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Market Equilibrium in the Presence of Green Consumers and Responsible Firms: a Comparative Statics Analysis

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Abstract

This paper analyzes how the interaction between green consumers and responsible firms affects the market equilibrium. The main result is that a higher responsibility by both producers and consumers can have different impacts on the efficiency of the firms' abatement activity, depending on the nature of the cleaning costs. When the abatement costs are fixed, the efficiency of the clean-up effort is always increasing in their degree of responsibility. On the other hand, when the abatement costs are variable, a higher level of responsibility may reduce social welfare. Finally, the first best allocation is never reached, even in the presence of the highest credible level of responsibility of both consumers and producers.

Key words: Green Consumers, Corporate Social Responsibility, Vertical Differentiation.

J.E.L. Classification: D62, L13, L21.

1 Introduction

In the late years a growing body of the environmental economics literature has been devoted to the analysis of the so called third generation instruments for the control of pollution. Indeed, the classic *command and control* approach can be substituted, or integrated, not only by economic instruments (i.e. taxes, subsidies and tradable permits) but also by the voluntary market choices of environmentally aware agents.[1] However, the current debate is far from a complete understanding of the actual capabilities of both individual and firm responsibility as a means to effectively promote environmental protection (see Bénabou and Tirole, 2010).

In many sectors firms try to increase their market share by advertising their production as environment-friendly. As noted by Kotchen (2005) and Besley and Ghatak (2007), environment-friendly goods can be viewed as impure public goods, in which private and public characteristics are bundled together. As emphasized by Bagnoli and Watts (2003), the form of this bundling can be explicit or implicit. The former corresponds to situations in which firms improve the environmental quality of the good they provide and, consequently, they increase their marginal costs of production. The latter corresponds to situations in which firms sustain environmental programs whose benefits and costs are not proportional to their sales.

There is a large evidence that many consumers are willing to pay a price premium to purchase environment-friendly goods. The premium paid represents a form of voluntary contribution to the provision of a public good. In the economic literature there are different ways to reconcile this behavior with the traditional assumption of self-interested agents. A first attempt is based on the assumption that green consumers obtain a direct utility by the environmental qualities of the goods they buy. In this view green consumers derive a *warm glow* from their responsible action (Andreoni, 1990), due to social approval or to their internal moral motivation. On the other hand, we could think that green consumers behave as *conditional cooperator*, who accept to sacrifice their utility conditional on expectations that others will do the same. Indeed, other authors (e.g. Ostrom, 2000) emphasized that in the presence of social dilemmas, if all the individuals seek to maximize their egoistic interest, they are unavoidably trapped in a suboptimal equilibrium. For this reason truly rational agents can choose to switch to more refined choice criteria. We follow such kind of opinion and for this reason in our model we assume that responsible citizens maintain the same kind of pref-

[1] See Khanna, 2001, for a good survey on this historical evolution.

erences, but adopt choice criteria that do not coincide with the maximization of their pay off. Coherently with this view, we consider the social welfare as the sum of agents' pay offs and not as the sum of their objective functions representing their choice criteria.

The economic literature traditionally has analyzed the green consumers phenomenon in the framework of vertically differentiated markets. A first group of paper focused on how the presence of green consumers interacts with the optimal environmental policy (see Arora and Gangopadhyay, 1995; Cremer and Thisse, 1999; Moraga-Gonzalez and Padron-Fumero, 2002; Bansal and Gangopadhyay, 2003; Lombardini-Riipinen, 2005). A second group dealt with the impact of a higher consumers' consciousness on the market equilibrium and the associated social welfare. Frequently the results of these models warn against a naive confidence in consumers' responsibility as a solution to environmental problems. Indeed, rarely the market equilibrium in the presence of green consumers approximates the maximization of social welfare (see Eriksson, 2004; Conrad, 2005). Moreover, some authors showed that it cannot be taken for granted that a higher level of consumers' responsibility is always associate to less pollution and higher welfare[2] (Rodriguez-Ibeas, 2007; Garcia-Gallego and Georgantzis, 2009).

Our paper can be considered as an extension of the vertically differentiated duopoly put forward by Garcia-Gallego and Georgantzis (2009). They assume that consumers have a different willingness to pay (hereafter WTP) for "clean" products and they study how an increase in their aggregate WTP affects the market equilibrium. As far as the production technology is concerned, they assume that the costs and the benefits of the abatement activity are increasing and convex in the level of clean-up and independent of the level of production. This assumption covers the case in which firms devote lump-sum expenditures to environmental protection activities not directly associated to their production of the private good. However there are cases in which the benefits and the costs of the abatement activity are proportional to the quantity produced, as happens when firms improve the environmental quality of their production process.[3] So a first extension consists in repeating their analysis with variable costs.

Moreover, the main novelty of our paper is that we allow firms to choose their

[2]Similar conclusions are reached in a different framework by Calveras et *al*. (2007). They consider a model in which citizens first vote the minimum environmental standard and then buy a good produced in perfectly competitive markets. According to their analysis, a higher level of activism in the society may imply a higher level of pollution.

[3]Many existing models adopt this assumption. See for instance Cremer and Thisse (1999), Eriksson (2004), Lombardini-Riipinen (2005), Conrad (2005) and Rodriguez-Ibeas (2007).

market strategy in accordance with an objective function that may not coincide with profit maximization. Indeed, in some markets, especially when the good traded is an impure public good, we can observe competition between firms with different aims: standard for profit firms may compete with non-profit firms, whose main objective is the maximization of the positive externality associated to their production.[4]

Recently many firms spend a lot of efforts in order to persuade consumers that their behavior is socially responsible. However, there is not a general consensus with regard to the exact concept of corporate social responsibility (CSR). We report two polar definitions that can appear in sharp contrast.[5] According to a first point of view, a firm is socially responsible when it takes environment-friendly actions not required by law. In this light, CSR can be defined without any regard neither to the motivation of the firm's choices nor to the impact of such choices on the firm's profit. From a different point of view, other authors believe that a firm is truly responsible only when it sacrifices its profit, at least in part, in order to carry out some social objective. Baron (2001) names the first behavior as *strategic* CSR and the second one as *altruistic* CSR.

In all the existing models regarding the influence of green consumers on the market equilibrium, firms are assumed to behave as standard profit maximizers. Consequently the current literature explores only the effect of the interaction between green consumers and firms engaged in strategic CSR. We propose a more general approach in which firms' objective function weighs together both profit and the social impact of their actions. In this view, firms' degree of CSR can be interpreted as the relative weight they assign to the second objective. Our purpose is to study the market equilibrium in the presence of green consumers and firms engaged in altruistic CSR. More specifically this work aims at analyzing:

1. whether a higher level of responsibility of both consumers and producers is always associated to a more efficient result in terms of pollution control;

2. whether a full responsibility of both producers and consumers is sufficient to attain the first best level of pollution.

The remaining part of the paper is organized as follows: in Section 2 we present the general model and introduce the concepts of green consumers and

[4]Becchetti and Huybrechts (2007) interpret in this way the Fair Trade sector.

[5]An interesting debate over this issue can be found in the first volume of the *Review of Environmental Economics and Policy*. In particular, see Lyon and Maxwell (2008) and Reinhardt et *al*. (2008).

responsible firms. In Section 3 we characterize the market equilibrium in the case in which the costs and the benefits of the cleaning technology are fixed (i.e.: independent of the quantity produced). In Section 4 we extend the same kind of analysis to the alternative case in which the costs and the benefits of clean-up are assumed to be proportional to the quantity produced. Section 5 concludes.

2 The general model

2.1 The technology

There is a physically homogeneous good, whose production generates pollution. The production costs depend on both the quantity produced, x, and the level of the abatement activity, e. Formally, the cost function for a generic firm i is:

$$C_i(e_i) = \frac{k}{2} e_i^2 x_i^{\gamma}, \quad \forall i = 1, ..., n, \tag{1}$$

where k is a constant and γ indicates how the quantity produced affects the abatement costs. Specifically, γ can assume two values: zero when the abatement costs are fixed, and one when these costs are variable.

The total emissions for a single firm are:

$$Y_i(e_i) = \bar{e} x_i - e_i x_i^{\gamma}, \quad \forall i = 1, ..., n; \tag{2}$$

$\bar{e}$ is the *unitary* level of emissions without clean-up activity. We assume that when the abatement costs are fixed, $\gamma = 0$, then the clean-up activity of a generic firm i is independent of x_i. In this case, according to the definition introduced by Bagnoli and Watts (2003), the private provision of the public good "abatement" is only implicitly linked to sales of the private good. On the other hand, when the abatement costs are variable, $\gamma = 1$, the clean-up activity of firm i is proportional to x_i. This case corresponds to a situation in which the provision of both public and private good are explicitly linked. Finally, let us define $Y = \sum_{i=1}^{n} Y_i$ the aggregate emissions.

