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## The persistence of environmental and social strategies under emission permits

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# The Persistence of Environmental and Social Strategies under Emission Permits<sup>\*</sup>

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## Abstract

In this paper, we analyse the long-run industry configuration of an oligopoly where profit-seeking (PS) and environmentally socially responsible (ECSR) firms compete in quantities. We adopt an evolutionary setting to determine the firms' endogenous choice of statute (PS or ECSR). Pollution is regulated through an Emission Trading System (ETS) scheme that allocates emissions rights to firms. Firms may also invest in emission abatement technology to reduce the cost of emission rights. Our findings show that the introduction of an ETS favours the persistence of the ECSR strategy by reducing the share of PS firms in the industry. In contrast, an increase in the stringency of the ETS policy makes the PS strategy more competitive.

**JEL Codes:** C73, H23, L13, L21, M14.

**Keywords:** Mixed oligopoly markets, emission reduction investment, environmentally concerned firms, emission trading system.

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# 1 Introduction

The Emission Trading System (ETS) is a policy that allows firms to purchase emissions rights. It provides an incentive to enhance internal emission abatement measures, thereby allowing for increased production while maintaining a set level of emission rights. The largest application of this policy is the European Union Emission Trading Scheme (EU ETS), started in 2005.

In this paper we investigate the interaction between an ETS scheme and environmental and socially responsible activities (ECSR) by firms: the latter refers to a business approach that integrates environmental concerns and social welfare into its operations and interactions with stakeholders. It entails a commitment by firms to conduct their business in a manner that not only generates profits but also minimises negative impacts on the environment and contributes positively to society. In practice, ECSR involves aims at reducing ecological footprints, conserving natural resources, minimising pollution in the production process. Given the nature of these activities, it seems natural to expect that the adoption of an ETS policy and of ECSR practices affect each other. A growing empirical evidence supports this statement ([Lee, 2011](#), [Gasbarro et al., 2013](#), [Hörisch, 2013](#), [Kong et al., 2014](#), [Doda et al., 2015](#), [Martin et al., 2016](#), among others), but how the interplay between ETS and ECSR works is left unanswered.

We analyse the introduction of an ETS policy in an evolutionary setting based on a mixed N-firms oligopoly model. In each time period, firms compete in quantities and invest in emissions technology simultaneously, choosing whether to adopt either a profit-seeking or an ECSR statute based on profitability. ECSR activities encompass both environmental and social concerns, modelled as concerns regarding own polluting emissions and consumer well-being, respectively.

As a benchmark, we analyse the mixed industry before the implementation of an ETS. The steady state industry configuration differs according to the ECSR commitment to environmental or social concern. When these commitments are very high, an ECSR firm is not sufficiently competitive compared to a PS firm; the industry configuration is homogeneous, showing only PS firm types. If instead both commitments are low, and social concern is stronger than environmental concern, the result reverses. This is explained by the fact that, as highlighted in the literature ([Lambertini and Tampieri, 2015](#), [Iannucci and Tampieri, 2023](#), among others), social concern induces a more aggressive production strategy than the PS one, while environmental

concern induces the ECSR firm to bear an additional emission cost. Thus, the strategy related to environmental concern is sustainable in a competitive context only if its effect is more than compensated by the strategy induced by social concern. Finally, mixed oligopolies emerge when social concern is further pushed while environmental concern remains low.

Next, we consider the implementation of ETS. Equilibria are qualitatively similar to the baseline model, with an important difference. While the entire region of existence of the steady-state interior solutions becomes smaller, the region of homogeneous industry configuration of PS firms drastically shrinks. From this it follows that, in the steady state, the emergence of industry configurations with only ECSR firms or mixed types is more likely. The intuition is simple. In the presence of ETS, socially responsible firms have a cost advantage. Indeed, their emission concern reduces the amount of permits they have to purchase compared to a profit seeking firm.

In terms of policy implications, we find that the introduction of an ETS pushes the share of ECSR firms, thus giving an incentive at adopting environmentally responsible practices. Clearly, without any policy in place, PS firms have no incentives in abating their polluting emissions. With the introduction of the ETS, a new extra cost emerges for both firms, which makes the additional cost of environmental concern relatively less important in terms of overall cost differences.

Finally, we perform a welfare analysis to evaluate the desirability of the policy. As a general assessment, we find that while the ETS decreases industry profits and consumer surplus, it also reduces environmental damage. Interestingly, the overall effect on social welfare is positive, so the reduction in environmental damage more than offsets the decrease in profits and consumer surplus. By examining the level of emissions restrictions, our results show that once the ETS is introduced, the stringency of the ETS policy (i.e., the reduction of the number of permits) favours the PS strategy over the ECSR strategy.

The remainder of the paper is organised as follows. [Section 2](#) briefly surveys the relevant literature linked to our contribution. [Section 3](#) describes the benchmark economy where no ETS is applied, while [Section 4](#) considers the introduction of the ETS in the industry. [Section 5](#) illustrates the welfare analysis before and after implementing the ETS, and [Section 6](#) concludes. All the proofs are developed in the Appendix.

## 2 Related literature

The present paper is linked to three strands of economic literature, namely, the literature on theoretical modelling of markets with ETS, the literature on strategic CSR, and the literature on evolutionary Cournot competition.

Regarding the first strand, a recent group of papers has investigated the functioning and impact of ETS in markets and their impact on welfare. Different contributions have focused either on analysing the optimal number of emission rights ([Grüll and Taschini, 2011](#), [Fell et al., 2012](#), [Kollemberg and Taschini, 2016](#), [Perino and Willner, 2016](#), among others), evaluating aggregate cost savings due to the implementation of new technology in the ETS ([Malueg, 1989](#), [Milliman and Prince, 1989](#), [Jung et al., 1996](#), [Unold and Requate, 2001](#), [Requate, 2005](#), among others) or examining the effects of firms' choices on investments in eco-innovation as a response to the implementation of an ETS ([Moreno-Bromberg and Taschini, 2011](#), [Antoci et al., 2019](#)). A relevant point that seems largely unexplored and addressed in this paper is how the introduction of ETS programs affects corporate strategy concerning environmental and social responsibility.

The literature on strategic ECSR models ECSR activities as a tool to reach some strategic advantage in the interaction with competitors. This strand has shown important developments in the last decade.

Some relevant contributions in a static framework are [Goering \(2008a\)](#), [Goering \(2008b\)](#) and [Goering \(2010\)](#), [Kopel and Brand \(2012\)](#), [Lambertini and Tampieri \(2015\)](#) and [Gioffré et al. \(2021\)](#), among others. A generally accepted result of these developments is that firms may strategically commit to CSR activities to obtain higher profits than their profit-seeking competitors. In non-evolutionary dynamic settings, relevant contributions to the literature of strategic CSR are [Wirl et al. \(2013\)](#), [Becchetti et al. \(2014\)](#) and [Lambertini et al. \(2016\)](#). In [Wirl et al. \(2013\)](#), CSR activities are an important determinant for a firm's reputation, and thus long run profit, while in [Becchetti et al. \(2014\)](#) the level of product position on the Hotelling line determines the "ethical value" of the good and thus its level of socially responsible features. Conversely, as in several static models and the present one, [Lambertini et al. \(2016\)](#) model a CSR behaviour as composed of profits, a share of consumer surplus and the own level of polluting emission.

With this literature, we share the feature, common of most of these papers, to model CSR behaviour as the maximisation, together with profits, also of a part of consumer surplus to represent social concern and the own share of polluting emission so as to represent the environmental concern.

Finally, the paper is related to the literature of evolutionary Cournot competition. The seminal paper of this literature is [Droste et al. \(2002\)](#), while other relevant contributions are [Bischi et al. \(2007\)](#) and [Bischi et al. \(2015\)](#), among others. This framework and has been already applied to the analysis of industry configurations with CSR firms in [Kopel et al. \(2014\)](#) and [Kopel and Lamantia \(2018\)](#), and to the analysis of ECSR in [Iannucci and Tampieri \(2023\)](#). The present analysis is mainly linked to the latter paper, which analyses an evolutionary setting with Cournot competition where firms can choose whether to adopt an ECSR or a profit-maximising behaviour. Nonetheless, [Iannucci and Tampieri \(2023\)](#) focuses on the interplay of ECSR activities with the implementation of a tax on emissions, while we focus on the effects of the implementation of an ETS policy.

