Working Papers
Quantitative Methods for Social Sciences

The interaction between emission tax and insurance in an evolutionary oligopoly

Ilaria Colivicchi, Gianluca Iannucci

Working Paper N. 02/2023

DISEI, Università degli Studi di Firenze
Via delle Pandette 9, 50127 Firenze (Italia) www.disei.unifi.it

The interaction between emission tax and insurance in an evolutionary oligopoly

Ilaria Colivicchi[a] and Gianluca Iannucci[a,*]

[a]Department of Economics and Management, University of Firenze, Florence, Italy

January 31, 2023

Abstract

It is now commonly accepted that polluting companies deeply contribute to climate change. Environmental losses significantly impact companies' profits so they have to manage them through different strategies to survive on the market. The model assumes two types of firms, polluting and non-polluting, playing a Cournot-Nash game. Due to the different impact on the environment, polluting firms have to pay an emission tax. Both types of firms are risk averse and can cover the potential climate change loss choosing an insurance coverage. From the comparative static analysis computed at the equilibrium, it emerges a substitution effect between insurance and taxation. Moreover, insurance can help clean firms to compete with dirty ones. Finally, we endogenize the market structure through an evolutionary setting and we perform comparative dynamics to confirm the interplay of taxation and insurance that arise from analytical results in order to nudge an ecological transition.

Keywords: Evolutionary game dynamics, Endogenous environmental loss, Insurance coverage, Oligopoly market, Emission tax.

JEL Classification: C73, G22, L13.

*Corresponding author. Via delle Pandette 9, 50127 Florence, Italy. *E-mails*: ilaria.colivicchi@unifi.it (I. Colivicchi), gianluca.iannucci@unifi.it (G. Iannucci).

1 Introduction

The economic impact that a company can have from potential loss due to climate change can be so significant as to cause it to go out of business. Indeed, several international organizations, as the World Meteorological Organization of the United Nations and the Climate Action of the European Commission, show how the rising of the global temperature, caused by production activities, has generated more frequent and severe extreme weather events, such as cold and heat waves, floods, droughts, wildfires and storms.[1] Moreover, the Working Group II contribution to the IPCC Sixth Assessment Report assesses the impacts of climate change, considering ecosystems, biodiversity, and human communities at different levels, global and regional (IPCC, 2022). Due to these reasons, recently, the number of papers dealing with natural disaster caused by climate change and their economic impact grows up in the economic literature. Among others, Tol (2018) reviews the economic impacts of climate change and the policy that can be implemented. Moreover, Botzen et al. (2019) collect theoretical and empirical methods to mitigate disaster impacts while Kolstad and Moore (2020) measure the effect of climate on economic outcomes estimating the costs of climate change impacts.

It is nowadays consolidated that polluting companies contribute to increase the probability of an environmental loss that all the firms operating on the market have to bear. Regulators around the world have adopted policies and instruments to mitigate the climate impact on the ecosystem and in this field a lot of studies consider emission taxes useful to regulate firm emission reductions and several papers analyze the potential interaction with green innovation (see, among others, Feess and Muehlheusser 2002, Feichtinger et al. 2016, Yi et al. 2021). In other studies, Chen et al. (2020) investigate how a carbon emissions taxation scheme can be structured to reduce emission considering different green technology investment while Bian and Guo (2022) focus on the effect of two environmental policies, the emission abatement subsidy and emission tax policies. It is a common assumption that polluting companies have to pay an emission tax imposed by the regulator to continue their activity. The cost for taxation contribute to decrease the total profit that could not be sufficient to bear a potential environmental loss. Recent researches deal with the effects of various financial incentives that compulsory insurance coverage could provide (Mol

[1] Click here for the World Meteorological Organization reports and here for the European Commission ones.

et al., 2020). Other scholars focus on financial instruments as adaptation to environmental risk (Castelli et al., 2019).