2.2 The market actors and the social welfare

On the demand side there is a unit mass of consumers that are interested in buying only one unit of the good. Their utility function is:

$$U = V - p - \rho Y, \tag{3}$$

where V is the (homogeneous) gross utility of consuming one unit of the product, p is its price and ρ is the marginal disutility that each consumer associates to the negative externality stemming from pollution. We assume that ρ is distributed according to $F(\rho)$ over the support $\left[\underline{\rho}, \bar{\rho}\right]$, where $F(\underline{\rho}) = 0$ and $F(\bar{\rho}) = 1$. As a consequence, the social benefits of the clean-up activity is equal to $\rho^T E^T$ where $E^T = \sum_{i=1}^n e_i x_i^\gamma$ is the total abatement and $\rho^T = \int \rho dF(\rho)$ is its marginal social benefit. We assume that V is higher than $\bar{e}$ and consequently the production of this good is always socially efficient, in the sense that the associated social benefits are higher than the correspondent social costs.

On the supply side there are n firms which share the common technology. Imagine that a generic firm i adopts a level of abatement e_i, charges a price equal to p_i and sells to a market share x_i. In such case its profit is:

$$\pi_i = p_i x_i - \frac{k}{2} e_i^2 x_i^\gamma, \quad i = 1, ..., n. \tag{4}$$

We assume that every consumer buys the product of one firm; consequently, $\sum_{i=1}^n x_i = 1$. The social welfare W is defined as the sum of consumers' surplus and firms' profit. In a generic market allocation it is equal to:

$$W = V - \rho^T \bar{e} + \sum_{i=1}^n \left[\rho^T e_i - \frac{k}{2} (e_i)^2 \right] x_i^\gamma. \tag{5}$$

It is worth noting that W corresponds to the difference between the social benefits and the total costs associated to the aggregate clean-up. Consequently we can identify the social welfare with the efficiency of the environmental protection activity.

The maximization of the social welfare entails that the first best level of clean-up of each firm is:

$$e^{FB} = \frac{\rho^T}{k}, \qquad \forall i = 1, ..., n, \tag{6}$$

which does not depend on the quantity produced. It is worth noting that whatever the total abatement is, its cost effective allocation requires that firms' marginal costs coincide, implying that total abatement should be shared equally among them.

We assume that the environmental regulator cannot force firms to adopt a positive level of clean-up. Consequently, if ρ^T is strictly higher than 0, then the public good "abatement" is too scarce and responsible citizens can choose to voluntarily contribute to its provision by producing or consuming more environment-friendly goods.

2.3 The green consumers

We assume that a share $0 \leq \beta \leq 1$ of consumers' population (labeled as *green*) takes into account firms' abatement efforts in choosing which product to buy, while all the others consumers acts as (radical) free riders. We let the WTP of green consumers be heterogeneous among them. Formally, a generic green consumer chooses a product to maximize:[6]

$$H = V - p - \rho Y + \theta e, \tag{7}$$

where θ is the individual WTP for the marginal increase in firms' abatement, assumed uniformly distributed in the interval $\left[0, \bar{\theta}\right]$.[7] Therefore the total WTP for more environment-friendly products, θ^T, is equal to $\frac{\beta\bar{\theta}}{2}$. Let us define μ as:

$$\mu = \frac{\theta^T}{\rho^T} = \frac{\beta\bar{\theta}}{2\rho^T}. \tag{8}$$

This ratio can be considered as an index of the social capital of the consumers' population because it represents how much their choices are driven by social rather than individualistic motivations. We limit the aggregate consumers' WTP to be lower than their aggregate marginal disutility of emissions, i.e. $\theta^T \leq \rho^T$. Given this assumption, it follows that $0 \leq \mu \leq 1$.

2.4 The responsible firms

Following Garcia-Gallego and Georgantzis (2009), we assume that on the supply side there is the coexistence of two kinds of firms. On the one hand, a fringe of firms who provide the good without employing any clean-up activity. Given that they sell an homogenous product and compete à la Bertrand, they charge a price equal to 0 (the marginal cost of production when $e = 0$), and they do not achieve extra profit. On the other hand, in the presence of green consumers, other firms can choose to employ the cleaning technology in order to differentiate their product and to obtain a strictly positive profit. We assume that there are only

[6]We assume that each consumer takes the total emissions Y as exogenous because her individual contribution to pollution is negligible.

[7]We choose to pay attention only to the case in which the lowest WTP is 0 in order to simplify our analysis of the market equilibrium. Indeed, the assumption that $\underline{\theta} = 0$ ensures that only an incomplete market coverage configuration can arise in equilibrium, as shown by Liao (2008) for the fixed costs case and by Ecchia and Lambertini (1998) for the variable costs case.

two firms that are able to carry out this abatement activity. We use H and L to denote the variables associated to the firms choosing the *high* and the *low* level of abatement.

These two firms are labeled as *responsible* because they overcomply the existing environmental regulation. As sustained by Kotchen (2009), environment-friendly innovations are frequently introduced by eco-entrepreneurs where eco-entrepreneurship can be defined as "the practice of starting new businesses in response to an identified opportunity to earn a profit and provide a positive environmental externality". So, the assumption regarding the existence of only two responsible producers can be justified by noting that frequently innovation processes are driven by a limited number of firms.

We assume that responsible firms can have a different willingness to sacrifice their profit in order to increase their clean-up. Formally, we allow them to have the following composite objective function that weighs the maximization of their profit and the maximization of the positive externality associated to their abatement activity:

$$J_i = \left(\pi_i\right)^{1-\alpha_i} \left(\rho^T e_i x_i^{\gamma}\right)^{\alpha_i} \quad s.t. \quad \pi_i \geq 0, \quad i = L, H, \tag{9}$$

where $\alpha_i \in [0, 1]$. When $\alpha_i = 0$ we have the standard case of a profit-maximizer firm; when $\alpha_i = 1$ we have the opposite case of a non-profit firm who simply wants to maximize the positive impact of its clean-up under the constraint of non-negative profit.[8] In general, a responsible firm i pursues two different objectives simultaneously and α_i is a parameter signaling the relative importance of the two objectives. More specifically, α_i can be interpreted as a measure of the degree of (altruistic) CSR of firm i. As explained in De Donder and Roemer (2009), such objective can be interpreted as the weighted Nash bargaining solution of an efficient negotiation between two different factions inside the firm: one aiming at maximizing profit and the other aiming at maximizing the positive externality associated to firm's production. Such interpretation is correct if i) the no agreement pay-offs are $(0, 0)$, as happens when a firm is part of the competitive fringe, that does not obtain any extra-profit and does not produce any positive externality, ii) the objective function of each faction is log-concave in firm's strategic choices of p and e (for our case this property is proved in the technical appendix). According to such interpretation, α_i represents the relative bargaining power of the faction supporting the abatement activity inside the firm i.

[8] We assume that when $\alpha_i = 1$ firm i maximizes the positive impact of its abatement activity even if its profit is equal to 0.

2.5 Firms' competition

We model competition between the two responsible firms according to the usual framework adopted in duopoly models of vertical differentiation. There are two stages: in the first one, the two firms simultaneously choose the clean-up level, that can be defined as the (environmental) quality of their product. In the second stage the two firms observe the choice of their competitor and simultaneously set the price. We know that when the lowest consumers' WTP is equal to 0, in equilibrium arises an uncovered market configuration (see Liao, 2008, and Ecchia and Lambertini, 1998). This means that in equilibrium a group of green consumers buys a standard good from the competitive fringe.

The market share of each firm can be calculated by identifying $\hat{\theta}$, the consumer which is indifferent between the high or the low quality product, and $\tilde{\theta}$, that which is indifferent between the low or the null quality product. Straightforward algebra, using equation (7), it is easy to see that: $\hat{\theta} = \frac{p_H - p_L}{e_H - e_L}$ and $\tilde{\theta} = \frac{p_L}{e_L}$. As known, in a vertically differentiated duopoly, the high (low) quality firm sells to green consumers included in $\left[\hat{\theta}, \bar{\theta}\right]$ $\left(\left[\tilde{\theta}, \hat{\theta}\right]\right)$. Then each firm's market share is:

$$x_H = \frac{\beta}{\bar{\theta}}\left[\bar{\theta} - \frac{p_H - p_L}{e_H - e_L}\right], x_L = \frac{\beta}{\bar{\theta}}\left[\frac{p_H - p_L}{e_H - e_L} - \frac{p_L}{e_L}\right], x_0 = 1 - x_H - x_L, \quad (10)$$

where x_0 is the total quantity sold by firms of the competitive fringe.

We apply the standard backward induction methodology by first analyzing the price equilibrium and then the environmental quality equilibrium.

