likely that profits will be lower than coupon payments, thus the firm will not fully benefit tax savings. Under U.S. tax codes, a necessary condition required to fully benefit tax savings, is that the firm's EBIT (earnings before interest and taxes) must cover payments required for coupons (see [7]). We now introduce the rate of EBIT and suppose it is related[3] to assets value in the following way:

$$EBIT := aV - k, \tag{38}$$

with $0 < a < 1, k > 0$, where k represents costs and a is a fraction of firm's current assets value. In this case the gross profit falls to 0 when V equals $\frac{k}{a}$. We assume that the corporate tax rate is τ_1 in case $EBIT - C \geq 0$, and τ_2 otherwise, with $\tau_2 \leq \tau_1$. Under this specification the switching barrier V_S depends upon the amount of debt issued by the firm:

$$V_S = k + \frac{1}{a}C. \tag{39}$$

In this scenario the switching barrier V_S increases with both k, C: a higher profit is required to cover higher costs k and/or higher interest payments, in order to benefit tax savings from issuing debt.

We now analyze how this different choice of the switching barrier V_S affects the endogenous failure level. The endogenous failure level is optimally chosen by equity holders by applying the smooth pasting condition; when applying the smooth pasting condition, we differentiate equity w.r.t. V and then evaluate this derivative at point V_B. We stress that definition (39) makes the switching barrier dependent and linear on C, but V_S does not depend on firm's current assets value V. Thus we can use results from subsection 3.1 about equity value in order to find the default boundary in this case. For $\delta = 0$, the endogenous failure level becomes:

$$V_B{}^c(C; \tau_1, \tau_2; 0; k, a) = \frac{2C(ak + C)(1 - \tau_2)}{(ak + C)(\sigma^2 + 2r) + 2aC(\tau_1 - \tau_2)}. \tag{40}$$

[3] EBIT is modeled in [7] as a linear function of V, and in [8] as a constant fraction of firm's assets value.

Look at increasing costs k or reducing a, the fraction of firm's value necessary to determine the rate of EBIT: this will bring default closer, rising the endogenous failure level in (40). A direct computation shows the following.

Proposition 3.11 *The endogenous failure level* ${V_B}^c(C; \tau_1, \tau_2; 0; k, a)$ *defined in (40) is:*
i) increasing and concave w.r.t. k;
ii) decreasing and convex w.r.t. a;
iii) increasing and concave w.r.t. C.

Consider a comparative static analysis: if k increases and/or a reduces, EBIT is lower for each firm's assets value V., thus the default boundary is higher since debt has a greater likelihood of losing its tax benefits, meaning for the firm is more likely to loose potential value.

We analyze the relation between total coupon payments supported by the firm and the endogenous failure level chosen by equity holders under this debt dependent asymmetry framework. A comparison between (32), (36) and (40) shows that under the assumption of tax benefits asymmetry, the endogenous failure level is an increasing and concave function of the coupon level, instead of being a linear increasing function of C, as it is in the case of a unique corporate tax rate. It is still true that the endogenous failure level is independent of firm's current assets value V and the fraction (of firm's value) α which is lost because of bankruptcy procedures [4]. When the corporate tax rate is unique, a change in the coupon level affects the optimal equity holders' choice in the same way for all coupon levels. A debt-dependent asymmetry introduces a different effect through modifying the shape of the endogenous failure level as function of C. As a consequence, a change in C modifies the endogenous failure level with different magnitudes, depending on the value of outstanding debt. If the firm is supporting very high interest payments, a reduction (increase) in the coupon level will produce a small effect on the failure level, while in case of low coupon payments, a variation in C will strongly affect the endogenous default boundary, producing a bigger impact on it.

Figure 1 shows the behavior of the endogenous failure level when different frameworks are considered: two constant tax benefits cases (alternatively a unique constant τ_1 or τ_2), two switching scenarios, one with V_S exogenous ($V_S = 90$), the other with the switching barrier debt dependent (${V_S}^c = 60 + 6C$). Leland's frameworks with constant τ_1, τ_2 represent two extreme boundaries between which both the endogenous failure levels obtained under asymmetric tax benefits lie.

We now compare (32) and (40): they are both increasing and concave w.r.t. coupon level C. When coupon payments are low, (32) is greater than (40), though their difference

[4]The independence w.r.t. α means that bankruptcy costs does not *directly* affect the endogenous failure level, since the strict priority rule holds. Bankruptcy costs will instead affect the optimal failure level through the choice of the optimal coupon C^* which maximizes total firm value.

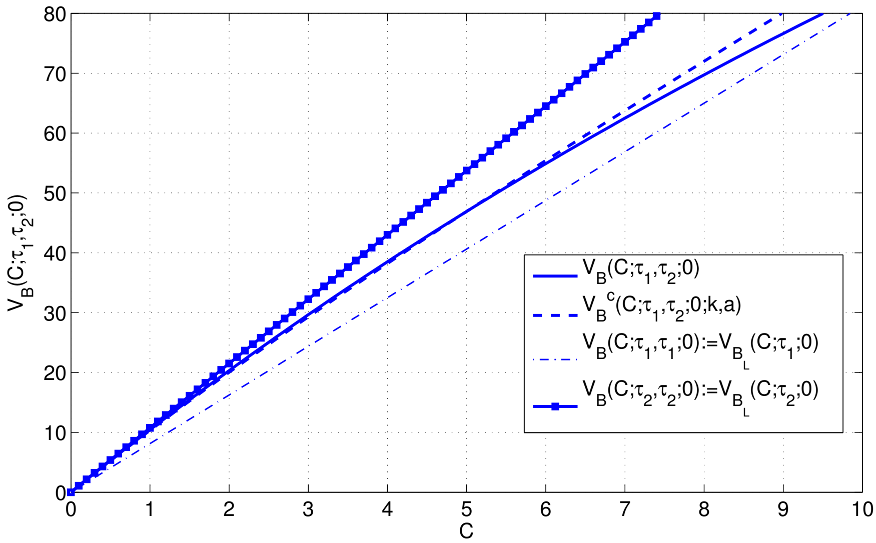

Figure 1: Endogenous failure level. This plot shows the behavior of the endogenous failure level w.r.t. coupon level C. Parameters values are: $\sigma = 0.2, r = 0.05, \delta = 0, \tau_1 = 0.35, \theta = 0.4, V_S = 90$. We then consider $k = 60, a = \frac{1}{6}$, giving $V_S{}^c = 60 + 6C$.