In this strand of the literature, the insurance sector is not present, although it plays an essential role in the transfer of risk due to climate change. Indeed, as pointed out by Schlesinger (2013), a risk averse firm can reduce the impact of the potential loss through an insurance coverage. Seog (2006) focuses on the corporate demand for insurance under duopoly. The paper highlights how firms purchase insurance in order to enhance their competitiveness. A higher level of corporate insurance makes a firm more aggressive respect to the others not insured. Colivicchi and Iannucci (2022) analyze whether it is possible that gaining an insurance coverage plays an active role towards an overall improvement of the environment. The insurer could offer, indeed, a coverage to reduce the economic impact of potential environmental loss and allow firms to remain alive on the market.

This work presents a two-stages game in which two types of companies, polluting and non-polluting, maximize their profit choosing the quantities of production. Firms are risk averse and they can protect themselves against the climate change loss through an insurance coverage, reducing the potential negative impact on profits. The optimal level of the coverage is chosen according to the expected utility criterion. Firms operate in a Cournot-Nash game and consider the loss of climate change as endogenous in the profit maximization problem. The study shows that different business strategies can coexist on the market. Even if the presence of insurance coverage is a strategic key to survive on the market, unfortunately the model points out that the insurance itself is not enough to push a polluting company to favor a more voluntary and environmental friendly business. An interaction with the emission tax is required and this represents a novelty of the paper.

The results of the one-shot game are confirmed endogenizing the market structure. Indeed, in the second part of the paper, firms can select their strategy according to a random matching evolutionary game. The dynamics is determined by the replicator equation, a selection process in which agents compare the expected payoffs of existing alternative strategies adopted on the market and choose the most profitable one (for further details see, among others, Weibull 1995, Cressman 2003, Sandholm 2010).

The paper is structured as follows. In Section 2 we introduce the model and in Section 3 we perfome comparative statics. The evolutionary setting is presented in Section 4 while Section 5 deals with comparative dynamics. Section 6 concludes.

2 The model

Let us consider a Cournot oligopoly composed of $N \geqslant 2$ firms that produce a unique homogeneous good. The firms can manage their business according to two different strategies: polluting and non-polluting. The first typology of companies uses a polluting production technology (in the following we will call them dirty firms, d) and for this reason these companies support an environmental tax over their production quantities. The other ones adopt a zero-carbon technology that allows them not to pay the emission tax (in the following these companies will be called clean firms, c).

Denoting $m \in \{0, 1, \ldots, N\}$ as the number of clean firms and $N - m$ as the number of dirty firms, the output price is given by the following linear inverse demand:

$$p = \gamma - \sum_{c=1}^{m} q_c - \sum_{d=1}^{N-m} q_d \tag{1}$$

where $\gamma > 0$ is the output market reservation price while $\sum_{c=1}^{m} q_c$ and $\sum_{d=1}^{N-m} q_d$ represent the quantities produced by clean and dirty firms, respectively. Dirty firms' production generates pollution which in turn causes natural extreme events. For the sake of simplicity, we assume that this risk can affect each firm with the same probability $\eta \in (0, 1)$ and with the same monetary damage. The loss function is distributed as follows:

$$\widetilde{L} = (l, \eta;\ 0, 1 - \eta) \tag{2}$$

where

$$l = \delta \sum_{d=1}^{N-m} q_d \tag{3}$$

with $\delta \in (0, 1)$ that represents an impact parameter. Both typology of firms are risk averse and can protect themselves from the potential loss through an insurance climate change coverage, according to the choice of a coinsurance rate $\beta_i \in [0, 1]$, with $i = c, d$, where $\beta_i = 0$ means that the firm is not insured while $\beta_i = 1$ means full coverage. The insurance premium is given by the expected value principle:

$$P_i = (1+\lambda)\beta_i \mathbb{E}(\widetilde{L}) \tag{4}$$

where $\lambda \in [0, 1)$ is the loading factor (if $\lambda = 0$ the premium is fair and the optimal choice by the firm is full coverage, see Mossin 1968), and $\mathbb{E}(\widetilde{L})$ is the expected value of the loss:

$$\mathbb{E}(\widetilde{L}) = \eta l = \eta\delta \sum_{d=1}^{N-m} q_d$$

The random profit function is composed of the operative profit (namely the difference between output revenues and production costs, denoted as R_i) minus the insurance premium and the random loss:

$$\widetilde{\pi}_i = R_i - P_i - (1-\beta_i)\widetilde{L}$$

The game develops in two stages with a discount rate equals to zero and it is solved in backward induction. Firms choose the quantity in the first stage and then select the coinsurance rate that maximizes the expected utility of their profit function, $Eu(\widetilde{\pi}_i)$. Adopting a logarithmic utility function (due to the risk aversion it is increasing and concave w.r.t. the risk), the problem of choosing the optimal coinsurance rate is:

$$\max_{\beta_i \in [0,1]} Eu(\widetilde{\pi}_i) = \ln(R_i - P_i - (1-\beta_i)\widetilde{L}) \tag{5}$$

Proposition 1 *The optimal insurance coverage is:*

$$\beta_i^* = \frac{(1-\eta)(1+\lambda)l - \lambda R_i}{[1-(1+\lambda)\eta](1+\lambda)l} \tag{6}$$

Proof. Substituting (4), the first order conditions of the maximization problem (5) are:

$$\frac{\partial Eu(\widetilde{\pi}_i)}{\partial \beta_i} = \left[\frac{-(1+\lambda)\eta l\beta_i + l\beta_i}{R_i - (1+\lambda)\eta l\beta_i - (1-\beta_i)l}\right]\eta + \left[\frac{-(1+\lambda)\eta l\beta_i}{R_i - (1+\lambda)\eta l\beta_i}\right](1-\eta) = 0$$

From which we obtain the value of (6). □

The optimal coinsurance rate can be interpreted as the firm's demand for insurance.

Corollary 2 *The denominator of (6) is positive if* $1-(1+\lambda)\eta > 0$*, that it is true if* $\lambda < \frac{1-\eta}{\eta}$*. Therefore, the condition* $\eta < \frac{1}{2}$ *guarantees that* $\lambda < 1$*.*

Remark 3 *If the market is composed of only clean firms, then the risk disappears and the optimal level of insurance coverage is zero:*

$$\lim_{m \to N} \beta_i^* = 0$$

Recall that only dirty firms pollute (see (3)).

The operating profits for each typology of firms are:

$$R_c = (p - c_c)q_c$$

$$R_d = (p - c_d - \tau)q_d$$

where $\tau > 0$ is the emission tax and c_i represents the production cost. We assume that $c_c > c_d$, namely the clean technology is more costly than the dirty one. The maximization problem of the random profit is:

$$\max_{q_i \in [0,+\infty)} \widetilde{\pi}_i = R_i - (1 + \lambda\beta_i^*)\eta l \tag{7}$$

Proposition 4 *The optimal values of the quantities are:*

$$\begin{aligned} q_c^* &= \begin{cases} 0 & \text{if } \tau \leqslant \frac{[c_c - c_d - (1+\lambda)\eta\delta](N-m) - (\gamma - c_c)}{N-m} \\ \frac{\gamma - c_c - [c_c - c_d - \tau - (1+\lambda)\eta\delta](N-m)}{N+1} & \text{if } \tau > \frac{[c_c - c_d - (1+\lambda)\eta\delta](N-m) - (\gamma - c_c)}{N-m} \end{cases} \\ q_d^* &= \begin{cases} 0 & \text{if } \tau \geqslant \frac{\gamma - c_d - (1+\lambda)\eta\delta + [c_c - c_d - (1+\lambda)\eta\delta]m}{m+1} \\ \frac{\gamma - c_d - \tau - (1+\lambda)\eta\delta + [c_c - c_d - \tau - (1+\lambda)\eta\delta]m}{N+1} & \text{if } \tau < \frac{\gamma - c_d - (1+\lambda)\eta\delta + [c_c - c_d - (1+\lambda)\eta\delta]m}{m+1} \end{cases} \end{aligned} \tag{8}$$

