Wage setting and unemployment in a general equilibrium model

Michele Gori
Dipartimento di Matematica per le Decisioni
Università degli Studi di Firenze
via delle Pandette 9, 50127, Firenze
e-mail: michele.gori@unifi.it

Antonio Villanacci
Dipartimento di Matematica per le Decisioni
Università degli Studi di Firenze
via delle Pandette 9, 50127, Firenze
e-mail: antonio.villanacci@unifi.it

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Abstract

The purpose of the paper is to argue that exogenous changes lowering wages may imply an increase of unemployment. To support that viewpoint, we use a general equilibrium approach. In that framework, we substitute the labour market clearing equation, which by very definition insures full employment, with the assumption that wages depend on employment and a variable representing all the other exogenous factors influencing the outcome of wage setting. We assume that an increase of that variable determines a decrease of wage for any given level of employment. We further assume that firms compete strategically in the labour market. After having proved there is a unique equilibrium for each economy, we show that an increase of the above mentioned residual variable may cause in equilibrium a decrease in real wages and wage bill along with an increase in unemployment rates and profits. That result suggests what are the effects of a large variety of policies analyzed in the current debate on the role of labour market institution.

Keywords: unemployment; wage setting function; income distribution; general equilibrium.

JEL classification: C72, D31, D51, J21.

1 Introduction

The goal of the paper is to study unemployment in a simple static general equilibrium model. In that framework, a necessary condition to get unemployment is to remove the condition which by very definition insures full employment, i.e., market clearing on the labour market. While that modeling choice is unavoidable given our research goal, it is quite arbitrary how to replace it. We decide to solve the problem adopting a truly textbook assumption[1]: we assume there is only one type of labour and the real wage is a function of the employment level and a residual variable representing all other factors which may influence the outcome of wage setting.

We assume that the considered function is positively related to the employment level. In fact, an increase in the unemployment rate does decrease wages as the higher is the unemployment rate the simpler is for firms to substitute a worker with another one having similar characteristics. Moreover, the considered wage setting function is assumed to be decreasing in the residual variable. In particular, that variable may be linked to any labour market policy like, for instance, those related to firing costs, hiring subsidies or unemployment benefits.

The main thrust of the paper is to study how equilibrium values of unemployment, wages and profits change when the above mentioned variable change. We argue that an increase of that variable may directly cause a decrease not only of real wages, but also of the employment level, together with an increase of profits.

[1]See the undergraduate textbook on macroeconomics by Blanchard (2003, pp.122-125); indeed, we assume a specific form of the wage function presented there.

The intuition can be simply described. Suppose there is a change in labour market institutions which reduces, for instance, firing restrictions or incentives to hire. Workers will think that it will be easier to loose their job (or not to find one) and they will be more willing to accept a lower wage to keep it. In other words, their bargaining power decreases. In the new situation firms will be able to pay lower wages and can moreover evaluate the possibility to further modify their choice of labour demand in order to further increase profits. An increase in the labour demand increases output, but it may cause an increase in wages which can more than offset the initial decrease. On the other hand, a decrease in the demand of labour decreases revenues, but it may imply a sharp fall in wages whose overall effect is a profit increase. Note that, since profits increase in any case, the described reasoning may also be an explanation of why deregulation policies on the labour markets are supported by some political parties sympathetic to firms interests.

Obviously, the above intuitive argument could be questioned on the following simple ground. Changes in exogenous variables do affect all endogenous ones, some of which are taken for given by a single firm. Therefore, the total effect of a change labour market institutions may be more difficult to determine than what said above. Nevertheless, we are going to show that the proposed "partial equilibrium analysis" intuition is consistent with a model where firms are described as players in a game. In that case, what it seems to be in one firm's interest, i.e., to decrease employment to induce a more than compensating decrease in wage, it turns out to be in each firm interest and, in fact, consistent with a Nash equilibrium behavior.

More formally, we consider a general equilibrium model with a finite number of potential workers who supply labour inelastically, only one good, a finite number of firm owners each of whom owns a unique firm. Firms are all equal and characterized by a production function whose sole input is labour. After having defined the obvious concept of competitive equilibrium in that framework, we introduce the definition of noncompetitive equilibrium obtained substituting the market clearing condition on the labour market with the fact that real wages are determined by a specific wage setting function W, a fact firms are aware of in their maximizing behavior. We then prove there is a unique equilibrium for each value of s, i.e., the above described residual variable, we make some comparative statics analysis of the effects of changes of s on some equilibrium variables and show that an increase in that variable can be associated with a decrease in wage, wage bill and employment, and an increase in profits. That can be also suggested by the main equation describing the working of our model[2], which can be written as follows

$$W(Fl, s) = \frac{F}{s+F} g'(l), \tag{1}$$

where F is the number of identical firms, l is the demand of labour by any single firm, Fl is the general level of employment, and g is the firm production function having only labour as input. An increase in s shifts down both the wage setting curve and the "modified demand curve", i.e., the graph of the function described in the right hand side of (1). Those movements cause a decrease in wage and an ambiguous change in the total employment level Fl that, as proved, is sometime negative. More precisely, we show that while the employment rate function decreases up to a threshold level s^* and increases above it, the wage and wage bill functions decrease, and the total profit function increases everywhere.

The above description of our model allows some comparison with the labour economics literature on unemployment. As it is well known, there exists a large theoretical, political and empirical literature which tries to explain the persistence of relatively high and across countries diversified rates of unemployment. A large number of phenomena and their effects on unemployment has been analyzed. Even though our modeling approach is not shared by any available contribution, the variable s is broadly enough defined to encompass basically any investigated phenomenon in the literature (once understood how a change of that phenomenon affects the variable s).