is very small. As coupon increases, the behavior completely changes: the debt dependent switching barrier causes the failure level to be higher than in case V_S constant, and the difference between the two levels increases too. The reason is that the debt dependent switching barrier (39) increases with coupon level, so a firm paying a high coupon C is facing a higher switching barrier, thus a greater probability of losing tax benefits, since now EBIT must cover a greater value of interest payments. In summary, let $V_B(C; \tau_1, \tau_2; 0)$ in (32) and $V_B{}^c(C; \tau_1, \tau_2; 0; k, a)$ in (40), then it holds

$$V_B(C; \tau_1, \tau_2; 0) \gtreqless V_B{}^c(C; \tau_1, \tau_2; 0; k, a), \quad \text{if} \qquad V_S \gtreqless \frac{C}{a} + k.$$

4 Optimal Capital Structure

In this section we determine the optimal capital structure within the model assuming the corporate tax function (3) in both cases of exogenous and debt dependent switching barrier, in the general framework with joint asymmetry and payouts. In particular, the case $\delta = 0$ is analyzed in order to isolate the asymmetry effect on corporate financing decisions.

We turn to the optimization of the total value of the firm depending on the endogenous failure level solution of the optimal stopping problem faced by equity holders. Once determined the endogenous default boundary, equity holders incorporate this decision into the total value of the firm. Then maximize it w.r.t. C in order to find the optimal amount of debt which guarantees the maximum total value of the firm due to the limited liability constraint. Thus the optimal coupon, namely C^*, maximizes total firm value. Once found, we replace C^* in all expressions of previous subsections in order to fully describe the optimal capital structure.

Finally we analyze each financial variable at its optimal level and study effects of both corporate tax asymmetry and payout rate on optimal coupon C^*, optimal debt value D^*, optimal equity value E^*, optimal default boundary ${V_B}^*$ and optimal total value of the firm v^*. We also analyze the optimal yield spread $R^* - r$ where $R^* := \frac{C^*}{D^*}$, and the optimal leverage ratio, defined as the ratio between optimal debt and optimal total value $L^* := \frac{D^*}{v^*}$ (when coupon is at its optimal level C^*).

4.1 Optimal capital structure with exogenous switching barrier

The optimal coupon C^* must be chosen in order to maximize the total value function

$$C \mapsto v(V, V_B(C; \tau_1, \tau_2; \delta), C),$$

where $v(V, V_B(C; \tau_1, \tau_2; \delta), C)$ is defined in (18). The optimal failure level is not given in closed form, nevertheless the following result allows us to study the optimal capital structure.

Proposition 4.1 *The function $V_B \mapsto C(V_B; \tau_1, \tau_2; \delta)$ is increasing, where V_B is implicitly given by equation (31).*

Remark 4.2 *Even if the analytical expression of the optimal coupon C^* is not given, it has to satisfy constraints (21) and (26). We numerically determine the optimal coupon and verify that these constraints are satisfied for our case studies.*

When $\delta = 0$ the endogenous failure level is given in closed form by equation (32). Then we study the optimal capital structure by maximizing the application

$$C \mapsto v(V, V_B(C; \tau_1, \tau_2; 0), C).$$

Theorem 4.3 *The function $C \mapsto v(V, V_B(C; \tau_1, \tau_2; 0), C)$ is a concave function achieving a maximum at point C^* solution of $\frac{\partial v(V, V_B(C; \tau_1, \tau_2; 0), C)}{\partial C} = 0$, under the condition*[5] *$\tau_1 < \frac{2}{3} + \frac{\tau_2}{3}$. Thus an optimal capital structure exists and is unique.*

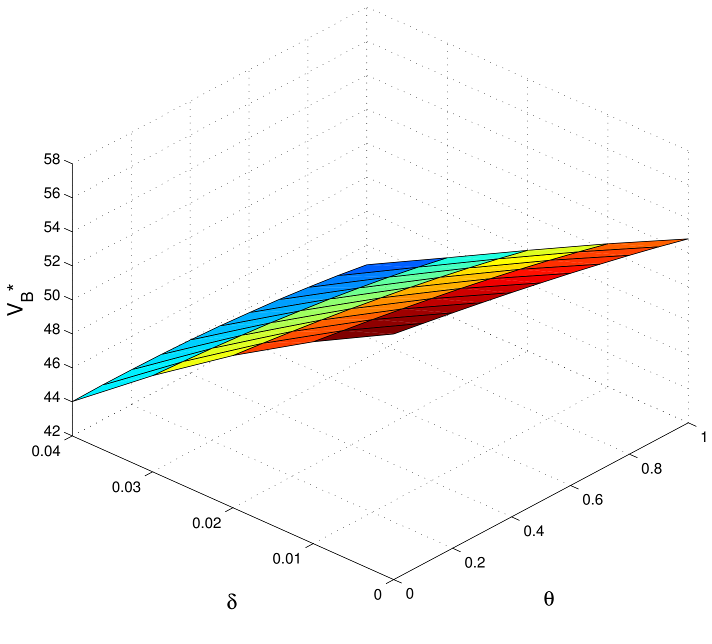

Figure 2: Optimal failure level V_B^* as function of δ, θ. The switching barrier V_S is exogenous and parameters values are: $\sigma = 0.2, r = 0.06, \tau_1 = 0.35, \alpha = 0.5, V_S = 90, V = 100$.