Proof. The first order conditions of maximization problem (7) are:

$$\frac{\partial \widetilde{\pi}_i}{\partial q_i} = \frac{\partial R_i}{\partial q_i} - \frac{\partial l}{\partial q_i}\eta - \left(\frac{\partial \beta_i^*}{\partial q_i} l + \frac{\partial l}{\partial q_i}\beta_i^* \right) \lambda\eta$$

where

$$
\begin{aligned}
\frac{\partial R_c}{\partial q_c} &= \gamma - 2q_c - \sum_{j=0}^{m-1} q_j - \sum_{d=1}^{N-m} q_d - c_c \\
\frac{\partial R_d}{\partial q_d} &= \gamma - 2q_d - \sum_{c=1}^{m} q_c - \sum_{s=0}^{N-m-1} q_s - c_d - \tau \\
\frac{\partial l}{\partial q_c} &= 0 \\
\frac{\partial l}{\partial q_d} &= \delta \\
\frac{\partial \beta_i^*}{\partial q_i} &= \frac{\left(\frac{\partial l}{\partial q_i} R_i - \frac{\partial R_i}{\partial q_i} l\right)\lambda}{[1-(1+\lambda)\eta](1+\lambda)l^2}
\end{aligned}
$$

in which $\sum_{j=0}^{m-1} q_j$ is the total output of the clean rivals while $\sum_{s=0}^{N-m-1} q_s$ is the total output of the dirty rivals. Invoking symmetry between firms of the same type, we get the values of (8). □

Corollary 5 *The condition $\tau \in (\underline{\tau}, \overline{\tau})$, where*

$$
\begin{aligned}
\underline{\tau} :&= \frac{[c_c - c_d - (1+\lambda)\eta\delta]N - (\gamma - c_c)}{N} \\
\overline{\tau} :&= \frac{\gamma - c_d - (1+\lambda)\eta\delta + [c_c - c_d - (1+\lambda)\eta\delta]N}{N+1}
\end{aligned}
\tag{9}
$$

guarantees positive quantities for all market composition (namely $q_c, q_d > 0 \; \forall m$).

Proof. Since

$$
\frac{\partial \left\{ \frac{[c_c - c_d - (1+\lambda)\eta\delta](N-m) - (\gamma - c_c)}{N-m} \right\}}{\partial m} = \frac{-(\gamma - c_c)}{(N-m)^2} < 0
$$

then the condition

$$
\tau > \underline{\tau} := \frac{c_c - \gamma + [c_c - c_d - (1+\lambda)\eta\delta]N}{N}
$$

is sufficient to guarantee $q_c^* > 0 \, \forall m$. Similarly, since

$$
\frac{\partial \left\{ \frac{\gamma - c_d - (1+\lambda)\eta\delta + [c_c - c_d - (1+\lambda)\eta\delta]m}{m+1} \right\}}{\partial m} = \frac{-(\gamma - c_c)}{(m+1)^2} < 0
$$

then the condition

$$\tau < \overline{\tau} := \frac{\gamma - c_d - (1+\lambda)\eta\delta + [c_c - c_d - (1+\lambda)\eta\delta]N}{N+1}$$

is sufficient to guarantee $q_d^* > 0 \, \forall m$. Notice that $\underline{\tau} < \overline{\tau}$ if $\gamma - c_c > 0$, that is always true. $\square$

The model therefore provides indications for the regulator regarding the amount of the emission tax that must be defined in the interval $(\underline{\tau}, \overline{\tau})$, independently from the composition in term of polluting and zero-emission firms on the market. From profit maximization's first order conditions we can rewrite the optimal random profits as:

$$\begin{aligned} \widetilde{\pi}_c^* &= (q_c^*)^2 - (1+\lambda\beta_c^*)(N-m)\eta\delta q_d^* \\ \widetilde{\pi}_d^* &= (q_d^*)^2 - (1+\lambda\beta_d^*)(N-m)\eta\delta q_d^* \end{aligned} \tag{10}$$

As standard in oligopoly theory we assume that profits are strictly positive.