### 3 An economy with no ETS

We first consider an economy with no environmental regulation. We outline the model features and then show the equilibrium results. In this way we are able later to highlight the effects of the introduction of an ETS.

#### 3.1 The model

Consider an economy composed of  $N \in \{2, 3, 4, \dots\}$  firms, producing a homogenous good. There are two types of firms: profit seeking (PS) and environmental and socially responsible (ECSR). The number of ECSR firms is  $m \in \{0, 1, 2, \dots, N\}$ , while the number of PS firms is  $N - m$ . The inverse demand is linear:

$$p = \gamma - \sum_{i=1}^m q_i - \sum_{j=1}^{N-m} q_j, \quad (1)$$

where  $\gamma > 0$  is the reservation price, while  $q_P$  and  $q_E$  are the quantities produced by PS and ECSR firms, respectively. Production is polluting: for simplicity, the level of emissions amounts

to the quantity of the good produced,  $q$ , minus an *end-of-pipe* abatement investment,  $z$ . Thus, we denote emissions as  $e = q - z$ .

The profit function of a generic firm  $k \in \{E, P\}$  is

$$\pi_k = (p - c)q_k - \frac{z_k^2}{2}, \quad (2)$$

where  $c > 0$  is the production cost. The emission reduction investment is chosen by firms simultaneously with production quantities.

While a PS firm's objective function is to maximise (2), an ECSR firm takes into account its impact in terms of emissions, as well as consumers' welfare. Hence, its objective function is (see Lambertini and Tampieri, 2015, Lambertini et al., 2016, Xu and Lee, 2023 and Iannucci and Tampieri, 2023, among others):

$$O_E = \pi_E + \beta CS - \delta e_E, \quad (3)$$

where  $\delta \in [0, 1]$  is the share of emissions  $e_E = q_E - z_E$  internalised by the ECSR during its production process, and  $\beta \in [0, 1]$  represents the ECSR firms sensitivity to consumer surplus  $CS$ , i.e.,

$$CS = \frac{\left(\sum_{i=1}^m q_i + \sum_{j=1}^{N-m} q_j\right)^2}{2}. \quad (4)$$

### 3.2 The static game

The static game takes as given the industry composition between PS and ECSR firms: firms choose simultaneously quantities  $q$  and abatement investment  $z$ . The maximum problem of an ECSR and PS firm are, respectively,

$$\begin{aligned} \max_{\substack{q_E, z_E \geq 0 \\ (q_E - z_E) \geq 0}} O_E &= (p - c)q_E - \frac{1}{2}z_E^2 + \beta CS - \delta e_E, \\ \max_{\substack{q_P, z_P \geq 0 \\ (q_P - z_P) \geq 0}} \pi_P &= (p - c)q_P - \frac{1}{2}z_P^2. \end{aligned} \quad (5)$$

In what follows, we define market size as the reservation price minus marginal cost (Shy, 1995), and we take it as the numeraire, i.e.,  $\gamma - c = 1$ . Solving for quantities and investment in emission reduction technology, we get the following results.

**Proposition 1** *In the static game, the equilibrium quantities and abatement of an ECSR and PS firm are, respectively,*

$$\begin{aligned} q_E^* &= \frac{1 - \delta + (\beta - \delta)(N - m)}{N - \beta m + 1}, \\ z_E^* &= \delta, \\ q_P^* &= \frac{1 - (\beta - \delta)m}{N - \beta m + 1}, \\ z_P^* &= 0. \end{aligned}$$

By Proposition 1, optimal profits can be rewritten as:

$$\begin{aligned} \pi_E^* &= q_P^* q_E^* - \frac{\delta^2}{2}, \\ \pi_P^* &= (q_P^*)^2. \end{aligned}$$

The following corollary summarises the conditions such that the equilibrium elements are positive, together with the equilibrium level of emissions,  $q_k^* - z_k^*$  for every firm type  $k$  and industry composition. For convenience, we define

$$\begin{aligned} \underline{\delta} &:= \max \left\{ 0, \frac{\beta N - 1}{N} \right\}, \\ \bar{\delta} &:= \min \left\{ \frac{(\beta N + 1)}{2(N + 1)}, \frac{1}{(1 - \beta)N + 2} \right\}. \end{aligned}$$

**Corollary 1** *Condition  $\delta \in (\underline{\delta}, \bar{\delta})$  ensures  $q_P^* > 0$  and  $q_E^* - z_E^* > 0$  for each  $m \in \{0, 1, 2, \dots, N\}$ .*

### 3.3 Evolutionary dynamics

In this section we introduce continuous time and let firms to choose their type. Accordingly, we assume an infinite population of firms composed of both ECSR and PS. In every instant,



two firms are randomly selected to play a duopoly game with the rules described above, so that  $N = 2$ , while  $m \in \{0, 1, 2\}$  (Droste et al., 2002). Therefore, the profit of adopting a strategy is a function of the probability of encountering. We denote  $x \in [0, 1]$  as the probability that a firm adopts the ECSR strategy. If the game is repeatedly played, the probability  $x$  may be interpreted as the share of ECSR firms in the population, and  $1 - x$  as the share of PS firms.

The expected profits can be written as linear functions of the probability  $x$  over the bounded and limited interval  $[0, 1]$ . In particular, the expected profit of the ECSR firm is:

$$\mathbb{E}(\pi_E^*(x)) = x\pi_{EE}^* + (1-x)\pi_{EP}^*, \quad (6)$$

with

$$\pi_{EE}^* = \frac{(1-\delta)[2(\delta-\beta)+1]}{(2\beta-3)^2} - \frac{\delta^2}{2}, \quad (7)$$

and

$$\pi_{EP}^* = \frac{(\delta-\beta+1)(\beta-2\delta+1)}{(\beta-3)^2} - \frac{\delta^2}{2}, \quad (8)$$

where  $\pi_{EE}^*$  is the payoff of the ECSR firm if it encounters another ECSR firm, while  $\pi_{EP}^*$  is the payoff of the profit firm if it encounters a PS firm. Equations (7) and (8) are obtained by the static equilibrium quantities of a ECSR firm in Proposition 1 when  $N = 2$  and  $m = 2$  and  $m = 1$ , respectively. The expected profit of the PS firm is:

$$\mathbb{E}(\pi_P^*(x)) = x\pi_{PE}^* + (1-x)\pi_{PP}^*,$$

with

$$\pi_{PE}^* = \left(\frac{\delta-\beta+1}{\beta-3}\right)^2, \quad (9)$$

and

$$\pi_{PP}^* = \frac{1}{9}, \quad (10)$$

where  $\pi_{PE}^*$  is the payoff of a PS firm if it competes against an ECSR firm, while  $\pi_{PP}^*$  is the PS firm's profit if it encounters a firm of the same type. Similarly to the previous case, Equations (9) and (10) are obtained by the static equilibrium quantities of a PS firm in Proposition 1 when

$N = 2$  and  $m = 1$  and  $m = 0$ , respectively. If the population of firms is large enough, which we assume, then by the law of large numbers we can take the expected profits to be a close approximation of realised profits (Weibull, 1995, pp 71-72).

The time evolution of the share  $x$  is given by the following replicator dynamics:

$$\dot{x} = x(1-x) [\mathbb{E}(\pi_E^*(x)) - \mathbb{E}(\pi_P^*(x))]. \quad (11)$$

The replicator equation (11) admits three types of steady states:  $x = 0$ , in which all firms are PS (“All PS” configuration),  $x = 1$  in which all firms are ECSR (“All ECSR” configuration), and an inner state,  $x \in (0, 1)$ , in which there is coexistence between firms. Only stable steady states are Nash equilibria so that, denoting  $x^*$  as a stable steady state, the corner solutions  $x^* \in \{0, 1\}$  are pure Nash equilibria, while the inner  $x^* \in (0, 1)$  is a mixed-strategy Nash equilibrium (Bomze, 1986). Therefore, if  $\pi_{EP}^* - \pi_{PP}^* < 0$  and  $\pi_{EE}^* - \pi_{PE}^* < 0$ , then  $\mathbb{E}(\pi_E^*(x)) < \mathbb{E}(\pi_P^*(x)) \forall x \in [0, 1]$  and so  $x^* = 0$ , while if  $\pi_{EP}^* - \pi_{PP}^* > 0$  and  $\pi_{EE}^* - \pi_{PE}^* > 0$ , then  $\mathbb{E}(\pi_E^*(x)) > \mathbb{E}(\pi_P^*(x)) \forall x \in [0, 1]$  and so  $x^* = 1$ . Differently, if