In order to study the asymmetry effect on these variables we analyze $\theta \geq 0$, measuring the *vertical degree of asymmetry* of the corporate tax schedule. Notice that $\theta = 1$ (symmetric case) and $\delta = 0$ lead exactly to Leland's results [7], while $\theta = 1$ and $\delta \neq 0$ are comparable with our analysis in [2].

Asymmetry increases as θ goes to 0, achieving its maximum for $\theta = 0$, representing a switching from a tax level τ_1 to zero-tax benefits for $V < V_S$.

First notice that tax asymmetry raises the optimal failure level $V_B^*(V;\tau_1,\tau_2;\delta)$: for any value of δ (and for values of $V_S < V$), as the tax asymmetry increases then the optimal failure level $V_B^*(V;\tau_1,\tau_2;\delta)$ increases. The opposite happens when considering the payout influence, given a degree of asymmetry θ. For any fixed value of θ, the optimal failure level decreases as δ increases from 0 to 0.04. Results are in Table 1, while Figure 2 shows the behavior of the optimal failure level as function of both δ, θ. From [2] we know that introducing payouts brings to a lower optimal failure level when the corporate tax rate is unique and constant through time. When both payouts and asymmetry in tax benefits are in force, our numerical analysis shows that the final joint effect can be quantitatively significant. Consider as extreme cases $\theta = 1, \delta = 0$ and $\theta = 0, \delta = 0.04$: passing from no asymmetry and no payouts, to a payout rate equal to a 4% of current assets value, brings to a reduction in optimal failure level of around 8.8%, from 52.82 to 44.02.

Further optimal leverage ratio is strongly affected by asymmetry in the corporate tax schedule as shown in Figure 3. In order to isolate the asymmetry effect, consider Table 1 in case $\delta = 0$: results are due only to the switching in tax benefits and bring to a reduction in optimal debt, optimal total value of the firm and also optimal leverage ratios.

Extending the analysis by considering the general case in which both $\delta > 0, 0 \leq \theta < 1$ shows that for each level of payouts, increasing the degree of asymmetry reduces optimal leverage and this effect is stronger when the the payout rate is higher. Consider the last column of Table 1: comparing the two extreme cases $\theta = 1$ and $\theta = 0$, the difference in optimal leverage ratio is 4.5% when $\delta = 0$, 6% when $\delta = 0.01$ and 12% when $\delta = 0.04$. Tax asymmetry has a negative effect on optimal leverage ratios L^*: for any value of δ considered, L^* decrease as the degree of asymmetry increases, that is as $\theta \to 0$. The decrease of the optimal leverage is quantitatively more significant as the payout rate rises. Analogously, as observed in [2] the capital structure of a firm is strongly affected by payouts. From [2] we know that introducing payouts in a structural model with a unique corporate tax rate τ has the effect of reducing optimal leverage ratios. Table 1 allows us to confirm this result also when the tax schedule is asymmetric. We consider as extreme cases to compare $\delta = 0$ and $\delta = 0.04$. Considering a unique corporate tax rate means $\theta = 1$: in such a case we know from [2] that the difference in optimal leverage ratio is quite 6%. Introducing convexity at $\theta = 0.8$, $\theta = 0.4$, $\theta = 0$: the difference in optimal leverage becomes respectively 7%, 11% and 13%. A higher payout lowers the optimal total value of the firm v^*, since a lower debt issuance can be supported because less assets

[5] This condition is always satisfied with our parameter values, since we always consider $\tau_1 < \frac{2}{3}$.

remain in the firm. As a consequence, this will bring down leverage ratios. But also the asymmetry effect is to reduce leverage ratios, since the potential loss in tax benefits due to the existence of the switching barrier makes debt less attractive: considering a scenario where both effects exist, will bring to a strong reduction in predicted optimal total value of the firm, optimal debt and optimal leverage ratios. We now consider Leland [7] case, i.e. $\theta = 1, \delta = 0$, and compare it with a scenario in which both payouts and asymmetry exists, meaning $\theta = 0, \delta = 0.04$ in order to capture the joint effect of these two realistic generalizations. Observe that moving from no asymmetry and no payouts, to a payout rate equal to a 4% of current assets value V, leads to a dramatic reduction in optimal leverage: in such a case, this joint influence of δ, θ brings to an optimal leverage ratio of 57.36%, with a significant reduction of 17% from Leland result of a 75%-leveraged firm, leading to a value which is more in line with historical norms[6]. This strong impact on optimal leverage ratios suggests that asymmetry and payouts seem to be important factors involved in the determinacy of corporate capital structure decisions.