Bentolila and Bertola (1990) propose a model of firms's optimal employment policies under linear firing and hiring costs where dynamics and uncertainty are taken into account. Using continuous-time stochastic control techniques, the authors found that firing costs slightly increase long-run employment. Moreover, calibrating the model with parameters for the four largest European economies, and still using a partial equilibrium model, they argue that high firing cost can explain the persistence of unemployment in those countries. Bertola (1994) uses a model of endogenous disaggregate growth, irreversible investment decision and uncertainty and shows that constraints on employment flexibility reduce production efficiency and

[2]See (2) in the statement of Proposition 3.

the value of firms. That decreases private incentives to invest, and reduced efficiency and slower capital accumulation might in turn have adverse effects on the level and rate of growth of product demand, unemployment, wages, and welfare.

Mortensen and Pissarides (1999) inquire the effects of market rigidities on employment, wages, growth and efficiency in the framework of their famous search model. They show that the presence of unions decreases the job destruction rate and increase the duration of unemployments with respect to the competitive benchmark search equilibrium. As these two effects are offsetting, the total effect on the unemployment rate is unclear. The authors then incorporate some policies in the model showing in particular that an increase in unemployment compensation and payroll taxes induce a rise in unemployment, while a hiring subsidy or a firing tax implies an ambiguous impact on unemployment. Moving then to an empirical analysis, the authors find that an increase in payroll taxes or unemployment benefits imply an increase in unemployment and a decrease in income. An increase in the firing costs lowers unemployment, while a hiring subsidy increases unemployment, both of them having an ambiguous effect on income. For an analysis of the effects of market institution reforms in a labour market search model, see Boeri (2011).

Acemoglu and Shimer (1999) study the role of unemployment insurance in a general equilibrium model of search with risk aversion. They show that an increase in unemployment insurance increases wages, unemployment, and investment. Moreover, moderate unemployment insurance not only improves risk sharing but also increases output. Pissarides (2001) starts his analysis observing that while labor market rigidity is often blamed for the apparently poor performance of European labor markets, no rigorous econometric testing has been able to support that conclusion. Indeed, the author argues, the debate has been conducted in a framework which is not suitable, because it does not justify the existence of employment protections as insurance of workers against income risk. After having explicitly modeled that insurance role, the author shows that well-designed employment protection does not reduce job creation.

Sierbert (1997), attempting to explain Europe's experience with unemployment since 1970, claims that exogenous economic changes are not a plausible reason for the difference between Europe's experience with unemployment and United States' one. Then he argues that institutional changes affecting Europe's labor markets occurred in the 1960s and 1970s are a central reason for Europe's poor labor market performance and that institutional differences between Europe and the United States can explain their different employment pictures. In particular, he observes that the combination of intensified competition in a global economy and of labor-saving technical progress requires flexibility in wages, but this flexibility is prevented by institutional conditions. Saint-Paul (1996, 2008) stresses that there is somewhat of a consensus among economists that labour market rigidities are responsible for high unemployment in Europe, and in particular for its most alarming aspects, such as its long duration and high incidence on youth. Moreover, he argues that many of those inflexibilities (and the underlying institutional regulations) can be understood as the outcome of political influence by incumbent employees. This is because many policies that increase unemployment actually benefit these insiders. Then he claims that an empirical investigation of the determinants of labour market institutions demonstrates that the observations are consistent with this view.

Layard et al. (2005) argue that 'the long-run equilibrium level of unemployment is affected by any variable which influences the ease with which unemployed individuals can be matched to available job vacancies, and any variable which tends to raise wages in a direct fashion despite excess supply in the labour market' (Layard et al., 2005, p.xiv). Then, the authors carefully list variables that affect the two above main causes of unemployment and make a very helpful effort to give a sign to the relationship between each of those variables and the level of unemployment. They claims that matching of jobs and vacancies is affected by unemployment benefits[3] (level [+], coverage [+], length [+], strictness [−]), active labour market policies (training [−], high-quality placement services [−]), availability of other resources to unemployed individuals (real interest rate [+]), and employment protection laws (professionalisation [−], job securities of existing employees [+], barriers to mobility [+]), while the wage rate is affected by unions (union power in wage bargaining [+], union coverage [+], degree of coordination of wage bargaining by workers unions [−], degree of coordination of wage bargaining by employees [−]), product market competition [−], real wage resistance as a consequence of adverse shift in terms of trade [+], fall in trend productivity growth [+], and increase in labour taxes paid by firms [+]. Of course, policies to cut unemployment are then easily derived from the above list.

[3]In what follows, the symbol [+] (resp. [−]) means that the related variable is positively (resp. negatively) correlated with unemployment.

Freeman (2005) observes instead the presence of a new orthodoxy making the deregulation of labour market institutions be one of main keys to economic success and, in particular, to tackle the unemployment problem. At the same time, he notes also that the models justifying the claim that labour institutions impair aggregate performance are non-robust and ill-specified. He identifies essentially two main reasons for inconclusive debate over that claim. On the one hand, economists who believe that labour markets operate nearly perfectly in the absence of institutions let their priors dictate their modeling choices and interpretation of empirical results. On the other hand, empirical evidence is too weak to decisively reject strong prior views or to convince those with weaker priors. Similarly, Howell et al. (2007) claim that the empirical evidence offers little support to the orthodox view. Indeed, the empirical research on the determinants of high unemployment in the developed world has been, to a disturbing degree, driven by efforts to verify, or confirm, orthodox theory, rather than by efforts to critically test it. Stiglitz (2009) observes unfettered markets are not efficient and that rigidity of wages is not the cause of market failure. Wages are instead too flexible and job protection is needed to make our economic systems more stable.

In our opinion, what clearly emerges from the above certainly incomplete survey of contributions is that the debate about the effects of deregulation policies on unemployment is still open and far from reaching a conclusion. Our theoretical contribution provides some further insight into the problem. Indeed, it suggests that the effect on unemployment of a change in labour institution is related to the preexisting institutional situation. In a country characterized by strict labour protection, a reduction of that protection implies an increase in unemployment. On the contrary, in a country characterized by weak labour protection, a reduction of that protection implies a decrease in unemployment. That seems to provide a partial explanation of the differences between Europe and US. Moreover, independently on the institutional structure of the country, our analysis suggests that a decrease in wage protection has always the effect to increase income distribution inequality. In particular, deregulating labour market may have an effect on unemployment which is the opposite to the one commonly claimed to be the goal of the intervention.