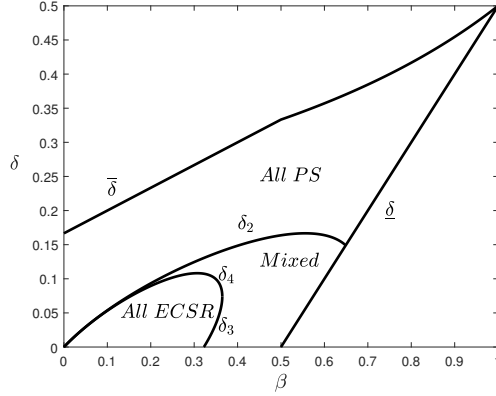
$$x = \frac{\pi_{EP}^* - \pi_{PP}^*}{\pi_{EE}^* - \pi_{EP}^* - \pi_{PE}^* + \pi_{PP}^*} \in (0, 1),$$

then the equilibrium is mixed and  $\mathbb{E}(\pi_E^*(x)) = \mathbb{E}(\pi_P^*(x))$ . This inner steady state is stable if  $\pi_{EP}^* - \pi_{PP}^* > 0$  and  $\pi_{EE}^* - \pi_{PE}^* < 0$  (“Mixed” configuration), while it is unstable if  $\pi_{EP}^* - \pi_{PP}^* < 0$  and  $\pi_{EE}^* - \pi_{PE}^* > 0$  (“Unstable” configuration).

The next proposition illustrates the possible industry configurations according to the values of the ECSR sensitivity to own emissions,  $\delta$ . To ease the exposition, we define  $\delta_{1,2}$  and  $\delta_{3,4}$  in the Appendix (Proof of Proposition 2), which help us to identify the different configurations. More specifically,  $\delta_2$  is one of the solution of the equation  $\pi_{EP}^* - \pi_{PP}^* = 0$  and  $\delta_{3,4}$  are the solution of the equation  $\pi_{EE}^* - \pi_{PE}^* = 0$ , while  $\hat{\delta} = \max\{\underline{\delta}, \delta_3\}$ .

**Proposition 2** *The possible industry configurations are*

1. “All ECSR” ( $x^* = 0$ ) for  $\delta \in (\hat{\delta}, \delta_4)$ ;
2. “All PS” ( $x^* = 1$ ) for  $\delta \in (\delta_2, \bar{\delta})$ ;



**Figure. 1.** Dynamic regimes in the plane  $(\beta, \delta)$ .

3. “Mixed” ( $x^* \in (0, 1)$ ) for  $\delta \in ((\underline{\delta}, \delta_2) \setminus (\hat{\delta}, \delta_4))$ .

Fig. 1 shows the regions of the plane  $(\beta, \delta)$  outlined by Proposition 2. The equilibrium industry structure varies depending on the degree of environmental and social responsibility, respectively. When ECSR commitments are extremely elevated, ECSR firms struggle to compete with PS firms, leading to a homogeneous industry configuration dominated by PS firms. Conversely, when both ECSR commitments are minimal and social concerns outweigh environmental concerns, the outcome is reversed. The intuition behind Proposition 2 can be explained as follows. The socially responsible production strategy is generally more aggressive, in terms of competition, than that of profit-seeking firms. This is true even if the socially responsible firm also attaches a weight to polluting emission, as long as the expansive effect of social concern more than offsets the related reduction in production.

## 4 The introduction of an ETS

An ETS generally operates on a cap-and-trade principle. This means that there is a cap, or limit, set on the total amount of certain greenhouse gases that can be emitted by allowances covered by the system. These allowances represent the right to emit a specific amount of greenhouse gases. The firms’ demand for allowances in the ETS, while the cap is exogenously established by the environmental agency. In equilibrium, demand and supply (the cap) of allowances determine an

ETS price, mirroring an auctioning system.

To introduce an ETS in the analysis, we now assume that firms must purchase emission permits (to which we will refer as “ETS” for brevity) that correspond to the emissions coming from their production at unit price  $a$ . The number of emission permits in the industry is established by the policy maker at  $\bar{E}$ . To reduce the cost of ETS, firms may invest into technology  $z$  aimed at reducing emissions.

Like before, each time period assumes the industry composition between PS and ECSR firms as given, but now the game is sequential rather than static. In particular, it is composed of two stages. In stage 1, the market of ETS clears and the price of permits is set. In stage 2, firms choose simultaneously quantities  $q$  and abatement investment  $z$ . The equilibrium concept is subgame perfect Nash equilibrium (SPNE), which is solved by backward induction.

In the second stage, the firms’ maximum problem amounts to

$$\begin{aligned} \max_{\substack{q_E^{ets}, z_E^{ets} \geq 0 \\ (q_E^{ets} - z_E^{ets}) \geq 0}} O_E^{ets} &= (p^{ets} - c)q_E^{ets} - \frac{1}{2}(z_E^{ets})^2 + \beta CS^{ets} - (\delta + a)(q_E^{ets} - z_E^{ets}), \\ \max_{\substack{q_P^{ets}, z_P^{ets} \geq 0 \\ (q_P^{ets} - z_P^{ets}) \geq 0}} \pi_P^{ets} &= (p^{ets} - c)q_P^{ets} - \frac{1}{2}(z_P^{ets})^2 - a(q_P^{ets} - z_P^{ets}), \end{aligned} \tag{12}$$

where  $a > 0$  is the price of one emission permit. In the first stage, the allowance price is determined by market clearing, i.e., the demand for permits equals the given supply:

$$(q_E^{ets} - z_E^{ets})m + (q_P^{ets} - z_P^{ets})(N - m) = \bar{E}. \tag{13}$$

The following proposition holds.

**Proposition 3** *The SPNE is given by:*

$$\begin{aligned}
a^* &= \frac{N - (N + 1)\bar{E} - [(N + 2)\delta + (\delta m + \bar{E})\beta]m}{(N - \beta m + 2)N}, \\
q_E^{ets*} &= \frac{1 - \delta - a + [(1 - a)\beta - \delta](N - m)}{N - \beta m + 1}, \\
z_E^{ets*} &= \delta + a^*, \\
q_P^{ets*} &= \frac{1 - a - [(1 - a)\beta - \delta]m}{N - \beta m + 1}, \\
z_P^{ets*} &= a^*.
\end{aligned} \tag{14}$$

By [Proposition 3](#), equilibrium profits may be rewritten as

$$\begin{aligned}
\pi_P^{ets*} &= (q_P^{ets*})^2 + \frac{(\alpha^*)^2}{2}, \\
\pi_E^{ets*} &= q_E^{ets*} q_P^{ets*} + \frac{(\alpha^*)^2}{2} - \frac{\delta^*}{2}.
\end{aligned} \tag{15}$$

Analogously to [Corollary 1](#), the following corollary summarises the conditions for interior solutions for each market composition. For convenience, define

$$\begin{aligned}
\underline{\delta}^{ets} &= \max \left\{ 0, \frac{(N + 2\bar{E})\beta N - (N + 2)\bar{E}}{2[(1 - \beta)N + 2]N} \right\}, \\
\bar{\delta}^{ets} &= \min \left\{ \frac{N - [(1 - \beta)N + 1]\bar{E}}{[(1 - \beta)N + 2]N}, \frac{(N + 2)\bar{E} + (N + \bar{E})\beta N}{2(N + 2)N}, 1 \right\}, \\
\underline{\beta}^{ets} &:= \max \left\{ 0, \frac{(N + 1)\bar{E} - N}{N\bar{E}} \right\}, \\
\bar{\beta}^{ets} &:= \min \left\{ \frac{(2N + \bar{E})(N + 2)}{[2(N + \bar{E}) + N]N}, 1 \right\}.
\end{aligned}$$

**Corollary 2** *The conditions  $\delta \in (\underline{\delta}^{ets}, \bar{\delta}^{ets})$  and  $\beta \in (\underline{\beta}^{ets}, \bar{\beta}^{ets})$  ensure positive emissions and allowance price for each  $m \in \{0, 1, 2, \dots, N\}$ .*