Figure 4 shows the behavior of optimal coupon C^* as function of δ and θ. Observe that for each degree of convexity $0 \leq \theta < 1$ the optimal coupon is decreasing w.r.t. δ, extending results in [2] to the case of asymmetric corporate tax schedule. We stress that in this general framework where both payouts and an asymmetric tax scheme interact, the negative effect of payouts on C^* is greater as the asymmetry in tax benefits increases, i.e. as $\theta \to 0$. From Figure 4 we can also observe that the optimal coupon C^* decreases as $\theta \to 0$ for each level of the payout rate δ. In fact introducing asymmetry in tax benefits makes debt less attractive for the firm, thus leading to a not negligible reduction in the optimal coupon level choice. The decrease in C^* due to the asymmetric tax benefits scheme is higher as payouts increase, as we note considering the slope in Figure 4 w.r.t. θ for each level of δ. Payouts and asymmetry in tax benefits influence each other by increasing the magnitude of their own effects on the optimal coupon, bringing to a joint influence on optimal coupon which is quantitative significant. To analyze the interaction between δ and θ on C^* consider for example three alternative scenarios $\theta = 1, 0.4, 0$: when δ goes from 0 to 0.04, the optimal coupon reduces from 6.5% to 6.23% in case $\theta = 1$, from 6.03% to 5.2% in case $\theta = 0.4$, from 5.78% to 4.32% in case $\theta = 0$. The reduction in C^* due to an increased payout is considerably higher as asymmetry in tax benefits increases: when tax benefits are completely lost under V_S the reduction of optimal coupon is more than 5 times the reduction in case of a constant tax schedule. Our analysis in this paper confirms our results in [2] and moreover extend their validity under asymmetry in tax benefits. Adding to this, the contribution of the present work is also to show how optimal capital structure is much more affected by payouts when considering a more realistic framework allowing also for asymmetry in the corporate tax schedule.

As it concerns optimal equity value and optimal spreads we note that the joint effect of asymmetry and payouts raises both optimal equity and optimal spreads. We can explain this as a consequence of two main insights arising from the model.

[6] Leland [7] in his Section D observes that a leverage of 52% is quite in line with historical norms.

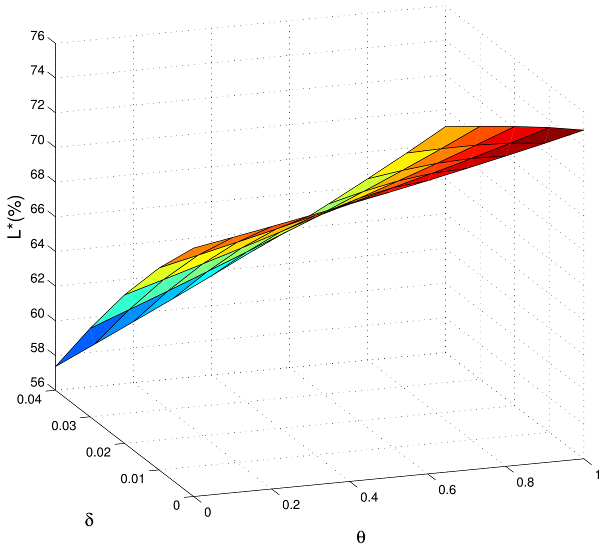

Figure 3: Optimal Leverage L^* as function of δ, θ. The switching barrier V_S is exogenous and parameters values are: $\sigma = 0.2, r = 0.06, \tau_1 = 0.35, \alpha = 0.5, V_S = 90, V = 100$.

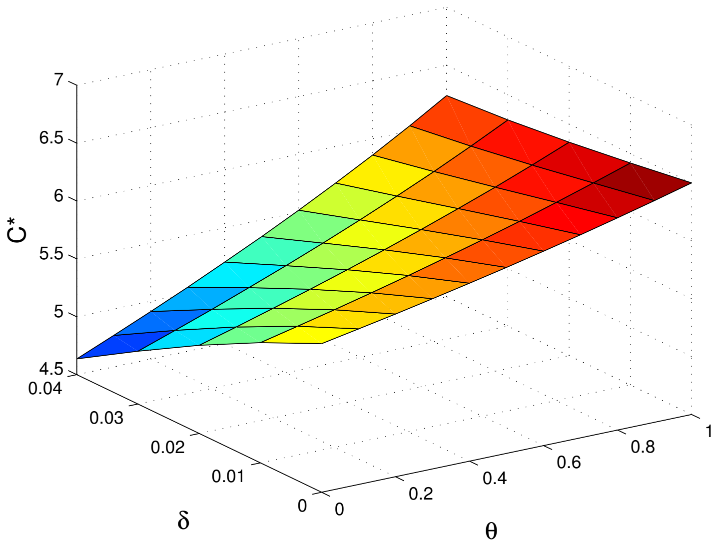

Figure 4: Optimal Coupon C^* as function of δ, θ. The switching barrier V_S is exogenous and parameters values are: $\sigma = 0.2, r = 0.05, \tau_1 = 0.35, \alpha = 0.5, V_S = 90, V = 100$.

Table 1: Effect of payouts and asymmetry in the tax schedule on all financial variables at their optimal level when the switching barrier V_S is exogenous. Base case parameter values: $V_0 = 100$, $\sigma = 0.2$, $\tau_1 = 0.35$, $r = 6\%$, $\alpha = 0.5$, $V_S = 90$ and three different cases: $\delta = 0$, $\delta = 0.01$, $\delta = 0.04$. Leverage is in percentage (%), spreads in basis points (bps).