3 Comparative statics

In this section we analyze the effect of emission tax and market composition on optimal insurance coverage and optimal profits.

Proposition 6 *Given the market composition, an increase of the emission tax decreases the optimal insurance coverage of clean firms while increases the optimal insurance coverage of dirty firms.*

Proof. Differentiating (6) w.r.t. the emission tax we get:

$$\frac{\partial \beta_i^*}{\partial \tau} = \frac{\left[\frac{\partial l}{\partial \tau}(q_i^*)^2 - \frac{\partial (q_i^*)^2}{\partial \tau}(N-m)\delta q_d^*\right]\lambda}{[1-(1+\lambda)\eta](1+\lambda)[(N-m)\delta q_d^*]^2}$$

Since

$$\frac{\partial q_c^*}{\partial \tau} = \frac{N-m}{N+1} \geqslant 0$$
$$\frac{\partial q_d^*}{\partial \tau} = \frac{-(m+1)}{N+1} < 0$$
$$\frac{\partial l}{\partial \tau} = (N-m)\frac{\partial q_d^*}{\partial \tau}\delta \leqslant 0$$

we derive that $\frac{\partial \beta_c^*}{\partial \tau} < 0$, for $m \neq N$ (see Remark 3). Differently,

$$\frac{\partial \beta_d^*}{\partial \tau} = \frac{(m+1)\lambda}{(N-m)(N+1)(1+\lambda)[1-(1+\lambda)\eta]\delta} > 0$$

for $m \neq N$ (see Remark 3). □

A higher emission tax provokes a decrease in the polluting production and consequently a low potential climate change loss. We can observe a substitution effect for the clean firms in terms of reduction of the coinsurance rate, while the dirty ones increase the coverage due to the reduction in the profit for the lower production. This income effect determines a lower ability to cover the potential loss and, therefore, an increase of the demand for insurance coverage. Indeed, lower operative profits would not be able to cover both the high costs of the tax and the possible environmental loss.

Proposition 7 *An increase of the number of clean firms decreases the demand for insurance of both clean and dirty firms.*

Proof. Differentiating (6) w.r.t. m, we get:

$$\frac{\partial \beta_i^*}{\partial m} = \frac{\left(\frac{\partial l}{\partial m}R_i - \frac{\partial R_i}{\partial m}l\right)\lambda}{[1-(1+\lambda)\eta](1+\lambda)l^2}$$

Therefore,

$$\begin{aligned}\text{sign}\left(\frac{\partial \beta_i^*}{\partial m}\right) &= \text{sign}\left([(N-m)\frac{\partial q_d^*}{\partial m} - q_d^*]\delta(q_i^*)^2 - (N-m)\frac{\partial q_i^*}{\partial m}2\delta q_i^* q_d^*\right) \\ &= \text{sign}\left((N-m)\left(\frac{\partial q_d^*}{\partial m}q_i^* - \frac{\partial q_i^*}{\partial m}2q_d^*\right) - q_d^* q_i^*\right)\end{aligned}$$

Since $\frac{\partial q_c^*}{\partial m} = c_c - c_d - \tau - (1+\lambda)\eta\delta = \frac{\partial q_d^*}{\partial m}$, then $\text{sign}\left(\frac{\partial \beta_c^*}{\partial m}\right) = \text{sign}\left(\frac{\partial \beta_d^*}{\partial m}\right)$. Differentiating β_d^* w.r.t. m, we obtain:

$$\frac{\partial \beta_d^*}{\partial m} = \frac{-\{\gamma - c_d - \tau - (1+\lambda)\eta\delta + [c_c - c_d - \tau - (1+\lambda)\eta\delta]N\}\lambda}{[1-(1+\lambda)\eta](1+\lambda)(N-m)^2(N+1)\delta} < 0$$

negative if $\tau < \overline{\tau}$. □

More clean firms means lower potential environmental loss and, therefore, all types of firms reduces the insurance cost.