We can now briefly describe the content of the remainder of the paper. In Section 2, we present the set-up of the model. Section 3 presents our main results, whose proofs are contained in the appendix. Proposition 3 shows that for any economy a unique, firm symmetric, noncompetitive equilibrium exists and it is characterized by equation (1) and by the presence of unemployment. We present the main results of the paper in Theorem 4. There, we show that if labour protection decreases, then below a threshold level, rate of employment decreases and, above that level, it increases. On the other hand, wage and wage bill decrease, while profits increase everywhere.

2 Set-up of the model and equilibrium concepts

We consider a market where there are a unique commodity, $F \geq 2$ firms, $H \geq 2$ workers and $F \geq 2$ firm owners. Define $\mathcal{F} = \{1, \ldots, F\}$, $\mathcal{H} = \{1, \ldots, H\}$ and $k = \frac{H}{F}$. Firms use labour as input and produce the unique commodity. Both workers and firm owners own a certain amount of initial endowments, have the same consumption space $\mathbb{R}_+$ and have preferences that can be represented by a utility function. Firm owners do not work but they own shares of firms that contribute to determine their wealth. We assume that each firm is owned by one and only one firm owner.

For every $h \in \mathcal{H}$, we assume that worker h is characterized by an initial endowment $e_h \in \mathbb{R}_+$ and a strictly increasing utility function $u_h : \mathbb{R}_+ \to \mathbb{R}$. Analogously, for every $f \in \mathcal{F}$, we assume that firm owner f is characterized by an initial endowment $\eta_f \in \mathbb{R}_+$ and a strictly increasing utility function $v_f : \mathbb{R}_+ \to \mathbb{R}$.

We assume that firms are identical and all characterized by a production function $g : \mathbb{R}_+ \to \mathbb{R}_+$ having amount of labour time as input and amount of produced commodity as output. We assume that $g \in C^2(\mathbb{R}_+)$, $g(0) = 0$ and, for every $l \in \mathbb{R}_+$, $g'(l) > 0$ and $g''(l) < 0$. The unit of measure of labour is the (maximum) amount of working time which is supposed to be the same for each worker. Firms are indexed by $f \in F$ and we assume that, for every f, firm owner f is the unique owner of firm f.

In the definition below, denote by p the commodity price, w the worker nominal wage, x_h the consumption of worker h, ξ_f the consumption of firm owner f, and l_f the labour employed by firm f.

Definition 1. *A competitive equilibrium is a vector*

$$\big(p^*, w^*, (x_h^*)_{h\in\mathcal{H}}, (\xi_f^*)_{f\in\mathcal{F}}, (l_f^*)_{f\in\mathcal{F}}\big) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+^H \times \mathbb{R}_+^F \times \mathbb{R}_+^F$$

such that:

- *for every $f \in \mathcal{F}$, l_f^* solves the problem*

$$\max_{l_f \in \mathbb{R}_+} p^* g(l_f) - w^* l_f$$

- *for every $h \in \mathcal{H}$, x_h^* solves the problem*

$$\max_{x_h \in \mathbb{R}_+} u_h(x_h) \quad s.t. \quad p^* x_h \leq p^* e_h + w^*$$

- *for every $f \in \mathcal{F}$, ξ_f^* solves the problem*

$$\max_{\xi_f \in \mathbb{R}_+} v_f(\xi_f) \quad s.t. \quad p^* \xi_f \leq p^* \eta_f + p^* g(l_f^*) - w^* l_f^*$$

- $\displaystyle\sum_{f \in \mathcal{F}} l_f^* = H$

- $\displaystyle\sum_{h \in \mathcal{H}} x_h^* + \sum_{f \in \mathcal{F}} \xi_f^* = \sum_{h \in \mathcal{H}} e_h + \sum_{f \in \mathcal{F}} \eta_f + \sum_{f \in \mathcal{F}} g(l_f^*)$

As it can be immediately noted, equilibrium commodity price has to be positive and price normalization is then possible. Therefore, from now on, we focus only on those competitive equilibria having the commodity price equal to 1, called *normalized competitive equilibria*. It is then immediate to prove that the unique normalized competitive equilibrium is the vector

$$\Big(1, g'(k), (e_h + g'(k))_{h \in \mathcal{H}}, (\eta_f + g(k) - g'(k)k)_{f \in \mathcal{F}}, (k)_{f \in \mathcal{F}}\Big),$$

where, as easily guessed, firms employ the same amount of labour force and the real wage is equal to the associated marginal productivity.

We introduce now a different equilibrium concept having two main additional characteristics. We assume that worker real wage is described by the function

$$W : [0, H] \times \mathbb{R}_{++} \to \mathbb{R}, \quad (t, s) \mapsto g'(k) \frac{t^s}{H^s},$$

where t represents the total amount of labour time, the variable s represents the relative bargaining power of firms over workers, and $g'(k)$ is the competitive wage. We assume that in the case of full employment, workers are able to get a wage which is the competitive one. The function W can be thought as the outcome of an unmodelled wage bargaining among workers and firms. Consistently with that interpretation, W has the following properties:

- for every $s \in \mathbb{R}_{++}$, $W(\cdot, s)$ is strictly increasing,
- for every $t \in [0, H]$, $W(t, \cdot)$ is strictly decreasing.

We finally need to introduce a function formalizing an institutional procedure to assign jobs in a world of homogeneous workers. Consider then

$$L : [0, H] \to \mathbb{R}_+^H, \quad t \mapsto (L_h(t))_{h \in \mathcal{H}},$$

where t again represents the total amount of labour time. Given $t \in [0, H]$ and $h \in \mathcal{H}$, $L_h(t)$ represents the amount of labour time worker h is allowed to work. We assume L satisfies the following properties:

- for every $t \in [0, H]$ and $h \in \mathcal{H}$, $L_h(t) \in [0, 1]$,
- for every $t \in [0, H]$, $\sum_{h \in \mathcal{H}} L_h(t) = t$.