	$\delta = 0$						
θ	C^*	D^*	$R^* - r$	E^*	V_B^*	v^*	L^*
0	5.784	84.149	87.389	35.519	56.435	119.668	70.318 %
0.1	5.844	85.136	86.467	35.253	56.142	120.389	70.717 %
0.2	5.906	86.163	85.489	34.975	55.837	121.138	71.128 %
0.3	5.971	87.234	84.454	34.683	55.518	121.918	71.552 %
0.4	6.038	88.352	83.357	34.378	55.186	122.730	71.989 %
0.5	6.107	89.519	82.195	34.057	54.838	123.576	72.441 %
0.6	6.179	90.741	80.965	33.719	54.474	124.461	72.908 %
0.7	6.254	92.022	79.660	33.363	54.092	125.386	73.391 %
0.8	6.333	93.367	78.278	32.987	53.691	126.355	73.893 %
0.9	6.415	94.783	76.812	32.590	53.267	127.372	74.414 %
1	6.501	96.274	75.256	32.167	52.820	128.442	74.956 %
	$\delta = 0.01$						
θ	C^*	D^*	$R^* - r$	E^*	V_B^*	v^*	L^*
0	5.539	79.524	96.489	38.047	53.868	117.571	67.639 %
0.1	5.613	80.658	95.868	37.683	53.604	118.341	68.157 %
0.2	5.689	81.838	95.167	37.306	53.325	119.144	68.688 %
0.3	5.768	83.069	94.384	36.914	53.030	119.983	69.234 %
0.4	5.850	84.355	93.515	36.506	52.720	120.861	69.795 %
0.5	5.935	85.701	92.554	36.080	52.391	121.781	70.373 %
0.6	6.024	87.112	91.497	35.634	52.043	122.746	70.969 %
0.7	6.116	88.594	90.336	35.166	51.673	123.760	71.585 %
0.8	6.212	90.155	89.067	34.674	51.281	124.829	72.223 %
0.9	6.313	91.802	87.680	34.154	50.865	125.957	72.884 %
1	6.419	93.545	86.169	33.604	50.420	127.149	73.571 %
	$\delta = 0.04$						
θ	C^*	D^*	$R^* - r$	E^*	V_B^*	v^*	L^*
0	4.637	63.933	125.230	47.526	44.023	111.459	57.360 %
0.1	4.772	65.630	127.059	46.690	44.048	112.321	58.431 %
0.2	4.911	67.390	128.708	45.844	44.043	113.234	59.514 %
0.3	5.054	69.220	130.171	44.981	44.007	114.201	60.612 %
0.4	5.203	71.129	131.439	44.100	43.942	115.229	61.729 %
0.5	5.357	73.129	132.503	43.194	43.845	116.323	62.867 %
0.6	5.517	75.230	133.352	42.261	43.716	117.491	64.031 %
0.7	5.684	77.447	133.974	41.292	43.553	118.739	65.224 %
0.8	5.860	79.794	134.354	40.284	43.356	120.078	66.452 %
0.9	6.044	82.291	134.477	39.227	43.121	121.518	67.719 %
1	6.239	84.957	134.326	38.114	42.847	123.072	69.031 %

First, when payouts are introduced, less assets remain in the firm, thus making possible only a lower optimal debt issuance. Adding to this, asymmetry makes debt less attractive,

Table 2: Effect of a change in the *vertical degree of asymmetry* of the corporate tax schedule on financial variables at optimal leverage ratio as $\theta \to 0$ when V_S is exogenous. We report the sign of change in each variable as the degree of asymmetry increases.

Fin. Var.	C^*	D^*	$R^* - r$	E^*	V_B^*	v^*	L^*
$\theta \to 0$	< 0	< 0	> 0	> 0	> 0	< 0	< 0

due to a possible switching to lower tax benefits, thus a potential loss of value has to be taken into account. As a consequence, the joint effect is to reduce both the optimal coupon C^* and the optimal amount[7] of debt D^*.

Equity value increases at its optimal level due to the joint effect of δ, θ on both C^*, V_B^*: recall that the optimal (endogenous) failure level increases as the degree of asymmetry is higher.

Secondly, we can also think about the joint effect of payouts and asymmetry in tax benefits as something which contributes to increase the *average riskiness* of the firm and moreover makes bankruptcy more likely. This is why, despite lower optimal leverage ratios, optimal spreads increase, in line with [7] suggestions. When $\theta \to 0$, the potential loss in tax benefits due to passing from τ_1 to τ_2 increases.

Optimal debt is lower, equity higher but the first effect always dominates the second one, bringing to lower optimal total values of the firm. In fact payouts and corporate tax asymmetry increase the likelihood of default. As debt holders must be compensated, then optimal spreads predicted by this model are considerably higher w.r.t. the case $\theta = 1, \delta = 0$, capturing all these economic insights.

Table 2 reports the behavior of financial variables at their optimal level when the exogenous switching barrier V_S and the payout rate δ are fixed, while the *vertical* degree of asymmetry increases, i.e. $\theta \to 0$, aiming at isolating and capturing only this asymmetry influence on optimal capital structure decisions made by the firm in a comparative static analysis.

Concerning the asymmetry of the corporate tax schedule a similar analysis could be done analyzing how optimal capital structure decisions are affected when the exogenous switching barrier V_S moves, meaning when the *horizontal degree of asymmetry* changes, fixing both θ and the payout rate δ. Numerical results show that different values of the barrier can significantly modify optimal choices, meaning the corporate tax schedule is an important determinant in leverage decisions. Table 3 shows numerical results for this

[7]As noted in [7] for $\delta = 0$, and also supported by results in [2] for $\delta > 0$, the firm will always chose a coupon level which is lower than that one corresponding to the maximum capacity of debt. As a consequence, a lower coupon means a lower debt value. Moreover, as in [7] we are assuming the face value of debt being constant.

case, Table 4 reports only the qualitative behavior of all financial variables at their optimal level as V_S increases. The switching barrier being exogenously given, results show in which direction a higher V_S will move optimal capital structure decisions.