We now introduce the evolutionary dynamics, following the same procedure as in the baseline

case. Substituting the equilibrium values by [Proposition 3](#), the optimal matching profits are

$$\begin{aligned}\pi_{EE}^{ets*} &= \frac{4(\beta - 4)\beta\bar{E}^2 + 16[(\beta - 4)\beta + 4]\delta\bar{E} - 4(2\beta + 1)\bar{E} - 4(4\beta - 3)}{32(\beta - 2)^2}, \\ \pi_{EP}^{ets*} &= \frac{4(\beta - 1)\bar{E} - (6\beta - 11)\bar{E}^2 + 4(9\beta - 4)\delta + 40(\beta - 2)\delta^2 + 2(\bar{E} + 12)\beta\bar{E} + 12}{8(\beta - 4)^2} \\ &\quad - \frac{[(1 - 2\delta)\bar{E} + 8(\bar{E} + \delta + 1) + 5\delta^2]\beta^2}{8(\beta - 4)^2}, \\ \pi_{PE}^{ets*} &= \frac{(\beta - 3)\bar{E} + (\beta - 4)\delta + 2}{2(2\beta - 8)^2} + \frac{\{(\delta - \beta + 1)(2\beta - 8) - (\beta - 1)[(\beta - 3)\bar{E} + (\beta - 4)\delta + 2]\}}{(2\beta - 8)^2(\beta - 3)^2}, \\ \pi_{PP}^{ets*} &= \frac{\bar{E}^2 - 4\bar{E} + 12}{128}.\end{aligned}$$

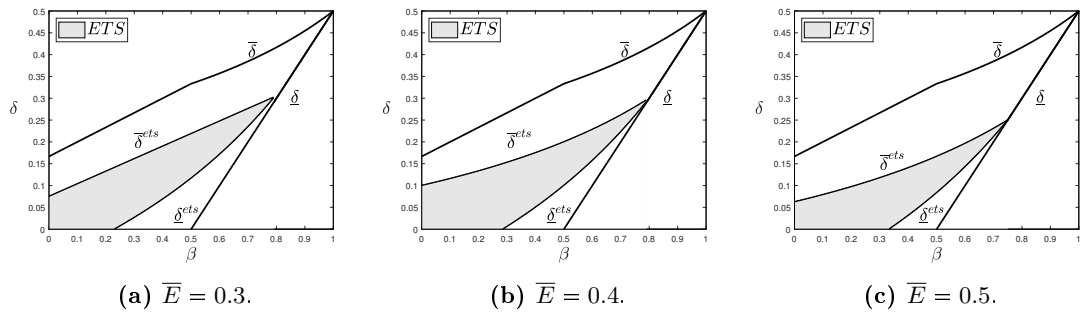
Following the same structure as in the previous section, we define  $\delta_{5,6}$  as the solution of the equation  $\pi_{EP}^{ets*} - \pi_{PP}^{ets*} = 0$  while  $\delta_{7,8}$  are the solutions of the equation  $\pi_{EE}^{ets*} - \pi_{PE}^{ets*} = 0$ , whose derivation can be found in the Appendix ([Proof of Proposition 4](#)). We also define  $\tilde{\delta}_1 = \max\{\underline{\delta}, \delta_7\}$ ,  $\tilde{\delta}_2 = \min\{\delta_6, \delta_8\}$ ,  $\tilde{\delta}_3 = \max\{\delta_6, \delta_8\}$ ,  $\tilde{\delta}_4 = \max\{\delta, \delta_5\}$ ,  $\tilde{\delta}_5 = \min\{\delta_6, \bar{\delta}\}$ . The following proposition outlines the steady state industry configuration in the presence of an ETS policy.

**Proposition 4** *Denoting  $x^{ets*}$  as a stable steady state, four dynamic regimes may arise:*

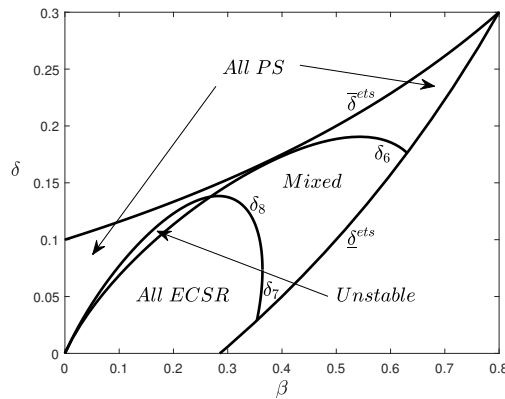
- “All ECSR” ( $x^{ets*} = 1$ ) for  $\delta \in (\tilde{\delta}_1, \tilde{\delta}_2)$ .
- “All PS” ( $x^{ets*} = 0$ ) for  $\delta \in (\tilde{\delta}_3, \bar{\delta})$ .
- “Mixed” ( $x^{ets*} \in (0, 1)$ ) for  $\delta \in ((\tilde{\delta}_4, \tilde{\delta}_5) \setminus (\tilde{\delta}_1, \tilde{\delta}_2))$ .
- “Unstable” ( $x^{ets*} \in \{0, 1\}$ ) for  $\delta \in (\delta_8, \delta_6)$ .

Compared to [Proposition 2](#), in [Proposition 4](#) a new region emerges with a different type of equilibrium, the “Unstable”: here, the type of steady state depends on the initial conditions in terms of industry configuration. If initially there are more PS than ECSR firms operating in the industry, it implies an “All PS” configuration. Conversely, if the initial conditions show more ECSR than PS firms, the unstable steady state leads to an “All ECSR” configuration. It is worth noting that, since ECSR practices are relatively recent, spanning their development over a few decades, we may reasonably assume that the initial conditions generally exhibit a higher proportion of PS firms, resulting in a collapse into an “All PS” configuration.

By [Proposition 2](#) and [Proposition 4](#), we are able to compare the two steady-state regions of existence before and after the introduction of an ETS scheme. The results are outlined in [Fig. 3](#), which appear robust to different values of the stringency  $\bar{E}$ : the region of existence of the steady-state equilibrium shrinks, with the larger reduction for a part of the region characterised by the “All PS” industry configuration. Hence, the introduction of an ETS makes the adoption of a PS strategy less likely. In addition, [Fig. 3](#) shows the equilibrium region with ETS, which appears qualitatively similar to that without ETS when we focus only on the region where both steady states exist. The intuition is similar as in the baseline case.



**Figure 2.** Regions of existence.



**Figure 3.** Dynamic regimes of the plane  $(\beta, \delta)$ ,  $\bar{E} = 0.4$ .

## 5 Welfare analysis

We are left with the task of evaluating how the introduction of an ETS and its stringency affect social welfare. First, we focus on the steady-state industry configurations and emissions. We then proceed to evaluate changes in social welfare. We may interpret the stringency of the ETS from the level of  $\bar{E}$ , which represents the supply of permits. In our model, this is exogenously determined by the environmental agency and can be treated as a parameter. The lower the  $\bar{E}$ , the higher the stringency level of the policy. In the numerical exercise, the chosen parameter values (i.e.,  $\beta = 0.38$  and  $\delta = 0.14$ ) ensure that [Corollary 1](#) and [Corollary 2](#) hold consistently, and that  $x^{ets*} \in (0, 1)$  across all figures.

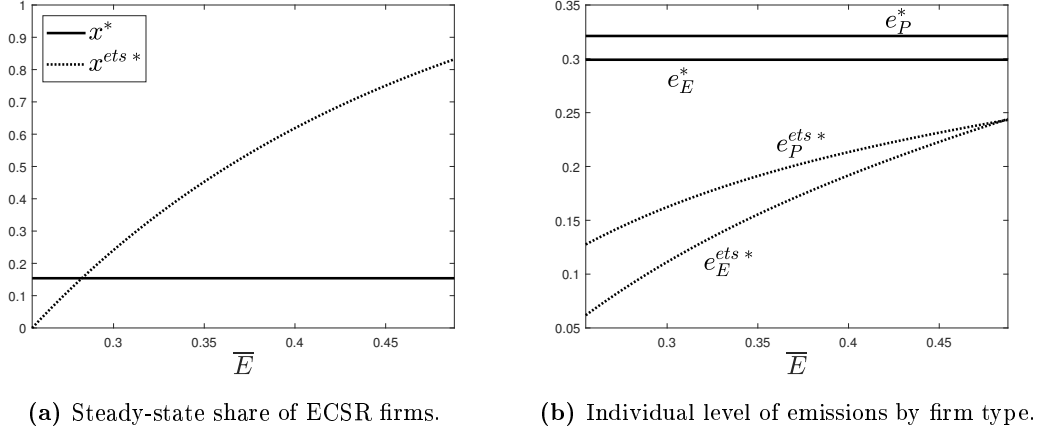
[Figure 4](#) shows the results: in both panels, the horizontal lines refer to the case without ETS, which is clearly not affected by the stringency of the policy. As shown in [Fig. 4\(a\)](#), the share of ECSR firms increases with the introduction of the ETS, provided that the policy is not strict. Indeed, the proportion of ECSR firms falls with increased stringency. Notice that, at the lowest admissible level of  $\bar{E}$ , the industry is composed of only of PS firms. This result may be explained as follows: the lower number of available permits increases their price, thus making it more difficult for ECSR firms to compete, as they also bear the internalisation cost of emission reduction.