3 Fixed costs of clean-up

3.1 The market equilibrium

In this section we study the case of fixed costs of clean-up, i.e. $\gamma = 0$. By using equations (4) and (9) we can specify the objective function of the responsible firms as:

$$J_i = \left(p_i x_i - \frac{k}{2}e_i^2\right)^{1-\alpha_i}\left(\rho^T e_i\right)^{\alpha_i} \quad s.t. \quad \pi_i \geq 0, \quad i = L, H. \quad (11)$$

It is worth noting that in case of fixed costs the generalization of the firms' objective function has no consequence on the price-setting stage. Indeed, at the second stage the abatement activity is considered as exogenous, and so the responsible firms choose their price strategy in order to maximize only their revenues, whatever their degree of CSR is. So the price equilibrium can be found by solving simultaneously the revenue maximization of both firms. As shown in the existing literature (Motta, 1993; Arora and Gangopadhyay, 1995), at this stage the unique Nash equilibrium is characterized by the following equations:

$$p_H^* = 2\bar{\theta} e_H \frac{e_H - e_L}{4e_H - e_L},$$

$$p_L^* = \bar{\theta} e_L \frac{e_H - e_L}{4e_H - e_L},$$

yielding profits:

$$\pi_H = 4\beta\bar{\theta} e_H^2 \frac{(e_H - e_L)}{(4e_H - e_L)^2} - \frac{k}{2} e_H^2; \tag{12}$$

$$\pi_L = \beta\bar{\theta} e_H e_L \frac{(e_H - e_L)}{(4e_H - e_L)^2} - \frac{k}{2} e_L^2. \tag{13}$$

In order to identify the duopolists' maximization problem at the first stage, equations (12) and (13) are substituted in equation (11). The equilibrium levels of clean-up corresponds to the solutions that solve simultaneously[9] the following unconstrained[10] maximization problems:[11]

$$\max_{e_H} \left[4\beta\bar{\theta} e_H^2 \frac{(e_H - e_L)}{(4e_H - e_L)^2} - \frac{k}{2} e_H^2\right]^{1-\alpha_H} \left(\rho^T e_H\right)^{\alpha_H};$$

$$\max_{e_L} \left[\beta\bar{\theta} e_H e_L \frac{(e_H - e_L)}{(4e_H - e_L)^2} - \frac{k}{2} e_L^2\right]^{1-\alpha_L} \left(\rho^T e_L\right)^{\alpha_L}.$$

The first order conditions require:

[9] As explained by Motta (1993), the solutions of this system are only the candidate equilibrium of the model. In the technical appendix we show that second order conditions hold, and consequently every solution represents effectively a local maximum. Moreover, we have checked that the firm choosing the high (low) quality has no incentive to "leapfrog" the rival firm and itself produce the lowest (highest) quality.

[10] We neglect the constrain that the firms' profit must be positive as we will verify that such condition is always satisfied in equilibrium.

[11] We use α_H and α_L to indicate the degree of CSR of the firms producing the *high* and the *low* level of clean-up. However, it is important to emphasize that we do not restrict the relative size of their degree of CSR.

$$\frac{\partial J_H}{\partial e_H} = 0 \Leftrightarrow \beta\bar{\theta}\frac{4e_H^3 - (3 + 2\alpha_H)e_H^2 e_L + (2 - \alpha_H)e_H e_L^2}{(4e_H - e_L)^3} = \frac{2 - \alpha_H}{8} k e_H; \quad (14)$$

$$\frac{\partial J_L}{\partial e_L} = 0 \Leftrightarrow \beta\bar{\theta}\frac{4e_H^3 - (7 - 2\alpha_L)e_H^2 e_L + \alpha_L e_H e_L^2}{(4e_H - e_L)^3} = \frac{2 - \alpha_L}{2} k e_L. \quad (15)$$

In order to study whether firms' abatement activities are strategically substitutes or complements we have to investigate the sign of the following cross derivatives:

$$\frac{\partial^2 J_H}{\partial e_H \partial e_L} = 4\beta\bar{\theta}\frac{-8\alpha_H e_H^2 + (10 - 12\alpha_H)e_H e_L + (2 - \alpha_H)e_L^2}{(4e_H - e_L)^4}; \quad (16)$$

$$\frac{\partial^2 J_L}{\partial e_L \partial e_H} = \beta\bar{\theta}e_L\frac{(16 - 8\alpha_L)e_H^2 + (14 - 12\alpha_L)e_H e_L - \alpha_L e_L^2}{(4e_H - e_L)^4}. \quad (17)$$

Straightforward, the cross derivatives of each firm's objective function is decreasing in its own degree of CSR. However, the cross derivative of firm L (eq. 17) is always positive (given that $\alpha_L \leq 1$): in equilibrium the optimal abatement for the low quality firm is always increasing in the abatement chosen by its rival. Conversely, the cross derivatives of firm H (eq. 16) is strictly positive for $\alpha_H = 0$, while is strictly negative when $\alpha_H = 1$. Hence, the best response of the high quality firm can be both increasing and decreasing in the abatement level chosen by its rival, depending on its own degree of CSR (and on the equilibrium levels of firms' abatement).

Following the definition of Bulow et *al.* (1985), if α_H is quite low, then e_H and e_L are strategic complements, while for higher values of α_H, we have neither strategic complementarity nor strategic substitutability at the second stage because the slopes of the two reaction functions have different sign.

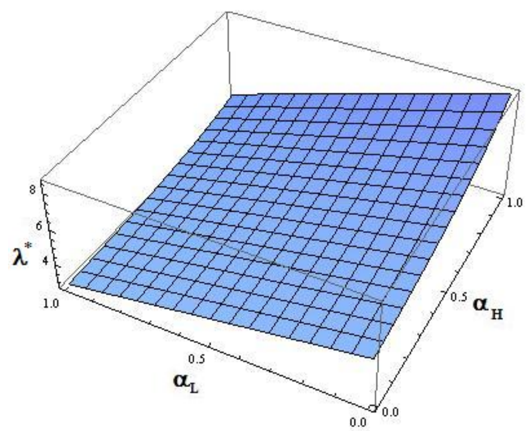
The solutions of the system given by equations (14) and (15) can be found making the ratio between them. We obtain:[12]

$$\begin{aligned} &4(2 - \alpha_H)\lambda^3 - (46 - 20\alpha_L - 7\alpha_H + 2\alpha_L\alpha_H)\lambda^2 + \\ &\qquad + (24 - 10\alpha_L + 16\alpha_H - 9\alpha_L\alpha_H)\lambda - 4(2 - \alpha_L)(2 - \alpha_H) = 0, \end{aligned}$$

[12] This equation is a generalization of equation (7) of Motta (1993, p. 117) to the case in which firms aim at maximizing not only their profit but also the positive externality implicitly associated to their production.

where λ is equal to $\frac{e_H}{e_L}$. λ can be interpreted as the degree of (environmental) differentiation. This equation has a unique acceptable solution $\lambda^* = g(\alpha_H, \alpha_L)$. In Figure 1 we show the three dimensional plot of λ^*. It is monotonically increasing (decreasing) in α_H (α_L), $\forall \alpha_H, \alpha_L \in [0, 1]$: hence, the higher the degree of CSR of the firm H (L) the higher (lower) the environmental differentiation. It's worth noting that λ^* has a maximum in $g(1, 0) = 8, 6164$ and a minimum in $g(0, 1) = 2, 7452$.

Figure 1: $\lambda^* = g(\alpha_H, \alpha_L)$



Substituting e_H with $\lambda^* e_L$ in (15) we achieve the equilibrium level of clean-up of both firms as a function of $\lambda^*, \alpha_L, \beta, \bar{\theta}, k$:

$$e_L^* = \frac{2\beta\bar{\theta}}{(2-\alpha_L)k} \frac{\lambda^*[4(\lambda^*)^2 - (7-2\alpha_L)\lambda^* + \alpha_L]}{(4\lambda^* - 1)^3};$$

$$e_H^* = \frac{2\beta\bar{\theta}}{(2-\alpha_L)k} \frac{(\lambda^*)^2[4(\lambda^*)^2 - (7-2\alpha_L)\lambda^* + \alpha_L]}{(4\lambda^* - 1)^3}.$$

By recalling and rearranging the equations (6) and (8) we obtain:

$$e_L^* = \frac{4\mu\lambda^*[4(\lambda^*)^2 - (7-2\alpha_L)\lambda^* + \alpha_L]}{(2-\alpha_L)(4\lambda^* - 1)^3} e^{FB}; \tag{18}$$

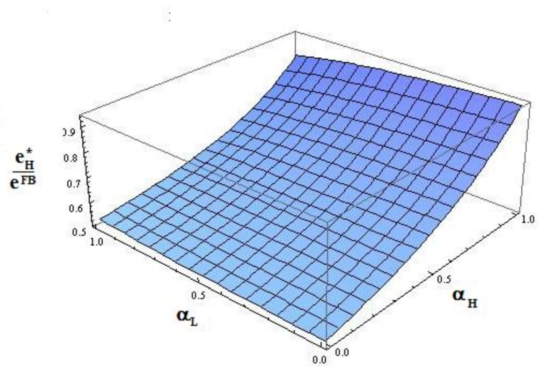
$$e_H^* = \frac{4\mu(\lambda^*)^2[4(\lambda^*)^2 - (7-2\alpha_L)\lambda^* + \alpha_L]}{(2-\alpha_L)(4\lambda^* - 1)^3} e^{FB}. \tag{19}$$

Trivially, if all consumers are radical free riders (i.e.: μ = 0), firms will not employ a cleaning technology, whatever their objective function is. It's worth noting that in this model in order to have an abatement activity in equilibrium the presence of green consumers is both necessary and sufficient. On the other hand, the mere existence of responsible firms is not sufficient.

Lemma 1. *In the presence of green consumers (i.e.: if $\mu > 0$), $\forall \alpha_H, \alpha_L, \mu \in [0,1], 0 < e_L^* < e_H^* < e^{FB}$.*

Proof. In Figure 2 we report the ratio[13] e_H over e^{FB} calculated by means of equation (19). It can be easily seen that it is always positive but less than 1. Moreover, given that λ^* is always strictly higher than 1, e_L^* is always less than e_H^* for $\mu > 0$.