A possible explanation of what we observe in Table 4 could be that, coeteris paribus, as V_S rises (decreases), the *horizontal* degree of asymmetry changes. As extreme case, our framework tends to a limit-model in which the tax sheltering value of interest payments is constant and equal to the lower $\tau_2 C$ (higher $\tau_1 C$). And this represents the limit-model for each degree of asymmetry θ and payout level δ. A reduction (increase) in the corporate tax rate produces exactly the effects shown by our results: each variable at its own optimal level decreases (increases), except equity value which instead rises (reduces). And this result is robust w.r.t. each payout level. The behavior we find in this limit-model is in line with [7] Table II, where $\delta = 0$: all variables except equity are increasing w.r.t. the constant corporate tax rate τ. Moreover, when payouts are introduced in a flat corporate tax schedule model, results are still in line with [2]. This behavior of financial variables holds for each level of payout δ and each *vertical* degree of asymmetry θ: what is different among alternative scenarios is only the magnitude of the effect, obviously depending on the joint influence. The joint influence is higher as payouts increase and $\theta \to 0$. Consider as an example a switching barrier of 95: in case $\delta = 0.04$ and $\theta = 0$ the model predicts a 56% optimal leverage, while in case $\delta = 0$ and $\theta = 0.8$ this optimal ratio is around 73%. This second case is very close to Leland's results [7] with constant τ_2, and the difference in leverage is quite negligible, i.e. 2%, while the first case brings to a huge 19%-reduction in leverage ratios.

4.2 Optimal capital structure with debt dependent switching barrier

In this section we study the optimal capital structure when the switching barrier is increasing with coupon C, being defined as $V_S := k + \frac{1}{a}C$.

If $\delta = 0$, we determine the optimal capital structure by substituting the endogenous failure level ${V_B}^c(C; \tau_1, \tau_2; 0; k, a)$ obtained in (40) into the total value of the firm v and then maximizing it w.r.t. C. This produces the optimal coupon C^*, allowing to analyze optimal leverage and optimal capital structure decisions as reported in Table 5. In the general case $\delta > 0$ we do not have a closed form for the endogenous failure level, and the smooth pasting condition is not linear w.r.t. C, since also the switching barrier depends on coupon payments. Thus we provide a numerical analysis of the optimal capital structure in this subsection.

The peculiarity of this model is that as the optimal coupon decreases for higher *vertical degree of asymmetry* $\theta \to 0$, then also the optimal switching barrier V_S^* decreases, meaning that also the *horizontal degree of asymmetry* is changed. From the opposite point of view, we observe that as $\theta \to 1$ the debt dependent switching barrier approaches current firm's

Table 3: Effect of asymmetry in the tax schedule on all financial variables at optimal level. Base case parameter values: $V_0 = 100$, $\sigma = 0.2$, $\tau_1 = 0.35$, $r = 6\%$, $\alpha = 0.5$, V_S varying from 80 to 95 and different scenarios for payouts and degree of asymmetry θ: $\delta = 0, \theta = 0.8$, $\delta = 0.01, \theta = 0.4$, $\delta = 0.04, \theta = 0$. Leverage is in percentage (%), spreads in basis points (bps).

	$\delta = 0, \theta = 0.8$						
V_S	C^*	D^*	$R^* - r$	E^*	V_B^*	v^*	L^*
85	6.371	93.831	78.983	32.818	53.810	126.649	74.087 %
86	6.363	93.736	78.845	32.855	53.787	126.591	74.046 %
87	6.356	93.642	78.705	32.891	53.763	126.533	74.006 %
88	6.348	93.549	78.564	32.925	53.739	126.474	73.967 %
89	6.340	93.458	78.422	32.957	53.715	126.415	73.930 %
90	6.333	93.367	78.278	32.987	53.691	126.355	73.893 %
91	6.325	93.278	78.132	33.017	53.665	126.295	73.857 %
92	6.318	93.189	77.985	33.044	53.640	126.234	73.823 %
93	6.311	93.102	77.837	33.070	53.614	126.172	73.789 %
94	6.303	93.015	77.686	33.095	53.587	126.110	73.757 %
95	6.296	92.929	77.534	33.119	53.560	126.047	73.725 %
	$\delta = 0.01, \theta = 0.4$						
V_S	C^*	D^*	$R^* - r$	E^*	V_B^*	v^*	L^*
85	5.971	85.731	96.453	35.908	53.168	121.640	70.480 %
86	5.946	85.448	95.868	36.037	53.080	121.485	70.336 %
87	5.922	85.168	95.281	36.161	52.992	121.330	70.196 %
88	5.898	84.894	94.694	36.280	52.902	121.174	70.059 %
89	5.874	84.622	94.106	36.395	52.812	121.018	69.926 %
90	5.850	84.355	93.515	36.506	52.720	120.861	69.795 %
91	5.827	84.091	92.923	36.613	52.626	120.704	69.667 %
92	5.804	83.830	92.328	36.716	52.531	120.546	69.542 %
93	5.781	83.572	91.732	36.816	52.435	120.388	69.419 %
94	5.758	83.316	91.132	36.912	52.338	120.228	69.299 %
95	5.736	83.063	90.530	37.005	52.238	120.068	69.179 %
	$\delta = 0.04, \theta = 0$						
V_S	C^*	D^*	$R^* - r$	E^*	V_B^*	v^*	L^*
85	4.848	66.084	133.629	46.254	45.218	112.339	58.826 %
86	4.804	65.635	131.884	46.525	44.976	112.160	58.519 %
87	4.760	65.196	130.174	46.787	44.737	111.982	58.220 %
88	4.718	64.766	128.497	47.040	44.498	111.806	57.927 %
89	4.677	64.346	126.850	47.286	44.260	111.632	57.641 %
90	4.637	63.933	125.230	47.526	44.023	111.459	57.360 %
91	4.597	63.528	123.636	47.759	43.786	111.287	57.085 %
92	4.558	63.129	122.065	47.987	43.549	111.116	56.814 %
93	4.520	62.736	120.515	48.210	43.312	110.946	56.546 %
94	4.483	62.348	118.985	48.428	43.075	110.776	56.283 %
95	4.446	61.965	117.473	48.643	42.838	110.608	56.022 %

Table 4: Effect of a change in the *horizontal degree of asymmetry* of the corporate tax schedule on financial variables at optimal leverage ratio as V_S increases when θ and δ are fixed. We report the sign of change in each variable as V_S increases.