Note that, if $t = H$, i.e., there is full employment, then for every $h \in \mathcal{H}$, $L_h(H) = 1$. On the other hand, if $t < H$, i.e., there is involuntary unemployment, the function L describes possible rationing schemes to decide who works and who does not. For example, we can think that each of the natural numbers from 1 to H are randomly assigned to the H potential workers. If a potential worker receives a natural number smaller or equal to the integer part of t, denoted by $\lfloor t \rfloor$, then she works 1 unit of labour, i.e., she is fully employed; if a potential worker receives the natural number equal to $\lfloor t \rfloor + 1$, then she works $t - \lfloor t \rfloor$ units of labour, i.e., she is partially employed (unemployed if t is an integer); if a potential worker receives a natural number greater than $\lfloor t \rfloor + 1$, then she does not work, i.e., she is unemployed. In that case we have that, for every $h \in \mathcal{H}$,

$$L_h(t) = \begin{cases} 0 & \text{if} \quad t \in [0, h] \\ t - \lfloor t \rfloor & \text{if} \quad t \in (h, h+1] \\ 1 & \text{if} \quad t \in (h+1, +\infty) \end{cases}$$

The main goal of the paper is to make a comparative static analysis of the effect of the parameter s on equilibrium values of some crucial endogenous variables. For such reason, in the following definition we decide to emphasize the role of s with respect to the other exogenous variables defining the economy.

In the definition below, we denote by w_h the nominal wage worker h expects to earn for one unit of labour time, t_h the labour time worker h expects to work, and γ_f the expected profits of firm owner f.

Definition 2. *A noncompetitive equilibrium associated with $s \in \mathbb{R}_{++}$ is a vector*

$$\left(p^*, (w_h^*)_{h\in\mathcal{H}}, (t_h^*)_{h\in\mathcal{H}}, (x_h^*)_{h\in\mathcal{H}}, (\gamma_f^*)_{f\in\mathcal{F}}, (\xi_f^*)_{f\in\mathcal{F}}, (l_f^*)_{f\in\mathcal{F}}\right) \in \mathbb{R}_+ \times \mathbb{R}_+^H \times [0,1]^H \times \mathbb{R}_+^H \times \mathbb{R}_+^F \times \mathbb{R}_+^F \times \mathbb{R}_+^F$$

such that:

- *for every $f \in \mathcal{F}$, l_f^* solves the problem*

$$\max_{l_f \in \mathbb{R}_+} p^* g(l_f) - p^* W\left(l_f + \sum_{f' \in \mathcal{F}, f' \neq f} l_{f'}^*, s\right) l_f, \quad s.t. \quad l_f \leq H - \sum_{f' \in \mathcal{F}, f' \neq f} l_{f'}^*$$

- *for every $h \in \mathcal{H}$, x_h^* solves the problem*

$$\max_{x_h \in \mathbb{R}_+} u_h(x_h) \quad s.t. \quad p^* x_h \leq p^* e_h + t_h^* w_h^*$$

- *for every $f \in \mathcal{F}$, ξ_f^* solves the problem*

$$\max_{\xi_f \in \mathbb{R}_+} v_f(\xi_f) \quad s.t. \quad p^* \xi_f \leq p^* \eta_f + \gamma_f^*$$

- $\displaystyle\sum_{h\in\mathcal{H}} x_h^* + \sum_{f\in\mathcal{F}} \xi_f^* = \sum_{h\in\mathcal{H}} e_h + \sum_{f\in\mathcal{F}} \eta_f + \sum_{f\in\mathcal{F}} g(l_f^*)$

- *for every $f \in \mathcal{F}$,*

$$\gamma_f^* = p^* g(l_f^*) - p^* W\left(\sum_{f\in\mathcal{F}} l_f^*, s\right) l_f^*$$

- *for every $h \in \mathcal{H}$,*

$$t_h^* = L_h\left(\sum_{f\in\mathcal{F}} l_f^*\right), \quad w_h^* = p^* W\left(\sum_{f\in\mathcal{F}} l_f^*, s\right)$$

Workers, firm owners and firms all know the commodity price. Moreover, firms know the function W and the maximum amount of labour that can be supplied by workers, i.e., H. Each firm has expectations about the labour employed by the other firms, each worker has expectations about the nominal wage and the labour time it will be allowed to work, and each firm owner has expectations about profits. On the basis of those information and expectations, firms, workers and firm owners decide their demands. The equilibrium is the situation in which expectations are confirmed and consistent with the known information and markets clear. In particular, the labour market rationing rule implies that in equilibrium it must be the case that

$$\sum_{f\in\mathcal{F}} l_f^* = \sum_{h\in\mathcal{H}} t_h^*.$$

Note also that in Definition 2, firms are price takers but not wage taker. Indeed, they all are assumed to know the relation linking employment level and wage as described by the wage setting function W. Thus, they take into account the way wage changes according to the general level of expected employment. As the feasible demand of labour and the profit of each firm depend on the decisions of the other firms, each firm has a strategic behavior in choosing its demand and in equilibrium they all contribute to determine both unemployment rate and wage without interacting with workers. On the contrary, individual workers and firm owners are both price and wage takers.