Figure 2: e_H^*/e^{FB}



□

This lemma allows us to conclude that, even if consumers and producers were fully responsible, the market equilibrium will not correspond to the first best allocation: both the responsible firms never adopt an efficient level of clean-up. Moreover the allocation of the aggregate abatement is not cost-effective because in equilibrium the two responsible firms never adopt the same level of clean-up and so their marginal costs differ.

[13]Figure 2 is plotted under the assumption that $\mu = 1$, so it indicates the maximum values of the ratio e_H over e^{FB}.

In order to analyze how the degree of responsibility of both consumers and producers affects the overall efficiency of the abatement activity we can now conduct some comparative statics. Note that given equations (18) and (19) there is a positive relationship between the social capital index and the equilibrium level of clean-up of both the responsible firms.

Lemma 2. *In the presence of green consumers (i.e.: if $\mu > 0$):*

1. *e_L^* is monotonically increasing in α_L and α_H, $\forall \alpha_H, \alpha_L \in [0, 1]$;*

2. *e_H^* is monotonically increasing in α_H, while it is monotonically increasing (decreasing) in α_L if α_H is close to 0 (1). For intermediate values of α_H, e_H is not monotone in α_L;*

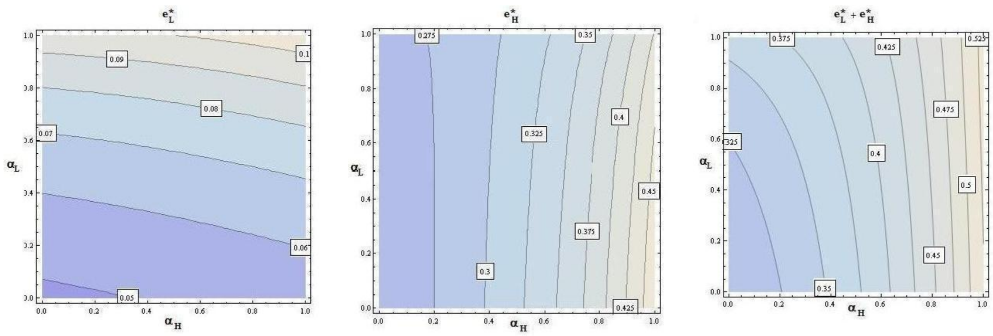
3. *$E^{T*} = e_H^* + e_L^*$ is monotonically increasing in α_L and α_H, $\forall \alpha_H, \alpha_L \in [0, 1]$.*

Proof. The influence of the degree of CSR of both the responsible firms on their equilibrium levels of abatement are proved by means of the contour plots shown in Figure 3. The left-hand side shows that e_L^* has iso-curves negatively sloped and that it reaches its maximum value in the point $(\alpha_H = 1, \alpha_L = 1)$. This means that e_L^* is monotone increasing in both α_H and α_L. On the other hand, the central contour plot shows that the iso-curve of e_H^* are decreasing in correspondence of low values of α_H and increasing for high values of α_H. Moreover, e_H^* reaches its maximum value in correspondence of the point $\alpha_H = 1, \alpha_L = 0$. This prove that a higher α_H always implies a higher e_H^*, while the influence of α_L on e_H^* depends on the value of α_H. Finally, the right-hand side shows that the sum of firms' abatement activities in equilibrium is monotone increasing in both α_H and α_L. Indeed, the iso-curves are negatively sloped and the maximum is reached in the point $(\alpha_H = 1, \alpha_L = 1)$.

□

Therefore increments in the degree of responsibility of a firm always entails an increase of its own abatement activity. On the other hand increments in the degree of responsibility of the rival firm may not have a clear-cut effect for both firms. Indeed, it is always true that the higher α_H, the higher e_L^*, while an increment of α_L may both increase and decrease e_H^*, depending on the level of α_H. This result is due to the different sign that the cross derivative of J_H can assume. As seen above, when firm H carries out a sufficiently high (low) degree of CSR, then its best response is decreasing (increasing) in the level of clean-up of firm L. As a consequence, given that an increment of α_L increases e_L^*, we have that when α_H

Figure 3: Contour Plot of e_L^* and e_H^*



Note: the lighter colors are associated with higher values.

is sufficiently high (low) e_H^* is decreasing (increasing) in α_L. However, the total level of abatement is monotonically increasing in the degree of CSR of each firm.

3.2 The social welfare

In the case of fixed costs of clean-up, the social welfare defined in equation (5) can be rewritten as:

$$W = \sum_{i=H,L} \rho^T e_i^* - \frac{k}{2}(e_i^*)^2. \tag{20}$$

Proposition 1. *The social welfare is monotonically increasing in* μ*,* α_L *and* α_H*,* $\forall\, \mu$*,* α_H*,* $\alpha_L \in [0, 1]$*.*

Proof. We can write the variation of W with respect to a generic exogenous parameter z as:

$$\frac{\partial W}{\partial z} = \sum_{i=H,L} (\rho^T - k e_i^*) \frac{\partial e_i^*}{\partial z}. \tag{21}$$

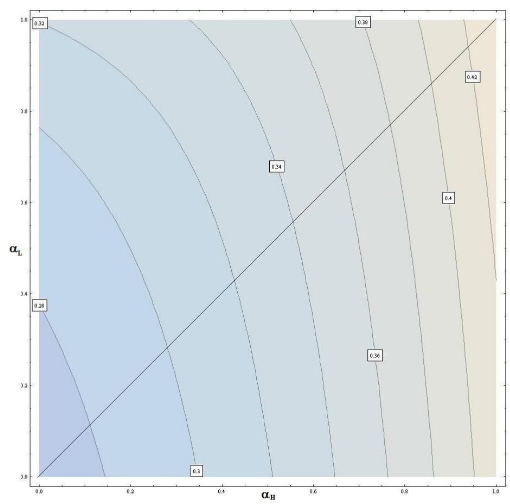
Given Lemma 1, $e_i^* < e^{FB} \Leftrightarrow \rho^T - k e_i^* > 0$. Therefore, the sign of the derivatives of the social welfare with respect to an exogenous parameter will depend only on how such parameter affects the equilibrium level of clean-up of both firms. If $\frac{\partial e_i^*}{\partial z}$ has the same sign, $\forall i = L, H$, then also $\frac{\partial W}{\partial z}$ will have that sign.

Therefore, given equations (18) and (19) and Lemma 2, the social welfare is everywhere increasing in μ and α_H. As far as α_L is concerned, rearranging equation 21 we obtain:

$$\frac{\partial W}{\partial z} = (\rho^T - ke_L^*)\frac{\partial E^{T*}}{\partial z} + k(e_H^* - e_L^*)\frac{\partial e_L^*}{\partial z}. \tag{22}$$

Applying Lemma 2 to formula 22 we can conclude that the social welfare is monotone increasing also in α_L. The following contour plot (Figure 4) confirms that the social welfare has its global maximum in ($\alpha_H = 1$, $\alpha_L = 1$).

Figure 4: Contour Plot of $W(\alpha_H, \alpha_L)$



□

This result is in line with Garcia Gallego and Georgantzis (2009), who had already shown that, in the uncovered market configuration, the social welfare is increasing in consumers' WTP. However, in this paper we show that the social welfare is also increasing in the degree of CSR of both the responsible firms.

Corollary 1. *The highest social welfare is attained when* $\mu = 1$, $\alpha_L = 1$ *and* $\alpha_H = 1$*, but it does not correspond to the first best solution.*

Proof. Proposition 1 ensures that the maximum welfare associated to the market equilibrium is reached when both μ, α_L and α_H have their maximum value. In such case, the social welfare calculated by means of equation (20) is equal to

$0,4366\frac{(\rho^T)^2}{k}$. However, if the level of abatement of both the responsible firms was $\frac{\rho^T}{k}$, i.e. its first best level, then the social welfare would be equal to $\frac{(\rho^T)^2}{k}$. Therefore in this case the social welfare attained in equilibrium is only the $43,66\%$ of its first best level. $\square$

Therefore, the market equilibrium never achieves the first best solution, even if both consumers and producers behave as fully responsible agents. The inefficiency of the market equilibrium is due to two different reasons: firstly, the clean-up of both the responsible firms is always lower than the first best level and consequently total abatement is below its optimal level. Moreover, the allocation of total abatement is not cost effective because it is not equally shared between the two responsible firms, whatever their degree of altruistic CSR is (both these facts are emphasized in Lemma 1).

To sum up, when the clean-up activity of the responsible firms is only implicitly linked to their production, the efficient level of the abatement is never reached in equilibrium (Corollary 1). However, an increase of both firms' CSR and consumers' WTP have always a positive effect on the social welfare (Proposition 1). In the next section we shall show that even this result is not guaranteed when the clean-up activity is explicitly associated to the production level (i.e. when both benefits and costs of the cleaning technology are variable).

Proposition 2. *Let us assume that* $\alpha_H = \alpha'_L > \alpha_L = \alpha'_H$*; then* $W(\alpha_H, \alpha_L) > W(\alpha'_H, \alpha'_L)$, $\forall \alpha_H, \alpha_L \in [0,1]$.