Fin. Var.	C^*	D^*	$R^* - r$	E^*	V_B^*	v^*	L^*
$V_S \nearrow$	< 0	< 0	< 0	> 0	< 0	< 0	< 0

activities value V: in the limit, for $\theta = 1$, asymmetry disappears, meaning the corporate tax schedule tends to a flat one.

If compared to the case considered in previous subsection, where V_S is exogenously given, this more realistic framework allows to analyze the joint effect of a change in both the *vertical* and *horizontal* degrees of asymmetry of the corporate tax schedule. As θ moves, the optimal coupon changes, and this in turns modifies the optimal switching barrier ${V_S}^*$. A change in the *vertical* degree of asymmetry will affect optimal capital structure decisions, both directly and indirectly, in this last case by changing the range of firm's values for which tax benefits depend alternatively on τ_1, τ_2.

When the switching barrier depends on the amount of debt issued, a higher profit is needed in order to have higher coupon payments fully deductible. Recall that we are assuming EBIT has to cover coupon payments in order to benefit from tax savings. In this framework, greater debt has a greater likelihood of losing its tax benefits, and optimal leverage drops significantly. This reduction is quantitatively higher in comparison to the case V_S being exogenously given, for each level of payout. The decrease in optimal leverage is a 19%-reduction in case $\delta = 0.04$, notably higher than the 12% increase obtained in the case V_S fixed. Leverage can reach a 52% in line with historical norms (see [7]). Table 6 shows that optimal credit spreads decreases in this scenario, reflecting the lesser leverage, in line with suggestions in [7]. In this simplified framework we model EBIT as a linear function of V and this allows to show that operational costs could be another variable to analyze in order to explain observed leverage ratios. As k and/or a rise, this will affect the optimal amount of debt issued, since a higher profit is necessary to fully benefit from coupon deductibility. An increase in k and/or a will drop predicted leverage.

5 Conclusions

We have extended a structural model with endogenous bankruptcy starting from Leland framework [7] in two main directions: introducing a payout rate and an asymmetric corporate tax schedule. Rather than considering a flat tax scheme, i.e. a unique corporate tax rate, an asymmetric tax code provisions allowing for a switching in corporate tax rates is considered. The switching from a corporate tax rate to the other is determined by the firm

Table 5: Effect of payouts and asymmetry in the tax schedule on all financial variables at their optimal level when the switching barrier V_S is debt dependent. Base case parameter values: $V_0 = 100$, $\sigma = 0.2$, $\tau_1 = 0.35$, $r = 6\%$, $\alpha = 0.5$, $k = 60$, $a = 1/6$. The exogenous barrier is set at $V_S = 90$. Leverage is in percentage (%), spreads in basis points (bps).

	$\delta = 0$							
θ	C^*	D^*	$R^* - r$	E^*	V_B^*	v^*	L^*	${V_S}^*$
0	5.079	76.816	61.192	42.294	50.969	119.110	64.491%	90.474
0.1	5.184	78.286	62.185	41.474	51.086	119.760	65.369%	91.104
0.2	5.296	79.847	63.270	40.608	51.221	120.455	66.288%	91.776
0.3	5.414	81.487	64.399	39.712	51.361	121.199	67.234%	92.484
0.4	5.539	83.219	65.592	38.779	51.511	121.998	68.213%	93.234
0.5	5.673	85.065	66.903	37.795	51.683	122.860	69.238%	94.038
0.6	5.815	87.015	68.275	36.776	51.861	123.791	70.292%	94.890
0.7	5.968	89.102	69.795	35.700	52.066	124.802	71.394%	95.808
0.8	6.133	91.336	71.475	34.568	52.298	125.905	72.544%	96.798
0.9	6.310	93.720	73.283	33.392	52.547	127.112	73.730%	97.860
1.0	6.501	96.274	75.256	32.167	52.820	128.442	74.956%	99.006
	$\delta = 0.01$							
θ	C^*	D^*	$R^* - r$	E^*	V_B^*	v^*	L^*	${V_S}^*$
0	4.855	72.545	69.236	44.808	48.414	117.353	61.818%	89.130
0.1	4.968	74.100	70.447	43.922	48.542	118.022	62.785%	89.808
0.2	5.088	75.745	71.729	42.993	48.682	118.738	63.792%	90.528
0.3	5.216	77.492	73.102	42.016	48.835	119.508	64.843%	91.296
0.4	5.352	79.342	74.545	40.995	48.998	120.338	65.933%	92.112
0.5	5.498	81.318	76.108	39.917	49.180	121.236	67.075%	92.988
0.6	5.655	83.431	77.804	38.780	49.383	122.211	68.268%	93.930
0.7	5.823	85.683	79.599	37.592	49.598	123.275	69.506%	94.938
0.8	6.006	88.115	81.611	36.326	49.848	124.441	70.808%	96.036
0.9	6.203	90.720	83.755	35.006	50.114	125.726	72.157%	97.218
1.0	6.419	93.545	86.169	33.604	50.420	127.149	73.571%	98.514
	$\delta = 0.04$							
θ	C^*	D^*	$R^* - r$	E^*	V_B^*	v^*	L^*	${V_S}^*$
0	4.121	58.897	99.700	53.172	39.483	112.068	52.554%	84.726
0.1	4.263	60.709	102.201	52.060	39.714	112.769	53.835%	85.578
0.2	4.416	62.651	104.860	50.877	39.964	113.528	55.185%	86.496
0.3	4.581	64.732	107.682	49.620	40.233	114.352	56.608%	87.486
0.4	4.758	66.955	110.622	48.296	40.511	115.252	58.095%	88.548
0.5	4.951	69.361	113.804	46.877	40.818	116.238	59.672%	89.706
0.6	5.161	71.959	117.210	45.364	41.148	117.323	61.334%	90.966
0.7	5.391	74.782	120.897	43.744	41.510	118.526	63.093%	92.346
0.8	5.644	77.858	124.910	42.008	41.905	119.866	64.954%	93.864
0.9	5.925	81.235	129.362	40.135	42.349	121.370	66.932%	95.550
1.0	6.239	84.957	134.326	38.114	42.847	123.072	69.031%	97.434