Finally, as equilibrium commodity price has to be positive, price normalization is allowed. Then, from now on we focus only on those noncompetitive equilibria having the commodity price equal to 1, called *normalized noncompetitive equilibria.*

3 Results

Proposition 3. *For every $s\in\mathbb{R}_{++}$, the unique normalized noncompetitive equilibria associated with s is the vector*

$$\Big(1, \left(W\left(Fl^*, s\right)\right)_{h\in\mathcal{H}}, \left(L_h\left(Fl^*\right)\right)_{h\in\mathcal{H}}, \left(e_h + L_h\left(Fl^*\right)W\left(Fl^*, s\right)\right)_{h\in\mathcal{H}}, \left(g(l^*) - W\left(Fl^*, s\right)l^*\right)_{f\in\mathcal{F}}, \\ \left(\eta_f + g(l^*) - W\left(Fl^*, s\right)l^*\right)_{f\in\mathcal{F}}, \left(l^*\right)_{f\in\mathcal{F}}\Big),$$

where l^* is the unique solution to the system

$$\begin{cases} W(Fl, s) = \frac{F}{s+F} g'\left(l\right), \\ l \in (0, k). \end{cases} \tag{2}$$

Observe that, for every $s\in\mathbb{R}_{++}$, the noncompetitive equilibrium associated with s is never consistent with full employment. With a slight abuse of notation, let us define the function

$$l^* : \mathbb{R}_{++} \to (0, k), \quad s \mapsto \text{ the unique solution to (2)}.$$

For every $s\in\mathbb{R}_{++}$, $l^*(s)$ represents the demand of labour of each firm in the normalized noncompetitive equilibrium associated with s. For simplicity, consider also the function

$$\alpha : \mathbb{R}_{++} \to \mathbb{R}, \quad s \mapsto \frac{l^*(s)}{k}.$$

Since, by definition of k, for every $s\in\mathbb{R}_{++}$, $\alpha\left(s\right) = \frac{Fl^*(s)}{H}$, we have that $\alpha(s)$ is the employment rate. Moreover, $\alpha(s)$ is the unique solution to the system

$$\begin{cases} g'(k)\alpha^s - \frac{F}{(s+F)} g'(k\alpha) = 0, \\ \alpha \in (0, 1). \end{cases} \tag{3}$$

Using α we can also express the following important equilibrium variables as a function of s:

- wage function: $\omega : \mathbb{R}_{++} \to \mathbb{R}_{++}$, $s \mapsto W(Fk\alpha(s), s)$,
- total firm profit function: $\pi : \mathbb{R}_{++} \to \mathbb{R}_{++}$, $s \mapsto F\big(g(k\alpha(s)) - W(Fk\alpha(s), s)k\alpha(s)\big)$,
- wage bill function: $\beta : \mathbb{R}_{++} \to \mathbb{R}_{++}$, $s \mapsto W(Fk\alpha(s), s)\, Fk\alpha(s)$.

We can state now the main result of the paper.

Theorem 4. *The functions α, ω, π and β have the following properties:*

- *they all belong to $C^1(\mathbb{R}_{++})$;*
- *there exists $s^* \in \mathbb{R}_{++}$ such that α is strictly decreasing on $(0, s^*]$ and strictly increasing on $[s^*, +\infty)$, and*
$$\lim_{s\to 0^+} \alpha(s) = 1, \quad \lim_{s\to +\infty} \alpha(s) = 1; \tag{4}$$
- *ω is strictly decreasing, and*
$$\lim_{s\to 0^+} \omega(s) = g'(k), \quad \lim_{s\to +\infty} \omega(s) = 0;$$
- *π is strictly increasing, and*
$$\lim_{s\to 0^+} \pi(s) = Fg(k) - Fg'(k)k, \quad \lim_{s\to +\infty} \pi(s) = Fg(k);$$
- *β is strictly decreasing, and*
$$\lim_{s\to 0^+} \beta(s) = g'(k)Fk, \quad \lim_{s\to +\infty} \beta(s) = 0.$$

A possible interpretation of Theorem 4 is that shifting the balance of power in wage bargaining in favor of firms is consistent with a decrease in the employment level, real wage and wage bill, along with an increase in (total) profits. Moreover, if workers have "complete power" in terms of s, then the noncompetitive equilibrium coincides with the competitive one: there is full employment, wages are equal to the marginal productivity of the employed labour, wage bill is equal to competitive wage times the available labour force and profits are the complement to the highest possible value of production. If instead all the power is in the hands of firms, then there is full employment at zero wage, and the profits are as high as possible.

4 Appendix

For every $s \in \mathbb{R}_{++}$ and $r \in [0, H)$, consider the function $\phi_{r,s} : (0, H - r] \to \mathbb{R}$, $\ l \mapsto g(l) - W(r + l, s)l$. Then we have that

$$\phi_{r,s} \text{ is concave,} \tag{5}$$
$$\text{there exists } \tilde{l} \in (0, H - r] \text{ such that } \phi_{r,s}(\tilde{l}) > 0. \tag{6}$$

The first property can be proved by computing the second derivative. The second property instead follows from the relation

$$\lim_{l\to 0^+} \frac{g(l) - W(l + r, s)l}{l} = g'(0) - W(r, s) \geq g'(0) - W(H, s) > 0.$$

Proof of Proposition 3. Fix $s \in \mathbb{R}_{++}$. First of all, note that the proof that (2) has a unique solution is straightforward. Consider now $(l^*_f)_{f\in\mathcal{F}} \in \mathbb{R}^F_+$ and the following conditions:

$c1$. for every $f \in \mathcal{F}$, l^*_f solves

$$\max_{l_f\in\mathbb{R}_+} g(l_f) - W\left(s, l_f + \sum_{f'\in\mathcal{F}, f'\neq f} l^*_{f'}\right) l_f, \quad \text{s.t.} \quad l_f \leq H - \sum_{f'\in\mathcal{F}, f'\neq f} l^*_{f'}, \tag{7}$$

$c2$. for every $f \in \mathcal{F}$, $l^*_f \in \mathbb{R}_{++}$ and solves

$$\max_{l_f\in\mathbb{R}_{++}} g(l_f) - W\left(s, l_f + \sum_{f'\in\mathcal{F}, f'\neq f} l^*_{f'}\right) l_f, \quad \text{s.t.} \quad l_f \leq H - \sum_{f'\in\mathcal{F}, f'\neq f} l^*_{f'}, \tag{8}$$

$c3$. $(l_f^*)_{f\in\mathcal{F}} \in \mathbb{R}^F_{++}$ and solves

$$\begin{cases} g'(l_f) - \frac{g'(k)}{H^s}\left(\sum_{f'\in\mathcal{F}} l_{f'}\right)^{s-1} sl_f - \frac{g'(k)}{H^s}\left(\sum_{f'\in\mathcal{F}} l_{f'}\right)^{s} = 0, \quad f\in\mathcal{F}, \\ \sum_{f'\in\mathcal{F}} l_{f'} < H, \end{cases} \tag{9}$$

$c4$. for every $f\in\mathcal{F}$, $l_f^* = l^*$, where l^* is the unique solution to (2).