Proof. Let us analyze the contour plot of the Welfare (Figure 4). It is straightforward that if we consider both a point such that $\alpha_H > \alpha_L$ and its symmetric point with respect to the $45°$ line, the former is always associated to a higher social welfare than the latter. $\square$

The existing literature regarding vertically differentiated duopolies has already stressed the existence of two *asymmetric* and *specular* Nash equilibria at the quality stage. However, in the present model these two equilibria are no more *specular* (if $\alpha_H \neq \alpha_L$) given the differences in the firms' objective functions. Proposition 2 states that the social welfare is always higher in the equilibrium in which the high quality firms is at the same time the firm with the highest degree of CSR.

Thanks to Propositions 1 and 2 we can compare a standard duopoly, where the firms are both profit maximizers, with a mixed duopoly, where a non-profit producer competes with a profit maximizer firm. When the costs of the abatement activity are only implicitly linked to the production level, then the presence of a non-profit firm is always welfare improving, and from the social welfare standpoint it is preferable the equilibrium in which the non-profit firm carries out the highest level of clean-up.

4 Variable costs of clean-up

4.1 The market equilibrium

In this section we study the case of variable costs of clean-up, i.e. $\gamma = 1$. Firms' objective function is:

$$J_i = \left(\pi_i\right)^{1-\alpha_i} \left(\rho^T e_i x_i\right)^{\alpha_i}, \quad i = L, H, \tag{23}$$

where:

$$\pi_i = \left(p_i - \frac{k}{2} e_i^2\right) x_i, \quad i = L, H, \tag{24}$$

and the market shares of each firm are still given by (10). In this case the prices affect not only firms' profit but also the size of their positive externality. Therefore, equilibrium prices are now dependent on the degree of CSR of both firms.

By computing the first derivatives of J_H and J_L with respect to prices and then solving the system we obtain the following equations for the equilibrium prices:

$$p_H^* = \frac{e_H \left[(2-\alpha_L) k e_H^2 + (1-\alpha_H) k e_L^2 + 2\bar{\theta}(1-\alpha_H)(2-\alpha_L)(e_H - e_L)\right]}{2[(2-\alpha_H)(2-\alpha_L)e_H - (1-\alpha_H)(1-\alpha_L)e_L]},$$

$$p_L^* = \frac{e_L[(1-\alpha_L) k e_H^2 + (2-\alpha_H) k e_L e_H + 2\bar{\theta}(1-\alpha_H)(1-\alpha_L)(e_H - e_L)]}{2[(2-\alpha_H)(2-\alpha_L)e_H - (1-\alpha_H)(1-\alpha_L)e_L]},$$

yielding profits:

$$\pi_H = (1-\alpha_H)(e_H - e_L) x_H^2 \frac{\bar{\theta}}{\beta}, \tag{25}$$

$$\pi_L = (1-\alpha_L)(e_H - e_L) \frac{e_L}{e_H} x_L^2 \frac{\bar{\theta}}{\beta}, \tag{26}$$

where:

$$x_H = \frac{\beta e_H\big[(2-\alpha_L)(2\bar{\theta} - ke_H) - ke_L)\big]}{2\bar{\theta}[(2-\alpha_H)(2-\alpha_L)e_H - (1-\alpha_H)(1-\alpha_L)e_L\big]}, \tag{27}$$

$$x_L = \frac{\beta e_H\big[(1-\alpha_H)(2\bar{\theta} - ke_L) + ke_H)\big]}{2\bar{\theta}[(2-\alpha_H)(2-\alpha_L)e_H - (1-\alpha_H)(1-\alpha_L)e_L\big]}. \tag{28}$$

We can now include equations (25) and (26) in the generic equation (23) and derive the first order conditions of the first stage obtaining the following system of equations:

$$\frac{\partial J_H}{\partial e_H} = 0 \Leftrightarrow (2-\alpha_H)(e_H - e_L)\frac{\partial x_H}{\partial e_H} + \frac{e_H - \alpha_H e_L}{e_H} x_H = 0; \tag{29}$$

$$\frac{\partial J_L}{\partial e_L} = 0 \Leftrightarrow (2-\alpha_L)(e_H - e_L)\frac{e_L}{e_H}\frac{\partial x_L}{\partial e_L} + \frac{e_H - 2e_L + \alpha_L e_L}{e_H} x_L = 0. \tag{30}$$

Moorthy (1988) has already shown that when vertically differentiated firms behave as profit maximizers, their reaction functions are both positively sloped and so their quality choices are strategic complements. However, it is easy to check that if the high quality firm is a non-profit firm, i.e. when $\alpha_H = 1$, its best response function is negatively sloped[14] while the best response function of firm L is still positively sloped. Therefore we may not record neither strategic complementarity nor strategic substitutability, as in the fixed costs case.

The system given by equations (29) and (30) can have at maximum one acceptable solution (i.e.: such that $e_H^* \geq e_L^*$). When one solution exists, such solution corresponds to the equilibrium levels of marginal clean-up of both firms[15], which depends on the parameters α_H and α_L. The solutions in the closed form are not analytically feasible. However, some clear results emerges when one of the duopolists is a non-profit firm and it produces either the high-quality or the low-quality good.

Proposition 3. *If $\alpha_L = 1$, then at the first stage no Nash equilibrium exists.*

[14]Indeed, in such case the cross derivative of the objective function of firm H is equal to $-\frac{\beta k}{2\bar{\theta}(2-\alpha_L)}$.

[15]In the technical appendix it is shown that in correspondence of the unique acceptable solution second order conditions hold also in the case of variable costs. However, not all the candidate solutions are valid because, as reported in the technical appendix, in some cases some firm has an incentive to leapfrog the "equilibrium" level of marginal abatement of its rival.

Proof. In this case the market shares of the responsible firms can be rewritten by substituting $\alpha_L = 1$ in formula (27) and (28). We obtain:

$$x_H = \frac{\beta(2\bar{\theta} - ke_L - ke_H)}{2\bar{\theta}(2 - \alpha_H)}, \tag{31}$$

$$x_L = \frac{\beta[(1 - \alpha_H)(2\bar{\theta} - ke_L) + ke_H]}{2\bar{\theta}(2 - \alpha_H)}. \tag{32}$$

The derivatives of J_L with respect to e_L is equal to:

$$\frac{\partial J_L}{\partial e_L} = e_L \frac{\partial x_L}{\partial e_L} + x_L = 2(1 - \alpha_H)(\bar{\theta} - ke_L) + ke_H \tag{33}$$

From equation (31) we can deduce that $x_H \geq 0 \Leftrightarrow ke_L \leq \bar{\theta}$. This implies that equation (33) is always strictly positive. Consequently, at the first stage the firm L would choose $e_L = e_H$, the maximum level of marginal abatement under the constraint that e_L must be weakly lower than e_H. On the other hand, if $\alpha_H < 1$, the firm H always would choose $e_H > e_L$, because if $e_H = e_L$ its profit is equal to 0. □

Therefore, when one of the two responsible producers is a non-profit firm, there is no equilibrium in which it chooses the low level of marginal abatement. Indeed, in such case the firm L would mimic the choice of its competitor, while the firm H would prefer level of marginal abatement strictly higher than the level of its rival.

As a consequence, in the presence of a non-profit firm, the only Nash equilibrium can be characterized by substituting $\alpha_H = 1$ in equations (29) and (30).

Lemma 3. *If $\beta > 0$, $\bar{\theta} > 0$, and $\alpha_H = 1$, then in equilibrium:*

1. *e_L^* is monotonically increasing in both $\bar{\theta}$ and α_L, decreasing in k and independent of β;*

2. *x_L^* is monotonically increasing in both β and α_L and independent of both $\bar{\theta}$ and k;*

3. *e_H^* is monotonically increasing in $\bar{\theta}$, decreasing in both k and α_L and independent of β;*

4. *x_H^* is monotonically increasing in β, decreasing in α_L and independent of both $\bar{\theta}$ and k;*

5. *$E^{T*} = e^*_H x^*_H + e^*_L x^*_L$ is monotonically increasing in both β and $\bar{\theta}$ and decreasing in both k and α_L.*

Proof. Note that if $\beta > 0$, $\bar{\theta} > 0$, $\alpha_H = 1$ and $\alpha_L \in [0, 1)$ then the solution of the system identifies the following equilibrium levels of marginal abatement:

$$e^*_L = \frac{2(2-\alpha_L)}{9-8\alpha_L+2\alpha_L^2}\frac{\bar{\theta}}{k}; \tag{34}$$

$$e^*_H = \frac{2(2-\alpha_L)^2}{9-8\alpha_L+2\alpha_L^2}\frac{\bar{\theta}}{k}. \tag{35}$$

Substituting these solutions in equations (27) and (28), the following equilibrium market shares are achieved:

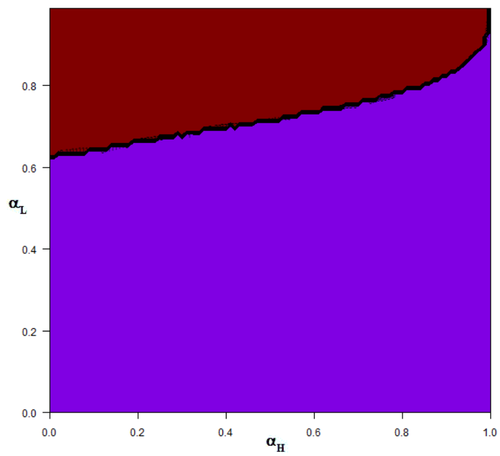
$$x^*_L = \frac{\beta(2-\alpha_L)}{9-8\alpha_L+2\alpha_L^2} \tag{36}$$

$$x^*_H = \frac{\beta(2-\alpha_L)^2}{9-8\alpha_L+2\alpha_L^2} \tag{37}$$

Straightforward algebra the parameters β, $\bar{\theta}$ and k affect e^*_L, x^*_L, e^*_H, x^*_H and E^{T*} in the way stated in the lemma. As far as the impact of the degree of CSR of the low-quality firm on firms' marginal abatement choices and on their market shares, the results stated in the proposition stem from the following derivatives: $\frac{\partial e^*_L}{\partial \alpha_L} = \frac{2\bar{\theta}}{\beta k}\frac{\partial x^*_L}{\alpha_L} > 0$; $\frac{\partial x^*_L}{\partial \alpha_L} = \beta\frac{2\alpha_L^2-8\alpha_L+7}{(9-8\alpha_L+2\alpha_L^2)^2} > 0$; $\frac{\partial e^*_H}{\partial \alpha_L} = \frac{2\bar{\theta}}{\beta k}\frac{\partial x^*_H}{\alpha_L} < 0$; $\frac{\partial x^*_H}{\partial \alpha_L} = -\beta\frac{2(2-\alpha_L)}{(9-8\alpha_L+2\alpha_L^2)^2} < 0$; $\frac{\partial E^{T*}}{\partial \alpha_L} = \frac{4\beta\bar{\theta}}{2k}\frac{2\alpha_L-4}{(9-8\alpha_L+2\alpha_L^2)^4} < 0$. □

In the presence of a non-profit firm, the unique Nash equilibrium is characterized by the fact that the other firm always adopts the lowest level of marginal abatement. Moreover, the higher the weight assigned by the low quality firm to its CSR, the higher its positive externality is. However, this effect is counterbalanced by the reduction of the positive externality of the high quality firm. This result is coherent with the slope of the best response function of firm H. If firm L becomes more careful with the environmental impact of its production, it will increase its level of abatement, but at the same time, it will reduce its mark-up, increasing its supply. On the other hand, firm H cannot reduce its price without reducing its level of clean-up because it always charges a price equal to its marginal costs. Hence, firm H finds it convenient to decrease its level of abatement in order to

Figure 5: Nash equilibrium existence



limit the reduction of its market share. The last statement shows us that the aggregate effect of an increase in α_L entails a reduction of the total abatement.

More general results regarding the market equilibrium in the presence of two responsible firms can be obtained by means of numerical simulations. The shaded region of Figure 5 identifies the set of couples (α_H, α_L) in which a Nash equilibrium (in pure strategies) does to exist: firm H would have an incentive to leapfrog[16] firm L by choosing a level of abatement lower than $e_L^*(\alpha_H, \alpha_L)$ in order to increase its market share and its total abatement. Consequently, at the first stage we can have zero, one or two Nash equilibria, depending on the weight that both the responsible firms assign to their profit. Indeed, making the symmetry of the shaded region with respect to the bisectrix we can identify three different regions. When the shaded region and its symmetric region coincide no Nash equilibrium exists. For all the other couples of values of (α_i, α_j) contained in the shaded region only a Nash equilibrium exists, in which $\alpha_H > \alpha_L$. Finally, for the couples of (α_i, α_j) contained neither in the shaded region nor in its symmetric counterpart, two Nash equilibria exist.

Hence, if both firms assign a high weight to the positive externality associated to their own production, the outcome of their strategic interaction is unpredictable. However, when a Nash equilibrium exists, we can analyze through numerical cal-

[16]The numerical calculations are available upon request.

culations how the market equilibrium is affected by consumers' and firms' degree of responsibility. It is worth noting that both e_L^* and e_H^* are linearly increasing in $\frac{\bar{\theta}}{k}$ and independent of β, while both x_L^* and x_H^* are linearly increasing in β and independent of both $\bar{\theta}$ and k. As far as the impact of firms' degree of responsibility on market equilibrium we resume the main observations in the following lemma:

Lemma 4. *When at least a Nash equilibrium exists, the following properties hold:*

1. e_L^* *is monotonically increasing in* α_L*, while it is increasing (decreasing) in* α_H *only when* α_L *is sufficiently low (high). For intermediate values of* α_L*,* e_L^* *is (almost) constant.* x_L^* *is monotone increasing in* α_L *and monotone decreasing in* α_H*. Finally,* $e_L^* x_L^*$ *is monotone increasing in* α_L *and monotone decreasing in* α_H*.*

2. e_H^* *is monotonically increasing (decreasing) in* α_H *only when* α_L *is sufficiently low (high). For intermediate values of* α_L*,* e_H^* *is first increasing and then decreasing. At the same time,* e_H^* *is monotonically increasing (decreasing) in* α_L *only when* α_H *is sufficiently low (high). For intermediate values of* α_H*,* e_H^* *is first decreasing and then increasing.* x_H^* *is monotone increasing in* α_H *and monotone decreasing in* α_L*. Finally,* $e_H^* x_H^*$ *is monotone increasing in* α_H *and monotone decreasing in* α_L*.*

3. $E^{T*} = e_H^* x_H^* + e_L^* x_L^*$ *is monotone increasing (decreasing) in* α_H *if* α_L *is sufficiently low (high) and monotone increasing (decreasing) in* α_L *if* α_H *is sufficiently low (high).*

4. $\lambda^* = e_H^*/e_L^*$ *is always monotone decreasing in* α_L *while it is monotone decreasing in* α_H *only if* α_L *is sufficiently high. When* α_L *is low,* λ^* *is first increasing and then decreasing in* α_H*.*

Proof. See the 3D plots in Figure 6. □

The equilibrium levels of marginal abatement are affected in several ways by firms' degree of responsibility. At the same time, both firms' market share and total abatement are always increasing in their own degree of CSR, and decreasing in the degree of responsibility of the rival firm. The overall impact on the aggregate level of clean-up is ambiguous: an increase in the CSR of firm L (H) can either increase or decrease the aggregate clean-up depending on the CSR of firm H (L). Finally, an increase in the degree of responsibility of firm L makes the allocation of the abatement activity more cost effective (i.e. it decreases the value of λ^*), while an increase in the degree of responsibility of firm H does not have a clear-cut effect on λ^*.

Lemma 5. *When at least a Nash equilibrium exists, $e^*_H \lesseqgtr e^{FB}$ and $e^*_L \lesseqgtr e^{FB}$, depending on the specific values of $\bar{\theta}, \alpha_L, \alpha_H$. At the same time, $\rho^T - \frac{k}{2}e^*_i > 0$, $\forall i = L, H$, $\forall \bar{\theta}, \alpha_L, \alpha_H$.*

Proof. In Figure 6 we set parameters as follows: β, $\bar{\theta}$ and k are equal 1, and there are couples (α_H, α_L) for which e^*_H and/or e^*_L are higher than 0.5. As a consequence, given the linear proportionality to $\bar{\theta}$, when maximum consumers' WTP is close to its maximum value (i.e. $2\rho^T$) both the responsible firms may exert a level of abatement higher than the first best level (i.e. $\frac{\rho^T}{k}$). At the same time, the equilibrium levels of clean-up do not assume a value higher than $\frac{\bar{\theta}}{k}$. Hence, the inequality contained in the lemma follows. □

Despite the fixed costs case, when the costs are variable in equilibrium both firms may choose to exert an inefficiently high level of abatement (i.e. higher than the first best level). Hence, the statements of Lemma 4 and 5 do not help us to draw any clear inference with regard to the influence of a higher degree of responsibility on the social welfare. Indeed, a higher green consumers' WTP always increases the aggregate abatement, but it may induce the responsible firms to adopt an inefficiently high level of marginal abatement. At the same time, an increase in the degree of CSR of one firm has not a clear-cut effect on the aggregate abatement and on the cost-effectiveness of the allocation of such activity.

4.2 The social welfare

In the case of variable costs of clean-up, the social welfare defined in equation (5) can be rewritten as:

$$W = \sum_{i=H,L} \left[\rho^T e^*_i - \frac{k}{2}(e^*_i)^2 \right] x^*_i. \tag{38}$$

Proposition 4. *The social welfare is monotone increasing in β while its partial (first) derivative with respect to $\bar{\theta}$ is never monotone.*

Proof. The variation of W with respect to a generic exogenous parameter z is:

$$\frac{\partial W}{\partial z} = \sum_{i=H,L} \left[(\rho^T - ke^*_i)x^*_i \frac{\partial e^*_i}{\partial z} + (\rho^T - \frac{k}{2}e^*_i)e^*_i \frac{\partial x^*_i}{\partial z} \right].$$

Consequently, $\frac{\partial W}{\partial \beta}$ is surely positive given that $\frac{\partial e_i^*}{\partial \beta} = 0$, $(\rho^T - \frac{k}{2}e_i^*) > 0$ (see Lemma 5) and $\frac{\partial x_i^*}{\partial \beta} > 0, \forall i = L, H$.