value crossing i) an exogenous barrier, ii) a debt dependent switching barrier (allowing to model EBIT). We investigate the joint effects of this corporate tax scheme and pay-

Table 6: Effect of a change in the degree of asymmetry of the corporate tax schedule on financial variables at optimal leverage ratio as $\theta \to 0$ when V_S is debt dependent. Results hold for each level of the payout rate $\delta \geq 0$. We report the sign of change in each variable as $\theta \to 0$.

Fin. Var.	C^*	D^*	$R^* - r$	E^*	V_B^*	v^*	L^*
$\theta \to 0$	< 0	< 0	< 0	> 0	< 0	< 0	< 0

outs on optimal default level and optimal capital structure. Our results support [4], [13] suggestion that tax-code provisions cannot be ignored when studying corporate financing decisions. Our findings are in line with [4]: when the corporate tax rate is higher, observed leverage ratios are higher. Nevertheless our model also analyzes how a change in tax-code provisions can affect a single firm's corporate decisions. The general result is that asymmetry always lowers optimal debt, optimal leverage ratios and the maximum total value of the firm since less tax savings (actual and/or potential) are available, meaning there is always a loss of potential value for the firm[8]. We observe that all financial variables at their optimal level are influenced by this asymmetric tax schedule. Our analysis extends [2] by showing how optimal capital structure is much more affected by the introduction of a payout rate $\delta > 0$ inside a more realistic framework allowing for asymmetry in tax-code provisions. These two factors influence each other with a resulting quantitative huge joint effect on optimal debt and leverage. The degree of *vertical* asymmetry in the corporate tax schedule ($\theta := \frac{\tau_2}{\tau_1}$) is a parameter imposed to the firm by external authorities and can vary i) strongly depending on the sector in which the firm is operating in, ii) in time, for example to encourage investments. Thus, our analysis provides a way to measure the economic influence (from both a qualitative and quantitative point of view) of such an important and external factor on internal and endogenous optimal choices made by the firm. Moreover, we also analyze effects on corporate decisions produced by its joint influence with payout rates, which are supposed to be constant (but we know that they should be even partly modified by the firm).

The economic insight we want to give is that this simple model is flexible to analyze the impact of many factors on optimal capital structure decisions, providing a framework to develop in the direction of a more empirical research, allowing to explain differences in observed leverage among firms facing different tax-code provisions.

[8]As in Leland [7] this model does not consider tax loss carryforwards which could be an interesting point to develop, since they will introduce path dependence, making the model even more realistic.

References

[1] L.H.R. Alvarez. Reward functionals, salvage values, and optimal stopping. *Mathematical Methods of Operations Research*, 54:315–337, 2001.

[2] F. Barsotti, M.E. Mancino, and M. Pontier. Capital structure with firm's net cash payouts. In *IV International MAF Conference - Mathematical and Statistical Methods for Actuarial Sciences and Finance*, Quantitative Finance Series. Springer, 2011.

[3] H.U. Gerber and E.S.W. Shiu. Martingale approach to pricing perpetual american options. *ASTIN Bulletin*, 24:195–220, 1994.

[4] J.R. Graham and C.W. Smith. Tax incentives to hedge. *The Journal of Finance*, 54:2241–2262, 1999.

[5] I. Karatzas and S. Shreve. *Brownian Motion and Stochastic Calculus.* Springer, Berlin, Heidelberg, New York, 1988.

[6] A.E. Kyprianou and M.R. Pistorius. Perpetual options and canadization through fluctuation theory. *The Annals of Applied Probability*, 13(3):1077–1098, 2003.

[7] H.E. Leland. Corporate debt value, bond covenant, and optimal capital structure. *The Journal of Finance*, 49:1213–1252, 1994.

[8] H.E. Leland. Agency costs, risk management, and capital structure. *The Journal of Finance*, 53:1213–1243, 1998.

[9] H.E. Leland and K.B. Toft. Optimal capital structure, endogenous bankruptcy and the term structure of credit spreads. *The Journal of Finance*, 51:987–1019, 1996.

[10] R.C. Merton. A rational theory of option pricing. *Bell Journal of Economics and Management Science*, 4:141–183, 1973.

[11] R.C. Merton. On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance*, 29:449–470, 1974.

[12] F. Modigliani and M. Miller. The cost of capital, corporation finance and the theory of investment. *American Economic Review*, 48:267–297, 1958.

[13] S. Sarkar. Can tax convexity be ignored in corporate financing decisions? *Journal of Banking & Finance*, 32:1310–1321, 2008.

[14] S. Sarkar and L. Goukasian. The effect of tax convexity on corporate investment decisions and tax burdens. *Journal of Public Economic Theory*, 8(2):293–320, 2006.

[15] M. Uhrig-Homburg. Cash-flow shortage as an endogenous bankruptcy reason. *Journal of Banking & Finance*, 29:1509–1534, 2005.