It is immediate to prove that we get the desired result proving $c1 \Leftrightarrow c4$. We are going to prove that equivalence showing that $c1 \Leftrightarrow c2$, $c2 \Leftrightarrow c3$, $c3 \Leftrightarrow c4$.

$c1 \Rightarrow c2$. Assume (7) and observe that surely $\sum_{f\in\mathcal{F}} l_f^* \leq H$. Assume now by contradiction that there exists $f'\in\mathcal{F}$ such that $l_{f'}^* = 0$. Then we have $\sum_{f\in\mathcal{F}, f\neq f'} l_f^* = H$, because, according to (6), if $\sum_{f\in\mathcal{F}, f\neq f'} l_f^* < H$, we can find $\tilde{l}_{f'} \in \left(0, H - \sum_{f\in\mathcal{F}, f\neq f'} l_f^*\right]$ such that

$$g(\tilde{l}_{f'}) - W\left(\tilde{l}_{f'} + \sum_{f\in\mathcal{F}, f\neq f'} \tilde{l}_f^*, s\right)\tilde{l}_{f'} > 0 = g(l_{f'}^*) - W\left(l_{f'}^* + \sum_{f\in\mathcal{F}, f\neq f'} l_f^*, s\right) l_f^*.$$

As a consequence, there exists $f''\in\mathcal{F}$ such that $l_{f''}^* \in (k, H]$. Applying Kuhn-Tucker first order conditions, we have, in particular, that it has to be

$$g'(l_{f''}^*) \geq g'(k)\left(\frac{s}{H} l_{f''}^* + 1\right).$$

However, as $l_{f''}^* > k$ and $\frac{s}{H} > 0$, the above inequality cannot hold true and the contradiction is found. Then we have proved that, for every $f\in\mathcal{F}$, $l_f^* > 0$ and $c2$ follows.
$c2 \Rightarrow c1$. It is sufficient to show that, for every $f\in\mathcal{F}$,

$$g(l_f^*) - W\left(l_f^* + \sum_{f\in\mathcal{F}, f\neq f'} l_{f'}^*, s\right) l_f^* > 0.$$

That follows from (6).

$c2 \Rightarrow c3$. Applying again Kuhn-Tucker first order conditions to problems (8), we have there exists $(\mu_f^*)_{f\in\mathcal{F}} \in \mathbb{R}^F$ such that $(l_f^*, \mu_f^*)_{f\in\mathcal{F}}$ solves the system

$$\begin{cases} g'(l_f) - \frac{g'(k)}{H^s}\left(\sum_{f'\in\mathcal{F}} l_{f'}\right)^{s-1} sl_f - \frac{g'(k)}{H^s}\left(\sum_{f'\in\mathcal{F}} l_{f'}\right)^{s} - \mu_f = 0, & f\in\mathcal{F} \\ \min\left\{\mu_f, H - \sum_{f'\in\mathcal{F}} l'_f\right\} = 0, & f\in\mathcal{F}. \end{cases} \tag{10}$$

Let us prove now that $\sum_{f\in\mathcal{F}} l_f^* < H$. If otherwise it were $\sum_{f\in\mathcal{F}} l_f^* = H$, then, for every $f\in\mathcal{F}$,

$$g'(l_f^*) = g'(k)\left(\frac{s}{H} l_f^* + 1\right) + \mu_f^*, \quad \text{and} \quad \mu_f^* \geq 0.$$

Arguing as before, for every $f\in\mathcal{F}$, it should be $l_f^* < k$ and the contradiction follows. That implies $c3$.
$c3 \Rightarrow c2$. Assume that $(l_f^*)_{f\in\mathcal{F}} \in \mathbb{R}^F_{++}$ and solves (9). Define, for every $f\in\mathcal{F}$, $\mu_f^* = 0$. Then $(l_f^*, \mu_f^*)_{f\in\mathcal{F}}$ solves (10). Since, because of (5), Kuhn-Tucker conditions are also sufficient for the problems (8), we immediately get $c2$.

$c3 \Rightarrow c4$. Consider $f, \tilde{f} \in\mathcal{F}$, $f\neq \tilde{f}$. Then

$$g'(l_f^*) - \frac{g'(k)}{H^s}\left(\sum_{f'\in\mathcal{F}} l_{f'}^*\right)^{s-1} sl_f^* - \frac{g'(k)}{H^s}\left(\sum_{f'\in\mathcal{F}} l_{f'}^*\right)^{s} = 0,$$

and

$$g'(l^*_{\tilde{f}}) - \frac{g'(k)}{H^s}\left(\sum_{f'\in\mathcal{F}} l^*_{f'}\right)^{s-1} sl^*_{\tilde{f}} - \frac{g'(k)}{H^s}\left(\sum_{f'\in\mathcal{F}} l^*_{f'}\right)^{s} = 0.$$

Then

$$g'(l^*_{f}) - g'(l^*_{\tilde{f}}) = \frac{g'(k)}{H^s} s(l^*_f - l^*_{\tilde{f}})\left(\sum_{f'\in\mathcal{F}} l^*_{f'}\right)^{s-1}.$$

The above relation immediately implies $l^*_f = l^*_{\tilde{f}}$ and $c4$ then follows.
$c4 \Rightarrow c3$. Straightforward. $\square$

Proof of Theorem 4. We divide the proof in several steps.