Given that $\frac{\partial x_i^*}{\partial \bar{\theta}} = 0$ and $\frac{\partial^2 e_i^*}{\partial \bar{\theta}^2} = 0$ we obtain that:

$$\frac{\partial^2 W}{\partial \bar{\theta}^2} = -k \sum_{i=H,L} x_i^* \left(\frac{\partial e_i}{\partial \bar{\theta}} \right)^2 < 0$$

Hence, W is concave in $\bar{\theta}$. Furthermore, as $\frac{\partial e_i^*}{\partial \bar{\theta}} > 0$ and $\frac{\partial x_i^*}{\partial \bar{\theta}} = 0$, we can note that the sign of $\frac{\partial W}{\partial \bar{\theta}}$ depends on the sign of $(\rho^T - ke_i^*)$, $\forall i = L, H$. Consequently, when $\bar{\theta}$ is close to 0 $(2\rho^T)$ both e_H^* and e_L^* are lower (higher) than e^{FB} and W is increasing (decreasing) in $\bar{\theta}$. Therefore, W is never monotone in $\bar{\theta}$. □

As far as the pattern of W with respect to α_L and α_H, it is not feasible to derive analytical results. Using numerical simulations, we show that W can be both increasing and decreasing in firms' CSR, depending on the specific values of α_L, α_H and $\bar{\theta}$ (see Figure 7).

Therefore, in the variable costs case, the impact of α_H and α_L on the social welfare depends crucially on their values and on the value of $\bar{\theta}$. In some cases a higher degree of responsibility of one firm can harm the social welfare. For instance, the social welfare turn out to be monotonically decreasing in α_L when α_H is very high and $\bar{\theta}$ is very low. At the same time, the social welfare is monotonically decreasing in α_H when α_L is very low and $\bar{\theta}$ is very high.

Proposition 5. *The highest social welfare is attained when $\beta = 1$, $\bar{\theta} \simeq 1,463\rho^T$, $\alpha_H = 1$ and $\alpha_L \simeq 0,85$, and it does not correspond to the first best solution.*

Proof. The maximum of W can be obtained through numerical calculations. In correspondence of such values $W = 0.33783\frac{(\rho^T)^2}{k}$. If all the existing firms could adopt a level of abatement equal to the first best level the social welfare would be equal to $0.5\frac{(\rho^T)^2}{k}$. Therefore, when the abatement cost are explicitly linked to sales, the maximum social welfare achievable in equilibrium is only the $0,67\%$ of its first best level. □

Proposition 6. *If $\bar{\theta}$ is sufficiently high, social welfare is higher in a standard duopoly than in a mixed duopoly.*

Proof. Thanks to Figure 5 we know that in the presence of a non-profit firm and of a profit-maximizing a unique Nash equilibrium exists, in which the former chooses the high level of abatement and the latter the low level. Observing the third graph in Figure 7 we can note that when $\bar{\theta}$ is very high, $W(0,0) > W(1,0)$. Therefore social welfare is higher in a standard duopoly (i.e. in the presence of two profit-maximizing firms) than in a mixed duopoly (i.e. in the presence a non-profit firm and a profit-maximizing firm) $\square$

Proposition 7. *Let us assume that $\alpha_H = \alpha'_L > \alpha_L = \alpha'_H$. There are cases in which $W(\alpha_H, \alpha_L) < W(\alpha'_H, \alpha'_L)$.*

Proof. Observing the third graph in Figure 7 we can note that when $\bar{\theta}$ is very high, the social welfare in $W(0,0)$ is decreasing in α_H and increasing in α_L. As a consequence, $W(y,0) < W(0,y)$ for any y strictly positive and close to 0. $\square$

Hence, when the abatement costs are proportional to firms' sales the conclusions are quite more confusing than in the fixed costs case. First of all, a higher degree of responsibility of consumers' and/or firms may decrease the social welfare. Moreover, the presence of a no-profit firm competing with a profit-maximizing firm may harm social welfare. Consequently is not always reasonable for consumers and share-holders to sacrifice their private utility in order to voluntarily contribute to the environmental protection. Finally, when two Nash equilibria exists, there are cases in which the social welfare is higher when the more responsible firm produces the low (environmental) quality good. Therefore in such cases the firm with some degree of altruism should choose a level of abatement lower than its profit-maximizing rival.

5 Conclusions

In this paper we draw on two so far unrelated strands of literature: the green consumers and the corporate social responsibility of firms. We develop a model where some consumers care about the environmental impact of goods they buy and some firms, following a multidimensional objective, weigh up both profit and abatement activity. Our analysis has focused mainly on the effects associated to exogenous changes in aggregate consumers' willingness to pay for cleaner goods or in the degree of firms' social responsibility.

In line with the existing literature, we have found that the presence of green consumers is sufficient to induce some firms to overcomply the minimum environmental standard. However we have also shown that the presence of green consumers is also necessary. Indeed, even if firms want to maximize their abatement effort, they would be forced to employ the standard technology if nobody is willing to pay an extra-premium for their environment friendly products.

A second result is that the nature of the abatement cost function influences how a higher level of responsibility of both producers and consumers affects the efficiency of aggregate clean-up. If the costs of the cleaning process are fixed, then social welfare is monotone increasing in consumers' WTP and in firms' CSR. On the other hand, if the abatement costs are variable, social welfare may be reduced by an increase of consumers' WTP and by a higher degree of firms' CSR. Therefore a higher responsibility does not necessarily mean a higher welfare. Moreover, if the abatement costs are fixed social welfare is always higher in a mixed than in a standard duopoly. Conversely, when the costs are variable, social welfare may be reduced by the presence of a non-profit firm.

Finally, we have found that in both cases, a full responsibility of consumers and producers is sufficient neither to implement the first best level of aggregate clean-up nor to achieve a cost effective allocation of the abatement activity. The existence of individuals who take care of the environment in their market decisions is usually a good news, but it cannot be considered a perfect substitute for environmental regulation.

Future research should extend our analysis in order to check the robustness of our results under different assumptions. For instance, firms could compete in a different market form: we could assume that one firm is a Stackelberg leader, and/or that the number of responsible firms is endogenous. Moreover, responsible firms could maximize other kind of objective functions. Finally, firms' degree of CSR could be endogenous: in such case we should analyze the dynamic properties of the interaction between green consumers and responsible firms.

6 Technical appendix

In this appendix we want to prove that at each stage the pair of candidate equilibrium prices or qualities (i.e. the solutions of the system given by the first order conditions stemming from firms' maximization problems) represents a Nash equilibrium. For this purpose we need to show that i) second order conditions are satisfied, and that ii) the low (high) quality firm has no incentive to leapfrog its rival by choosing a level of abatement higher than e_H^* (lower than e_L^*).

We start by introducing the following lemma: let J_i be an objective function given by the weighted product of two different functions: $J_i = [\pi_i(z_i)]^{1-\alpha_i}[E_i(z_i)]^{\alpha_i}$

Lemma 6. *If both π_i and E_i are log-concave in z_i, then the solution of the first order condition, z_i^*, represents a local maximum.*

Proof. We recall that the solution of a maximization problem is invariant wrt monotone transformation of the objective function, so:

$$\max_{z_i}[\pi_i(z_i)]^{1-\alpha_i}[E_i(z_i)]^{\alpha_i} \equiv \max_{z_i} \log[\pi_i(z_i)]^{1-\alpha_i}[E_i(z_i)]^{\alpha_i} \equiv$$
$$\equiv \max_{z_i}(1-\alpha_i)\log[\pi_i(z_i)] + \alpha_i \log[E_i(z_i)]$$

Hence, if both π_i and E_i are log-concave in z_i, then the second order condition of the maximization problem holds. □

Proposition 8. *At each stage the solutions of first order conditions represent always local maxima.*

Proof. In order to guarantee that the solution of each first order condition is indeed a local maximum we need to prove that second order conditions always hold. Thanks to Lemma 6 we have to show that at each stage the profit and the positive externality are log-concave in each firm's strategic choice.

As far as the fixed costs case is concerned, we know from the existing literature (see for instance Arora and Gangopadhyay, 1995) that each firm's profit function is concave in firm's price strategy at the second stage (whatever the quality equilibrium) and in each firm's quality choice at the first stage. However, concavity of the profit functions imply also their log-concavity. At the same time, the positive externality of each firm is equal to e_i which is obviously log-concave in itself.

With regard to the variable costs case, the log-concavity of firms' objective function is shown below.