[Figure 4\(b\)](#) shows that, quite intuitively, (i) ECSR firms always emit less than PS ones, (ii) the introduction of the ETS reduces emissions regardless of the stringency of the policy, and (iii) emissions decrease for each firm type as the policy becomes stricter. Interestingly, the difference in the level of emissions between PS and ECSR firms increases with the stringency of the policy. This may be attributed to the internalisation of emission costs; the higher price of permits prompts greater investment in emission reduction technology by ECSR firms compared to PS firms.

We now turn to social welfare. This is given by the sum of industry profits, consumer surplus and permits revenue  $PR$  (if the policy is in place) minus environmental damage  $ED$ :

$$\begin{aligned} W^* &= 2E\pi_k^* + CS^* - ED^*, \\ W^{ets*} &= 2E\pi_k^{ets*} + CS^{ets*} + PR^* - ED^{ets*}. \end{aligned} \tag{16}$$





**Figure 4.** The introduction of an ETS and variation of its stringency.

In (16), the permits revenue  $PR$  corresponds to

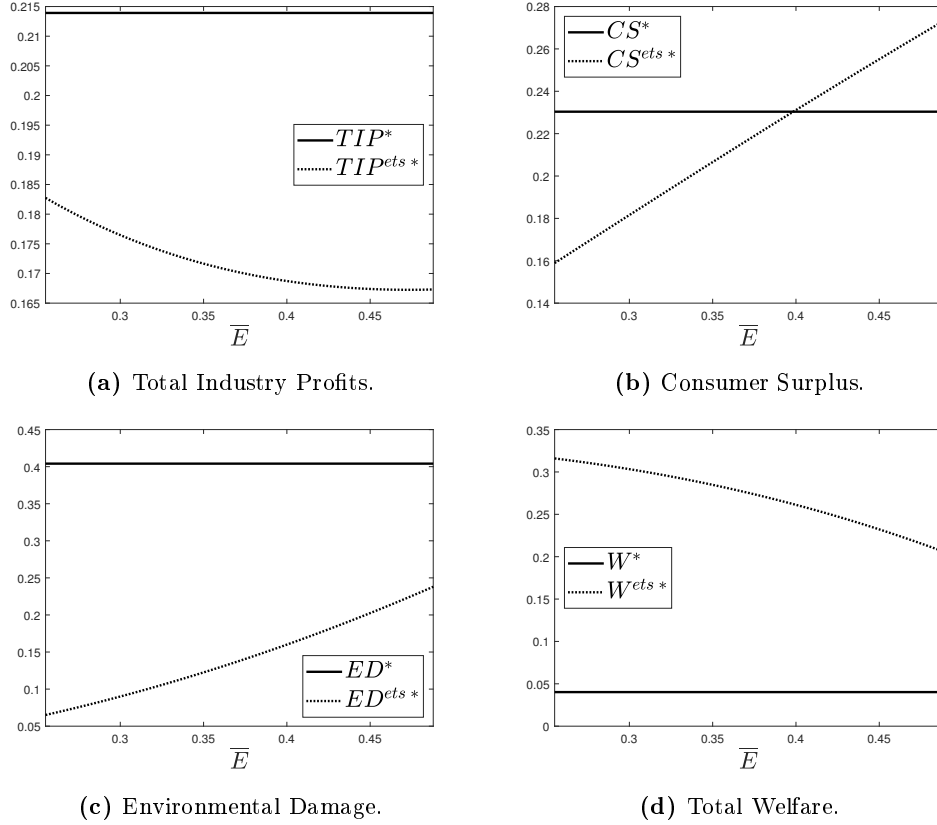
$$PR^* = a [(q_E^{ets*} - z_E^{ets*})x^{ets*} + (q_P^{ets*} - z_P^{ets*})(1 - x^{ets*})] N, \quad (17)$$

while environmental damage  $ED$  is a quadratic function in the polluting emissions,

$$\begin{aligned} ED^* &= d [(q_E^* - z_E^*)x^* + (q_P^* - z_P^*)(1 - x^*)]^2 N, \\ ED^{ets*} &= d [(q_E^{ets*} - z_E^{ets*})x^{ets*} + (q_P^{ets*} - z_P^{ets*})(1 - x^{ets*})]^2 N, \end{aligned} \quad (18)$$

where  $d$  is normalised to 1. Finally, notice that, in steady state  $x^{ets*} \in (0, 1)$ , the expected profits of ECSR and PS firms are equal.

Figure 5 illustrates our findings: in Fig. 5(a), total industry profits are negatively affected by the introduction of an ETS, but their level increases with the increase in stringency. In Fig. 5(b), consumer surplus may increase with the introduction of the ETS if the policy is sufficiently lenient, while it decreases otherwise. Environmental damage is systematically reduced with the ETS, and intuitively falls further as the policy becomes stricter (Fig. 5(c)). In contrast, overall social welfare reaches its maximum level when the stringency is strongest (Fig. 5(d)). This result implies that what determines social welfare at the margin is indeed the level of environmental damage.



**Figure. 5.** The introduction of an ETS and variation of its stringency.

## 6 Concluding remarks

We have analysed the steady-state industry configuration of an oligopoly composed of profit-seeking and environmentally socially responsible firms in an evolutionary setting. Pollution in the industry is regulated through an Emission Trading System scheme that allocates emissions rights to firms.

Our findings show that: (i) the share of ECSR firms increases with the implementation of the ETS policy, (ii) the number of ECSR firms decreases with the stringency of the ETS policy, (iii) in the presence of the ETS, the difference in emissions between PS and ECSR firms widens with increasing stringency, leading to a general reduction in emissions, and (iv) equilibrium profits decline with the introduction of the ETS policy and its increasing stringency. We hope that our results may assist environmental regulators in addressing their policy agenda.

A potentially interesting avenue for future research is to include in the methodology a type of process-integrated abatement technologies. Such abatement methods are relevant across various industries (Hartman et al., 1997). Examples include enhancing automation process control, substituting raw and auxiliary materials, recycling waste, employing low-waste technological processes, and extending the lifespan of materials and process liquids. Typically, an emission abatement technology that alters the production process is modeled such that the cost of emissions reduction is convex in the level of production (Subramanian et al., 2007, Christin et al., 2014, Anand and Giraud-CARRIER, 2020, and Lambertini et al., 2020, among others). While incorporating this assumption may increase the complexity of the analysis, it may not necessarily yield qualitatively similar results to the current approach. The inclusion of process-integrated abatement technologies remains a topic for future investigation.