Step 1: regularity of $\alpha, \omega, \beta, \pi$. The function α is implicitly defined by the C^1 function

$$\mathcal{F} : (0, +\infty) \times (0, 1) \to \mathbb{R}, \quad (s, \alpha) \mapsto g'(k)\alpha^s - \frac{Fg'(k\alpha)}{s + F}.$$

Since

$$\frac{\partial \mathcal{F}}{\partial \alpha}(s, \alpha) = g'(k)s\alpha^{s-1} - \frac{Fkg''(k\alpha)}{s + F} > 0,$$

we have that α belongs to $C^1(\mathbb{R}_{++})$. Moreover, since

$$\frac{\partial \mathcal{F}}{\partial s}(s, \alpha) = g'(k)\alpha^s \ln(\alpha) + \frac{Fg'(k\alpha)}{(s + F)^2},$$

by the implicit function theorem and (3), we have that, for every $s \in \mathbb{R}_{++}$,

$$\alpha'(s) = -\frac{g'(k)\alpha(s)^s \ln(\alpha(s)) + \frac{Fg'(k\alpha(s))}{(s+F)^2}}{g'(k)s\alpha(s)^{s-1} - \frac{Fkg''(k\alpha(s))}{s+F}}. \tag{11}$$

As, for every $s \in \mathbb{R}_{++}$, $\alpha(s) \in (0, 1)$ and $W : (0, H) \times \mathbb{R}_{++} \to \mathbb{R}$ is C^1, we immediately get that ω, β, and π belong to $C^1(\mathbb{R}_{++})$, as well.

Step 2: monotonicity property of α. Using (11) we have that, for every $s \in \mathbb{R}_{++}$,

$$\text{sign}\,(\alpha'(s)) = \text{sign}\left(-g'(k)\alpha(s)^s \ln(\alpha(s)) - \frac{Fg'(k\alpha(s))}{(s + F)^2}\right).$$

¿From (3), we have

$$-g'(k)\,\alpha(s)^s \ln(\alpha(s)) - \frac{Fg'(k\alpha(s))}{(s+F)^2} = -g'(k)\,\alpha(s)^s \ln(\alpha(s)) - \frac{g'(k)\,\alpha(s)^s}{s+F}$$

$$= g'(k)\,\alpha(s)^s\left(-\ln(\alpha(s)) - \frac{1}{s+F}\right),$$

and therefore

$$\text{sign}\,(\alpha'(s)) = \text{sign}\left(-\ln(\alpha(s)) - \frac{1}{s+F}\right).$$

Consider now the function $\theta : \mathbb{R}_{++} \to \mathbb{R}$ defined, for every $s \in \mathbb{R}_{++}$, as $\theta(s) = e^{-\frac{1}{F+s}}$. We have, for every $s \in \mathbb{R}_{++}$, $-\ln(\theta(s)) - \frac{1}{F+s} = 0$. As a consequence, for every $s \in \mathbb{R}_{++}$,

$$\text{sign}\,(\alpha'(s)) = \text{sign}\left(-\ln(\alpha(s)) - \frac{1}{F+s}\right) = \text{sign}(\theta(s) - \alpha(s)) =$$

$$= \text{sign}\left((s+F)g'(k)\theta(s)^s - Fg'(k\theta(s))\right).$$

Then we can study the sign of $\alpha'(s)$ studying the sign of the function $G : \mathbb{R}_{++} \to \mathbb{R}$ defined, for every $s \in \mathbb{R}_{++}$, as

$$G(s) = e^{-\frac{s}{F+s}} g'(k)(s+F) - Fg'\left(ke^{-\frac{1}{F+s}}\right).$$

Of course, $G = G_1 - G_2$ where $G_1 : \mathbb{R}_{++} \to \mathbb{R}$ and $G_2 : \mathbb{R}_{++} \to \mathbb{R}$ are defined, for every $s \in \mathbb{R}_{++}$, as

$$G_1(s) = e^{-\frac{s}{F+s}} g'(k)(s+F), \quad G_2(s) = Fg'\left(ke^{-\frac{1}{F+s}}\right).$$

It is immediate to verify that G_1 is strictly increasing, G_2 is strictly decreasing on $\mathbb{R}_{++}$ and

$$\lim_{s\to 0^+} G_1(s) = Fg'(k), \quad \lim_{s\to 0^+} G_2(s) = Fg'(ke^{-\frac{1}{F}}), \quad \lim_{s\to+\infty} G_1(s) = +\infty, \quad \lim_{s\to+\infty} G_2(s) = Fg'(k).$$

Then we have that there exists $s^* \in \mathbb{R}_{++}$ such that $\alpha'(s^*) = 0$, for every $s \in (0, s^*)$, $\alpha'(s) < 0$ and, for every $s \in (s^*, +\infty)$, $\alpha'(s) > 0$. As a consequence, α is strictly decreasing on $(0, s^*]$ and strictly increasing on $[s^*, +\infty)$. In particular, it has an absolute minimum in s^*.

Step 3: limit results. For every $s \in \mathbb{R}_{++}$,

$$\alpha(s)^s = \frac{Fg'(k\alpha(s))}{(s+F)g'(k)} > \frac{Fg'(k)}{(s+F)g'(k)} = \frac{F}{(s+F)}$$

and then

$$1 \geq \alpha(s) > \left(\frac{F}{(s+F)}\right)^{\frac{1}{s}}.$$

The above relation implies $\lim_{s\to+\infty} \alpha(s) = 1$. By monotonicity of the function α, there exists $R \in [0,1]$ such that

$$\lim_{s\to 0^+} \alpha(s) = R.$$

Let us recall also that $g' : \mathbb{R}_+ \to \mathbb{R}$ is positive, strictly decreasing and continuous. Then $g'(\mathbb{R}_+) = (\inf(g'), g'(0)]$ where $\inf(g') \geq 0$. Then $(g')^{-1} : (\inf(g'), g'(0)] \to \mathbb{R}_+$ is strictly decreasing and continuous. We surely have $R \in (\alpha(s^*), 1]$ and then

$$\lim_{s\to 0^+} g'(k)\alpha(s)^s = g'(k), \quad \lim_{s\to 0^+} \frac{Fg'(k\alpha(s))}{F+s} = g'(kR).$$

Using (3), we obtain that it has to be $g'(kR) = g'(k)$ and then $R = 1$. Then (4) is proved.