- *Price stage* - Using formulas (24) and (10) we obtain:

$$\frac{\partial^2 \log[\pi_H]}{\partial {p_H}^2} = \frac{\partial^2 \log\left(p_H - \frac{k}{2}e_H^2\right)}{\partial {p_H}^2} + \frac{\partial^2 \log x_H}{\partial {p_H}^2};$$
$$= -\frac{1}{\left(p_H - \frac{k}{2}e_H^2\right)^2} - \frac{1}{\left(\bar{\theta}(e_H - e_L) - p_H + p_L\right)^2} < 0.$$
$$\frac{\partial^2 \log[e_H x_H]}{\partial {p_H}^2} = -\frac{1}{\left(\bar{\theta}(e_H - e_L) - p_H + p_L\right)^2} < 0.$$

$$\frac{\partial^2 \log[\pi_L]}{\partial {p_L}^2} = \frac{\partial^2 \log\left(p_L - \frac{k}{2}e_L^2\right)}{\partial {p_L}^2} + \frac{\partial^2 \log x_L}{\partial {p_L}^2};$$
$$= -\frac{1}{\left(p_L - \frac{k}{2}e_L^2\right)^2} - \frac{e_H^2}{\left(p_H e_L - p_L e_H\right)^2} < 0.$$
$$\frac{\partial^2 \log[e_L x_L]}{\partial {p_L}^2} = -\frac{e_H^2}{\left(p_H e_L - p_L e_H\right)^2} < 0.$$

- *Quality stage* - Recalling formulas (25) and (26) we can write:

$$\log \pi_H = \log{(1-\alpha_H)(e_H - e_L)} + 2\log x_H;$$
$$\log \pi_L = \log{(1-\alpha_L)(e_H - e_L)} + \log e_L - \log e_H + 2\log x_L;$$

where, thanks to formulas (27) and (28) we can know that:

$$\log x_H = \log \beta e_H + \log{[(2-\alpha_L)(2\bar{\theta} - ke_H) - ke_L]} -$$
$$- \log 2\bar{\theta}[(2-\alpha_H)(2-\alpha_L)e_H - (1-\alpha_H)(1-\alpha_L)e_L];$$

$$\log x_L = \log \beta e_H + \log{[(1-\alpha_H)(2\bar{\theta} - ke_L) + ke_H]} -$$
$$- \log 2\bar{\theta}[(2-\alpha_H)(2-\alpha_L)e_H - (1-\alpha_H)(1-\alpha_L)e_L];$$

Consequently we can calculate the following derivatives:

$$\frac{\partial^2 \log[\pi_H]}{\partial {e_H}^2} = -\frac{1}{(e_H - e_L)^2} - \frac{1}{e_H^2} - 2\frac{[(2-\alpha_L)k]^2}{[(2-\alpha_L)(2\bar{\theta} - ke_H) - ke_L]^2} -$$
$$- 2\frac{[(2-\alpha_H)(2-\alpha_L)]^2}{[(2-\alpha_H)(2-\alpha_L)e_H - (1-\alpha_H)(1-\alpha_L)e_L]^2} < 0;$$

$$\frac{\partial^2 \log[\pi_L]}{\partial {e_L}^2} = -\frac{1}{(e_H - e_L)^2} - \frac{1}{e_L^2} - 2\frac{[(1-\alpha_H)k]^2}{[(1-\alpha_H)(2\bar{\theta} - ke_L) + ke_H]^2} - \\ - 2\frac{[(2-\alpha_H)(2-\alpha_L)]^2}{[(2-\alpha_H)(2-\alpha_L)e_H - (1-\alpha_H)(1-\alpha_L)e_L]^2} < 0;$$

$$\frac{\partial^2 \log[e_H x_H]}{\partial {e_H}^2} = -\frac{2}{e_H^2} - 2\frac{[(2-\alpha_L)k]^2}{[(2-\alpha_L)(2\bar{\theta} - ke_H) - ke_L]^2} - \\ - 2\frac{[(2-\alpha_H)(2-\alpha_L)]^2}{[(2-\alpha_H)(2-\alpha_L)e_H - (1-\alpha_H)(1-\alpha_L)e_L]^2} < 0;$$

$$\frac{\partial^2 \log[e_L x_L]}{\partial {e_L}^2} = -\frac{1}{e_L^2} - 2\frac{[(1-\alpha_H)k]^2}{[(1-\alpha_H)(2\bar{\theta} - ke_L) + ke_H]^2} - \\ - 2\frac{[(2-\alpha_H)(2-\alpha_L)]^2}{[(2-\alpha_H)(2-\alpha_L)e_H - (1-\alpha_H)(1-\alpha_L)e_L]^2} < 0.$$

□

Finally, in order to guarantee that (e_L^*, e_H^*) is indeed a Nash equilibrium we have to check that the firm choosing e_L^* has no incentive to "leapfrog" its rival by choosing a quality higher than e_H^*. Likewise, we have to verify that firm choosing the highest quality, e_H^*, has no incentive to deviate by producing a quality lower than e_L^*. Formally, we must check that:

$$J_L(e_L^*, e_H^*) > J_H(e_1^*, e_H^*) \quad \forall \alpha_H, \alpha_L \in [0, 1], \tag{39}$$

where e_1^* in the fixed costs case is the solution of equation (14) (for the variable costs case we have to consider equation (29)) when $e_L = e_H^*$, and:

$$J_H(e_H^*, e_L^*) > J_L(e_2^*, e_L^*), \quad \forall \alpha_H, \alpha_L \in [0, 1], \tag{40}$$

where e_2^* in the fixed costs case is the solution of equation 15 (for the variable costs case we have to consider equation (30)) when $e_H = e_L^*$

From the numerical calculations we can observe that in the fixed costs case no firm has an incentive to leapfrog its rival in equilibrium. However, in the variable costs case, firm H has an incentive to leapfrog firm L when α_L is sufficiently high and α_H is sufficiently low (see Figure 5). The file with the numerical calculations is available upon request.

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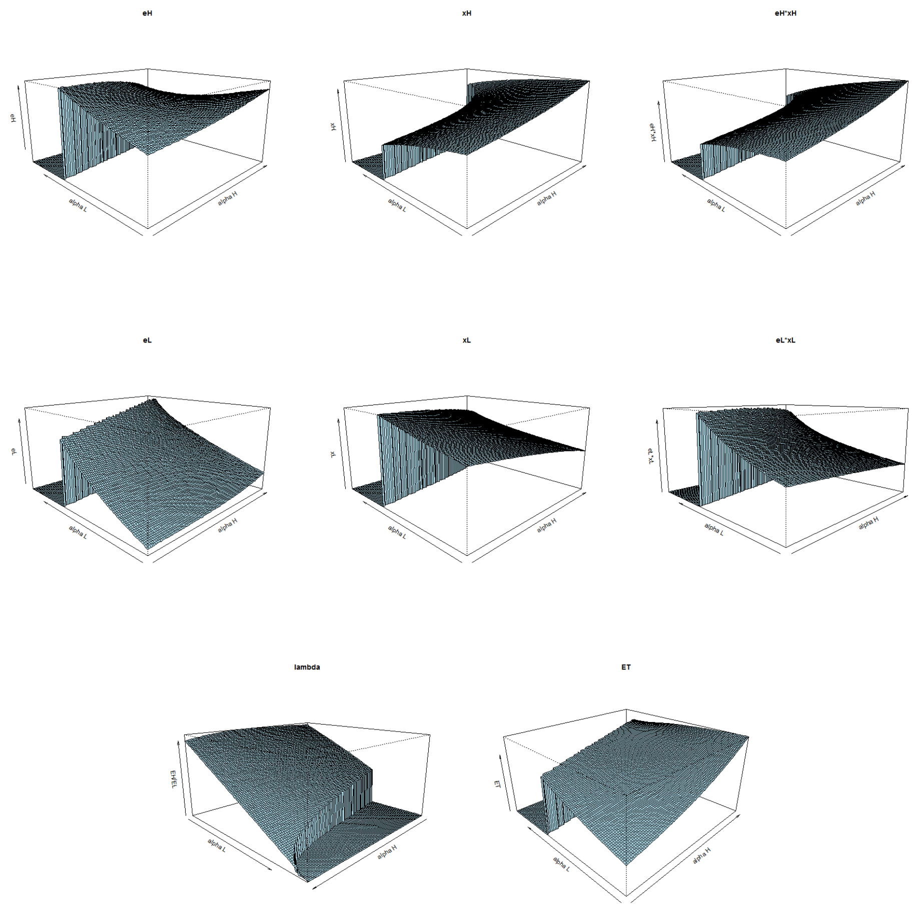
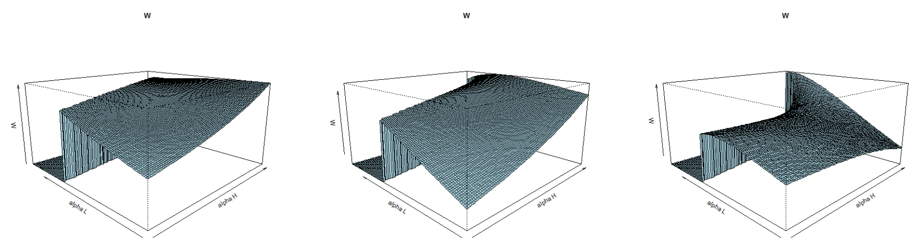
Figure 6: Illustration of Lemma 4



Figures are drawn assuming $\bar{\theta} = 1$, $k = 1$ and $\beta = 1$.

Figure 7: $W(\alpha_H, \alpha_L)$



Social welfare pattern when $\bar{\theta} = 0.2; 1; 1.8$.