## Appendix

### Proof of Proposition 1

The Lagrangian functions associated with the problems (5) are:

$$\begin{aligned}\mathcal{L}_E &= O_E + \lambda_1 q_E + \lambda_2 z_E + \lambda_3 (q_E - z_E), \\ \mathcal{L}_P &= \pi_P + \lambda_4 q_P + \lambda_5 z_P + \lambda_6 (q_P - z_P).\end{aligned}$$

where  $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6 \geq 0$  are the Kuhn-Tucker multipliers. The first order conditions with respect to  $q_k$  and  $z_k$  are ( $\gamma - c = 1$ ):

$$\begin{aligned}\frac{\partial \mathcal{L}_E}{\partial q_E} &= 1 - 2q_E - \sum_{i=0}^{m-1} q_i - \sum_{j=1}^{N-m} q_j + \left( q_E + \sum_{i=0}^{m-1} q_i + \sum_{j=1}^{N-m} q_j \right) \beta - \delta + \lambda_1 + \lambda_3 = 0, \\ \frac{\partial \mathcal{L}_E}{\partial z_E} &= -z_E + \delta + \lambda_2 - \lambda_3 = 0, \\ \frac{\partial \mathcal{L}_P}{\partial q_P} &= 1 - 2q_P - \sum_{j=0}^{N-m-1} q_j - \sum_{i=1}^m q_i + \lambda_4 + \lambda_6 = 0, \\ \frac{\partial \mathcal{L}_P}{\partial z_P} &= -z_P + \lambda_5 - \lambda_6 = 0.\end{aligned}\tag{19}$$

Invoking symmetry among firms of the same type, from (19) we obtain the optimality conditions for ECSR firms:

$$\left\{ \begin{array}{l} 1 - (m + 1)q_E - (N - m)q_P + [mq_E + (N - m)q_P] \beta - \delta + \lambda_1 + \lambda_3 = 0, \\ -z_E + \delta + \lambda_2 - \lambda_3 = 0, \\ \lambda_1 q_E = 0, \lambda_1 \geq 0, \\ \lambda_2 z_E = 0, \lambda_2 \geq 0, \\ \lambda_3 (q_E - z_E), \lambda_3 \geq 0, \\ q_E \geq 0, z_E \geq 0, q_E - z_E \geq 0, \end{array} \right. \quad (20)$$

and for PS firms:

$$\left\{ \begin{array}{l} 1 - mq_E - (N - m + 1)q_P + \lambda_4 + \lambda_6 = 0, \\ -z_P + \lambda_5 - \lambda_6 = 0, \\ \lambda_4 q_P = 0, \lambda_4 \geq 0, \\ \lambda_5 z_P = 0, \lambda_5 \geq 0, \\ \lambda_6 (q_P - z_P), \lambda_6 \geq 0, \\ q_P \geq 0, z_P \geq 0, q_P - z_P \geq 0, \end{array} \right. \quad (21)$$

Solving the system (20)-(21), we get:

$$\begin{aligned}
q_E &= \begin{cases} \frac{1-\delta+(\beta-\delta)(N-m)}{N-\beta m+1}, & \text{if } \delta < \frac{[(N-m)\beta+1]}{2(N+1)-(1+\beta)m}, \\ \frac{2+(N-m)\beta}{2(N-\beta m+2)}, & \text{if } \delta \geq \frac{[(N-m)\beta+1]}{2(N+1)-(1+\beta)m}; \end{cases} \\
z_E &= \begin{cases} \delta, & \text{if } \delta < \frac{[(N-m)\beta+1]}{2(N+1)-(1+\beta)m}, \\ \frac{2+(N-m)\beta}{2(N-\beta m+2)}, & \text{if } \delta \geq \frac{[(N-m)\beta+1]}{2(N+1)-(1+\beta)m}; \end{cases} \\
q_P &= \begin{cases} \frac{1-(\beta-\delta)m}{N-\beta m+1}, & \text{if } \delta > \frac{\beta m-1}{m}, \\ \frac{2-\beta m}{2(N-\beta m+2)}, & \text{if } \delta \leq \frac{\beta m-1}{m}; \end{cases} \\
z_P &= \begin{cases} 0, & \text{if } \delta > \frac{\beta m-1}{m}, \\ \frac{2-\beta m}{2(N-\beta m+2)}, & \text{if } \delta \leq \frac{\beta m-1}{m}. \end{cases}
\end{aligned}$$

The condition

$$\delta \in \left( \max \left\{ 0, \frac{\beta m-1}{m} \right\}, \frac{[(N-m)\beta+1]}{2(N+1)-(1+\beta)m} \right), \quad (22)$$

ensures interior solutions. Notice that

$$\frac{[(N-m)\beta+1]}{2(N+1)-(1+\beta)m} < 1.$$

□

## Proof of Corollary 1

From (22), we derive that  $q_E^* - z_E^* > 0$  if

$$\delta < \frac{[(N-m)\beta+1]}{2(N+1)-(1+\beta)m}.$$

Therefore,

$$\delta < \bar{\delta} := \min \left\{ \frac{(\beta N+1)}{2(N+1)}, \frac{1}{(1-\beta)N+2} \right\},$$

is such that  $q_E^* - z_E^* > 0 \forall m \in \{0, 1, 2, \dots, N\}$ . Analogously,  $q_P^* > 0$  if

$$\delta > \frac{[\beta m - 1]}{m}.$$

Therefore, the condition

$$\delta > \underline{\delta} := \max \left\{ 0, \frac{(\beta N - 1)}{N} \right\}$$

is such that  $q_P^* > 0 \forall m \in \{0, 1, 2, \dots, N\}$ . □

## Proof of Proposition 2

Equation  $\pi_{EP}^* - \pi_{PP}^* = 0$  can be rewritten as a quadratic form

$$A_1 \delta^2 + B_1 \delta + C_1 > 0, \tag{23}$$

with

$$A_1 = -\frac{2}{(\beta - 3)^2} - \frac{1}{2} < 0, \quad B_1 = \frac{3\beta - 1}{(\beta - 3)^2}, \quad C_1 = -\frac{2(5\beta - 3)\beta}{9(\beta - 3)^2}.$$

The discriminant of equation (23) is

$$\Delta_1 = \frac{-20\beta^2 + 12\beta + 1}{9(\beta - 3)^2},$$

which is positive for  $\beta \in [0, \hat{\beta}]$ , with  $\hat{\beta} \approx 0.6741$  and negative for  $\beta \in (\hat{\beta}, 1]$ . The solutions of equation (23) are

$$\delta_{1,2} = \frac{3(3\beta + 1) \pm (\beta - 3)\sqrt{-20\beta^2 + 12\beta + 1}}{3(\beta^2 - 6\beta + 13)},$$

with  $\delta_1 < \delta_2$ . Notice that  $\delta_1 \leq 0 \forall \beta \in [0, \check{\beta}_1]$ , with  $\check{\beta}_1 \approx 0.6$ , and  $\delta_1 \geq 0 \forall \beta \in [\check{\beta}_1, \hat{\beta}]$ , while  $\delta_2 \geq 0 \forall \beta \in [0, \hat{\beta}]$ . It always occurs that  $\delta_1 < \underline{\delta}$  while  $\delta_2 > \underline{\delta}$ . Therefore, if  $\beta \in [0, \hat{\beta}]$ , then  $\pi_{EP}^* - \pi_{PP}^* > 0$  for  $\delta \in (\underline{\delta}, \delta_2)$  and  $\pi_{EP}^* - \pi_{PP}^* < 0$  for  $\delta \in (\delta_2, \bar{\delta})$ . Finally, if  $\beta \in (\hat{\beta}, 1]$ , then  $\pi_{EP}^* - \pi_{PP}^* < 0$ .

Analogously, the equation  $\pi_{EE}^* - \pi_{PE}^* = 0$  can be rewritten as a quadratic form

$$A_2 \delta^2 + B_2 \delta + C_2 > 0, \tag{24}$$

with

$$A_2 = -\frac{1}{(\beta-3)^2} - \frac{2}{(2\beta-3)^2} - \frac{1}{2} < 0, \quad B_2 = \frac{10\beta^3 - 43\beta^2 + 54\beta - 9}{(2\beta^2 - 9\beta + 9)^2},$$

and

$$C_2 = -\frac{2\beta-1}{(2\beta-3)^2} - \left(\frac{\beta-1}{\beta-3}\right)^2.$$

The discriminant of (24) is

$$\Delta_2 = \frac{-8\beta^4 + 36\beta^3 - 47\beta^2 + 10\beta + 1}{(2\beta^2 - 9\beta + 9)^2},$$

which is positive for  $\beta \in [0, \tilde{\beta})$ , with  $\tilde{\beta} \approx 0.3647$ , and negative for  $\beta \in (\tilde{\beta}, 1]$ . The solutions of (24) are

$$\delta_{3,4} = \frac{10\beta^3 - 43\beta^2 + 54\beta - 9 \pm (2\beta^2 - 9\beta + 9)\sqrt{-8\beta^4 + 36\beta^3 - 47\beta^2 + 10\beta + 1}}{4\beta^4 - 36\beta^3 + 129\beta^2 - 210\beta + 135},$$

with  $\delta_3 < \delta_4$ . Notice that  $\delta_3 > 0$  for  $\beta \in (\check{\beta}_2, \tilde{\beta})$ , with  $\check{\beta}_2 \approx 0.3223$ , while  $\delta_4 > 0 \forall \beta \in [0, \tilde{\beta})$ .