Finally, from (3) and (4), we get

$$\lim_{s\to 0^+} \alpha(s)^s = 1, \quad \lim_{s\to 0^+} \alpha(s)^{s+1} = 1, \quad \lim_{s\to\infty} \alpha(s)^s = 0, \quad \lim_{s\to\infty} \alpha(s)^{s+1} = 0,$$

and all the other desired limits follow.

The above results on α' and the limit behavior of α allow to prove the desired monotonicity properties of all the other significant functions mostly using the following strategy: write the expression of the desired function using its definition and also the key equilibrium equation (3); compute the derivative with respect to s of both expressions and apply (3) if not done before; exploit the fact that α' is positive or negative in one or the other of the two ways of writing the desired derivative.

Step 4: monotonicity property of ω. Using the analytical expression of W and (3), we have

$$\omega(s) = g'(k)\alpha(s)^s = \frac{F}{F+s} g'(k\alpha(s)),$$

and then

$$\omega'(s) = g'(k)\alpha(s)^s \left(\ln(\alpha(s)) + s\frac{\alpha'(s)}{\alpha(s)}\right), \tag{12}$$

$$\omega'(s) = -\frac{F}{(F+s)^2} g'(k\alpha(s)) + \frac{F}{F+s} g''(k\alpha(s))k\alpha'(s). \tag{13}$$

To get the desired results, we use (12) and (13) and the results on the sign of α' obtained in Step 2. For every $s \in (0, s^*)$, $\alpha'(s) < 0$, and then (12) implies that, for every $s \in (0, s^*)$, $\omega'(s) < 0$; moreover, for every $s \in (s^*, +\infty)$, $\alpha'(s) > 0$, and then (13) implies that, for every $s \in (s^*, +\infty)$, $\omega'(s) < 0$. Then, ω is strictly decreasing in $\mathbb{R}_{++}$.

Step 5: monotonicity property of π. Using the analytical expression of W and (3), we have

$$\pi(s) = Fg(k\alpha(s)) - Fg'(k)k\alpha(s)^{s+1} = Fg(k\alpha(s)) - \frac{F^2}{F+s}g'(k\alpha(s))k\alpha(s).$$

Then, using again (3), we get both the equalities

$$\pi'(s) = -Fkg'(k\alpha(s))\left(\frac{(F-1)s\alpha'(s) + F\alpha(s)\ln(\alpha(s))}{s+F}\right), \tag{14}$$

$$\pi'(s) = kF\left(1 - \frac{F}{F+s}\right)g'(k\alpha(s))\alpha'(s) + \frac{kF}{F+s}\alpha(s)\left(\frac{1}{F+s}g'(k\alpha(s)) - g''(k\alpha(s))k\alpha'(s)\right). \tag{15}$$

Using the same strategy adopted in Step 4, we have that (14) implies that, for every $s \in (0, s^*)$, $\pi'(s) > 0$, while (15) implies that, for every $s \in (s^*, +\infty)$, $\pi'(s) > 0$. Then, π is strictly increasing in $\mathbb{R}_{++}$.

Step 6: monotonicity property of β. Using the analytical expression of W and (3), we have

$$\beta(s) = Fg'(k)k\alpha(s)^{s+1} = \frac{F^2}{F+s}g'(k\alpha(s))k\alpha(s).$$

Then, we get both the equalities

$$\beta'(s) = Fg'(k)k\alpha(s)^{s+1}\left(\ln(\alpha(s)) + (s+1)\frac{\alpha'(s)}{\alpha(s)}\right) \tag{16}$$

$$\beta'(s) = \frac{kF^2}{F+s}g'(k\alpha(s))\left(-\frac{1}{F+s}\alpha(s) + \alpha'(s)\right) + \frac{kF^2}{F+s}g''(k\alpha(s))k\alpha(s)\alpha'(s). \tag{17}$$

We have that (16) implies that, for every $s \in (0, s^*)$, $\beta'(s) < 0$. We are now going to prove that, for every $s \in (s^*, +\infty)$, $\beta'(s) < 0$: that allows to conclude that β is strictly decreasing in $\mathbb{R}_{++}$. Note at first that (17) implies that we get the desired result proving that, for every $s \in (s^*, +\infty)$,

$$\alpha'(s) < \frac{1}{F+s}\alpha(s). \tag{18}$$

Recalling (11) and using (3), we can write

$$\alpha'(s) = -\frac{g'(k\alpha(s))\alpha(s)\left(\ln(\alpha(s)) + \frac{1}{F+s}g'(k\alpha(s))\right)}{g'(k\alpha(s))s - kg''(k\alpha(s))\alpha(s)},$$

that implies that (18) is equivalent to

$$g'(k\alpha(s))\left(-\ln(\alpha(s)) - \frac{s+1}{s+F}\right) < -\frac{k}{F+s}g''(k\alpha(s))\alpha(s).$$

As a consequence, we obtain the desired condition on β' proving that, for every $s \in (s^*, +\infty)$,

$$-\ln(\alpha(s)) < \frac{s+1}{s+F},$$

that is,

$$\alpha(s) > e^{-\frac{s+1}{s+F}}. \tag{19}$$

Using the properties of the function that implicitly defines α, we have that (19) is equivalent to the inequality $H_1(s) < H_2(s)$, where

$$H_1(s) = (s+F)g'(k)e^{-\frac{s+1}{s+F}s}, \quad H_2(s) = Fg'\left(ke^{-\frac{s+1}{s+F}}\right).$$

It is immediate to prove that H_1 is strictly decreasing, H_2 is strictly increasing and

$$\lim_{s\to 0^+} H_1(s) = Fg'(k), \quad \lim_{s\to 0^+} H_2(s) = Fg'(ke^{-\frac{1}{F}})$$

Then, for every $s \in \mathbb{R}_{++}$, $H_1(s) < H_2(s)$. That implies that in particular, for every $s \in (s^*, +\infty)$, $\beta'(s) < 0$. $\square$

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