Since  $\underline{\delta} > 0$  if  $\beta > \frac{1}{2}$ , then both  $\delta_3$  and  $\delta_4$  are always greater than  $\underline{\delta}$ . Therefore, if  $\beta \in [0, \tilde{\beta})$  then

$\pi_{EE}^* - \pi_{PE}^* > 0$  for  $\delta \in (\hat{\delta}, \delta_4)$ , with  $\hat{\delta} = \max\{\underline{\delta}, \delta_3\}$  and  $\pi_{EE}^* - \pi_{PE}^* < 0$  for  $\delta \in (\underline{\delta}, \delta_3) \cup (\delta_4, \bar{\delta})$ .

Finally, if  $\beta \in (\tilde{\beta}, 1]$ , then  $\pi_{EE}^* - \pi_{PE}^* < 0$ .

Since  $\delta_2 > \delta_4 \forall \beta \in [0, \tilde{\beta})$ , we have:

- if  $\beta \in [0, \tilde{\beta})$ :

- $\pi_{EP}^* - \pi_{PP}^* > 0$  and  $\pi_{EE}^* - \pi_{PE}^* > 0$  for  $(\underline{\delta}, \delta_2) \cap (\hat{\delta}, \delta_4) = (\hat{\delta}, \delta_4)$ ;
- $\pi_{EP}^* - \pi_{PP}^* < 0$  and  $\pi_{EE}^* - \pi_{PE}^* < 0$  for  $(\delta_2, \bar{\delta}) \cap ((\underline{\delta}, \delta_3) \cup (\delta_4, \bar{\delta})) = (\delta_4, \bar{\delta})$ ;
- $\pi_{EP}^* - \pi_{PP}^* > 0$  and  $\pi_{EE}^* - \pi_{PE}^* < 0$  for  $(\underline{\delta}, \delta_2) \cap ((\underline{\delta}, \delta_3) \cup (\delta_4, \bar{\delta})) = (\underline{\delta}, \delta_3) \cup (\delta_4, \bar{\delta})$ ;
- $\pi_{EP}^* - \pi_{PP}^* < 0$  and  $\pi_{EE}^* - \pi_{PE}^* > 0$  for  $(\delta_2, \bar{\delta}) \cap (\hat{\delta}, \delta_4) = \emptyset$ .

- if  $\beta \in (\tilde{\beta}, \hat{\beta})$ :

- $\pi_{EP}^* - \pi_{PP}^* > 0$  and  $\pi_{EE}^* - \pi_{PE}^* < 0$  for  $(\underline{\delta}, \delta_2) \cap (\underline{\delta}, \bar{\delta}) = (\underline{\delta}, \delta_2)$ ;
- $\pi_{PE}^* - \pi_{EE}^* < 0$  and  $\pi_{EE}^* - \pi_{PE}^* < 0$  for  $(\delta_2, \bar{\delta}) \cap (\underline{\delta}, \bar{\delta}) = (\delta_2, \bar{\delta})$ .

- if  $\beta \in (\hat{\beta}, 1]$ , then  $\pi_{EP}^* - \pi_{PP}^* < 0$  and  $\pi_{PE}^* - \pi_{EE}^* < 0$  for  $\delta \in (\underline{\delta}, \bar{\delta})$ .

Thus the inner steady state, when existing, is always stable.  $\square$

### Proof of Proposition 3

The Lagrangian functions associated with the problems (12) are:

$$\begin{aligned}\mathcal{L}_E^{ets} &= O_E^{ets} + \lambda_1^{ets} q_E^{ets} + \lambda_2^{ets} z_E^{ets} + \lambda_3^{ets} (q_E^{ets} - z_E^{ets}), \\ \mathcal{L}_P^{ets} &= \pi_P^{ets} + \lambda_4^{ets} q_P^{ets} + \lambda_5^{ets} z_P^{ets} + \lambda_6^{ets} (q_P^{ets} - z_P^{ets}).\end{aligned}$$

where  $\lambda_1^{ets}, \lambda_2^{ets}, \lambda_3^{ets}, \lambda_4^{ets}, \lambda_5^{ets}, \lambda_6^{ets} \geq 0$  are the Khun-Tucker multipliers. The first order conditions with respect to  $q_k^{ets}$  and  $z_k^{ets}$  are ( $\gamma - c = 1$ ):

$$\begin{aligned}\frac{\partial \mathcal{L}_E^{ets}}{\partial q_E^{ets}} &= 1 - 2q_E^{ets} - \sum_{i=0}^{m-1} q_i^{ets} - \sum_{j=1}^{N-m} q_j^{ets} + \left( q_E^{ets} + \sum_{i=0}^{m-1} q_i^{ets} + \sum_{j=1}^{N-m} q_j^{ets} \right) \beta - \delta - \alpha + \lambda_1^{ets} + \lambda_3^{ets} = 0, \\ \frac{\partial \mathcal{L}_E^{ets}}{\partial z_E^{ets}} &= -z_E^{ets} + \delta + \alpha + \lambda_2^{ets} - \lambda_3^{ets} = 0, \\ \frac{\partial \mathcal{L}_P^{ets}}{\partial q_P^{ets}} &= 1 - 2q_P^{ets} - \sum_{j=0}^{N-m-1} q_j^{ets} - \sum_{i=1}^m q_i^{ets} - \alpha + \lambda_4^{ets} + \lambda_6^{ets} = 0, \\ \frac{\partial \mathcal{L}_P^{ets}}{\partial z_P^{ets}} &= -z_P^{ets} + \alpha + \lambda_5^{ets} - \lambda_6^{ets} = 0.\end{aligned}\tag{25}$$

Invoking symmetry among firms of the same type, from (25) we obtain the optimality conditions for ECSR firms:

$$\left\{ \begin{array}{l} 1 - (m+1)q_E^{ets} - (N-m)q_P^{ets} + [mq_E^{ets} + (N-m)q_P^{ets}] \beta - \delta - \alpha + \lambda_1^{ets} + \lambda_3^{ets} = 0, \\ -z_E^{ets} + \delta + \alpha + \lambda_2^{ets} - \lambda_3^{ets} = 0, \\ \lambda_1^{ets} q_E^{ets} = 0, \lambda_1^{ets} \geq 0, \\ \lambda_2^{ets} z_E^{ets} = 0, \lambda_2^{ets} \geq 0, \\ \lambda_3^{ets} (q_E^{ets} - z_E^{ets}), \lambda_3^{ets} \geq 0, \\ q_E^{ets} \geq 0, z_E^{ets} \geq 0, q_E^{ets} - z_E^{ets} \geq 0, \end{array} \right.\tag{26}$$



and for PS firms:

$$\left\{ \begin{array}{l} 1 - mq_E^{ets} - (N - m + 1)q_P^{ets} - \alpha + \lambda_4^{ets} + \lambda_6^{ets} = 0, \\ -z_P + \alpha + \lambda_5^{ets} - \lambda_6^{ets} = 0, \\ \lambda_4^{ets} q_P^{ets} = 0, \lambda_4^{ets} \geq 0, \\ \lambda_5^{ets} z_P^{ets} = 0, \lambda_5^{ets} \geq 0, \\ \lambda_6^{ets} (q_P^{ets} - z_P^{ets}), \lambda_6^{ets} \geq 0, \\ q_P^{ets} \geq 0, z_P^{ets} \geq 0, q_P^{ets} - z_P^{ets} \geq 0, \end{array} \right. \quad (27)$$

Solving the system (26)-(27), we get:

$$\begin{aligned} q_E^{ets} &= \begin{cases} \frac{1-\delta-a+[(1-a)\beta-\delta](N-m)}{N-\beta m+1}, & \text{if } \delta < \frac{1+(N-m)\beta-(N-2m)a\beta-(N+1)a}{2N-(1+\beta)m+2}, \\ \frac{(N-m)\beta+2}{2(N-\beta m+2)}, & \text{if } \delta \geq \frac{1+(N-m)\beta-(N-2m)a\beta-(N+1)a}{2N-(1+\beta)m+2}; \end{cases} \\ z_E^{ets} &= \begin{cases} \delta + \alpha, & \text{if } \delta < \frac{1+(N-m)\beta-(N-2m)a\beta-(N+1)a}{2N-(1+\beta)m+2}, \\ \frac{(N-m)\beta+2}{2(N-\beta m+2)}, & \text{if } \delta \geq \frac{1+(N-m)\beta-(N-2m)a\beta-(N+1)a}{2N-(1+\beta)m+2}; \end{cases} \\ q_P^{ets} &= \begin{cases} \frac{1-a-[(1-a)\beta-\delta]m}{N-\beta m+1}, & \text{if } \delta > \frac{(N+2)a-1+(1-2a)\beta m}{m}, \\ \frac{2-\beta m}{2(N-\beta m+2)}, & \text{if } \delta \leq \frac{(N+2)a-1+(1-2a)\beta m}{m}; \end{cases} \\ z_P^{ets} &= \begin{cases} \alpha, & \text{if } \delta > \frac{(N+2)a-1+(1-2a)\beta m}{m}, \\ \frac{2-\beta m}{2(N-\beta m+2)}, & \text{if } \delta \leq \frac{(N+2)a-1+(1-2a)\beta m}{m}. \end{cases} \end{aligned}$$

Condition

$$\delta \in \left( \max \left\{ 0, \frac{(N+2)a-1+(1-2a)\beta m}{m} \right\}, \min \left\{ \frac{1+(N-m)\beta-(N-2m)a\beta-(N+1)a}{2N-(1+\beta)m+2}, 1 \right\} \right), \quad (28)$$

ensures interior solutions. Notice that if condition (28) is not satisfied, then the allowance price is equal to zero (the border solutions of the system (20)-(21) with no ETS, are the same of system (26)-(27) with ETS).

We now turn to the first stage, where the permit price  $a$  is such that

$$(q_E^{ets} - z_E^{ets})m + (q_P^{ets} - q_P^{ets})(N - m) = \bar{E}$$

Solving, we obtain:

$$a = \frac{N - (N + 1)\bar{E} - [(N + 2)\delta + (\delta m + \bar{E})\beta]m}{(N - \beta m + 2)N} > 0.$$

□

## Proof of Corollary 2

The equilibrium price of the permit  $a^*$  is strictly positive if

$$\delta < \min \left\{ \frac{N - (N + 1 - \beta m)\bar{E}}{(N - \beta m + 2)m}, 1 \right\}.$$

Therefore, the condition

$$\delta < \min \left\{ \frac{N - [(1 - \beta)N + 1]\bar{E}}{[(1 - \beta)N + 2]N}, 1 \right\},$$

guarantees positive allowance price for each market composition. Notice that

$$\frac{N - [(1 - \beta)N + 1]\bar{E}}{[(1 - \beta)N + 2]N} > 0,$$

if

$$\beta > \max \left\{ 0, \frac{(N + 1)\bar{E} - N}{\bar{E}N} \right\}.$$

Substituting the value of  $a^*$  in (28), then  $q_E^{ets*} - z_E^{ets*} > 0$  if

$$\delta < \min \left\{ \frac{[N + 2 + (N - 2m)\beta]\bar{E} + (N - m)\beta N}{2(N - m)(N - \beta m + 2)}, 1 \right\}.$$

Therefore, the condition

$$\delta < \min \left\{ \frac{(N + 2)\bar{E} + (N + \bar{E})\beta N}{2(N + 2)N}, 1 \right\},$$

ensures positive ECSR emissions for each market composition. Analogously,  $q_P^{ets*} - z_P^{ets*} > 0$  if

$$\delta > \max \left\{ 0, \frac{\beta(2\bar{E} + N) - (N + 2)\bar{E}}{2(N - \beta m + 2)m} \right\}.$$

Therefore, the condition

$$\delta > \max \left\{ 0, \frac{(N + 2\bar{E})\beta N - (N + 2)\bar{E}}{2[(1 - \beta)N + 2]N} \right\},$$

guarantees positive PS emissions for each market composition. Notice that

$$\frac{(N + 2\bar{E})\beta N - (N + 2)\bar{E}}{2[(1 - \beta)N + 2]N} < 1,$$

if

$$\beta < \min \left\{ \frac{(2N + \bar{E})(N + 2)}{[2(\bar{E} + N) + N]N}, 1 \right\}.$$

Summarising, we obtain the results of [Corollary 2](#). □

### Proof of [Proposition 4](#)

Equation  $\pi_{EP}^{ets*} - \pi_{PP}^{ets*} = 0$  can be rewritten as a quadratic form

$$A_3\delta^2 + B_3\delta + C_3 > 0, \tag{29}$$

with

$$A_3 = -\frac{5}{8}, \quad B_3 = -\frac{(\beta - 3)\bar{E} + 2(2\beta - 1)}{4(\beta - 4)},$$

and

$$C_3 = -\frac{(8\bar{E} + 70\beta + 27\beta\bar{E} - 48)(\bar{E} + 2)\beta}{128(\beta - 4)^2}.$$

The discriminant of [\(29\)](#) is

$$\Delta_3 = \frac{(56\beta - 144)\bar{E}^2 + 96(5\beta - 2)\bar{E} - (119\bar{E}^2 + 492\bar{E} - 444)\beta^2 + 32(7\beta - 2)}{256(\beta - 4)^2}.$$

Thus, the solutions of (29) are

$$\delta_{5,6} = \frac{-8(2\beta - 1) - 4(\beta + 3) \pm \sqrt{(56\beta - 144)\bar{E}^2 + 96(5\beta - 2)\bar{E} - (119\bar{E}^2 + 492\bar{E} - 444)\beta^2 + 32(7\beta - 2)}}{20(\beta - 4)},$$

with  $\delta_5 < \delta_6$ . Therefore, if  $\Delta_3 > 0$ , then  $\pi_{ep}^* - \pi_{pp}^* > 0$  for  $\delta \in (\tilde{\delta}_5, \tilde{\delta}_6)$ , with  $\tilde{\delta}_1 = \max\{\underline{\delta}, \delta_5\}$  and  $\tilde{\delta}_2 = \min\{\bar{\delta}, \delta_6\}$ . Conversely, if  $\Delta_3 < 0$ , then  $\pi_{ep}^* - \pi_{pp}^* < 0$ .

Similarly, equation  $\pi_{EE}^{ets*} - \pi_{PE}^{ets*} = 0$  can be rewritten as a quadratic form

$$A_4\delta^2 + B_4\delta + C_4 > 0, \quad (30)$$

with

$$A_4 = -\frac{3}{8}, \quad B_4 = \frac{-(\beta + 3)\bar{E} - 2(2\beta - 1)}{4(\beta - 4)},$$

and

$$C_4 = \frac{-(2 + \bar{E})[(49\beta - 8)\bar{E} + 2(73\beta - 24) + (16 + 8\bar{E})\beta^3 - 8(11 + 5\bar{E})\beta^2]\beta}{32(\beta^2 - 6\beta + 8)^2}.$$

The discriminant of (30) is

$$\Delta_4 = \frac{[2(2\beta - 1) + (\beta + 3)\bar{E}]^2}{16(\beta - 4)^2} - \frac{3(2 + \bar{E})[(49\beta - 8)\bar{E} + 2(73\beta - 24) + (16 + 8\bar{E})\beta^3 - 8(11 + 5\bar{E})\beta^2]\beta}{64(\beta^2 - 6\beta + 8)^2}.$$

Hence, the solutions of (30) are

$$\delta_{7,8} = \frac{-(\beta - 6)\bar{E} + (10 - \bar{E}\beta)\beta \pm 8(\beta^2 + 1)\sqrt{F}}{2(3\beta^2 - 18\beta + 24)},$$

with

$$F = -4(10\bar{E}^2 + 16\bar{E} + 8)\beta^4 + 8(16\bar{E}^2 + 57\bar{E} + 26)\beta^3 - (191\bar{E}^2 + 972\bar{E} + 348)\beta^2 - 8(9\bar{E}^2 - 76\bar{E} + 4)\beta + 144\bar{E}^2 + 192\bar{E} + 64.$$

Notice that, if  $\Delta_4 > 0$ , then  $\pi_{EE}^* - \pi_{PE}^* > 0$  for  $\delta \in (\tilde{\delta}_3, \tilde{\delta}_4)$ , with  $\tilde{\delta}_3 = \max\{\underline{\delta}, \delta_7\}$  and  $\tilde{\delta}_4 = \min\{\bar{\delta}, \delta_8\}$ . Conversely, if  $\Delta_4 < 0$ , then  $\pi_{ee}^* - \pi_{pe}^* < 0$ .  $\square$

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