Proposition 8 *If the emission tax is relatively low, then clean firms demand more insurance than dirty firms.*

Proof. From (6) it holds that $\beta_c^* > \beta_d^*$ if and only if $q_c^* < q_d^*$. According to (8), $q_c^* < q_d^*$ holds only if

$$\tau < \breve{\tau} := c_c - c_d - (1+\lambda)\eta\delta \tag{11}$$

with $\breve{\tau} \in (\underline{\tau}, \overline{\tau})$. □

In the case of a low emission tax, polluting companies will have an incentive to increase the quantity of goods produced compared to clean ones. The latter will therefore have to bear a decrease in their profits which may not be able to cover an expected climate loss. The plan of action that emerges for clean companies is to protect themselves by increasing the coinsurance rate. This result can be interpreted as a support of the insurance market for clean firms. The opposite plan of action will occur in the case of an increase of the emission tax.

Proposition 9 *An increase of the emission tax favors the clean strategy, increasing the optimal profit of clean firms and decreasing the optimal profit of dirty firms.*

Proof. From (10), we get:

$$\frac{\partial \widetilde{\pi}_i^*}{\partial \tau} = \frac{\partial q_i^*}{\partial \tau} 2q_i^* - \left[\frac{\partial \beta_i^*}{\partial \tau}\lambda q_d^* + (1+\lambda\beta_i^*)\frac{\partial q_d^*}{\partial \tau}\right](N-m)\eta\delta$$

Since $\frac{\partial q_c^*}{\partial \tau} \geqslant 0$, $\frac{\partial \beta_c^*}{\partial \tau} < 0$, and $\frac{\partial q_d^*}{\partial \tau} < 0$ (see Proposition 6), it is easy to check that $\frac{\partial \widetilde{\pi}_c^*}{\partial \tau} > 0$. Differently, to understand the behavior of dirty profits w.r.t. the emission tax, we re-arrange the first derivative as:

$$\frac{\partial \widetilde{\pi}_d^*}{\partial \tau} = [2q_d^* - (1 + \lambda \beta_d^*)(N-m)\eta\delta] \frac{\partial q_d^*}{\partial \tau} - (N-m)\frac{\partial \beta_d^*}{\partial \tau}\lambda\eta\delta q_d^*$$

since $\frac{\partial \beta_d^*}{\partial \tau} > 0$ (see Proposition 6), it holds that $\frac{\partial \widetilde{\pi}_d^*}{\partial \tau} < 0$ if $2q_d^* - (N-m)(1+\lambda\beta_d^*)\eta\delta > 0$, that is always true assuming positive optimal random profits. □

From Proposition 9 we have an interesting indication about firm behavior in presence of taxation. An increase of the emission tax favors the clean strategy, increasing the optimal profit of clean firms and decreasing the one of the dirty's. In this case dirty firms could not be able to support even a potential environmental loss having no sufficient positive profit. The presence of an insurance coverage can bear this loss. We can observe in this case that an interesting interaction between the regulator and the insurance sector is necessary. Only a combined action of both will have a positive effect on the market to nudge the agents to play in compliant with an environmental transition.

4 Evolutionary dynamics

Assume now an infinite population of firms composed of both clean and dirty. At each time period, N firms are randomly selected to play the game described above. Therefore, the payoff (i.e., the profit) of adopting a strategy is a function of the market composition. Before choosing β_i^* and q_i^*, each firm observes the composition of the $N-1$ rivals and computes the value of both payoffs ($\widetilde{\pi}_c^*$ and $\widetilde{\pi}_d^*$) for every possible market composition. This approach has been introduced by Droste et al. (2002) to a duopoly and recently developed, among others, by De Giovanni and Lamantia (2016), Hommes et al. (2018), Kopel and Lamantia (2018), Lamantia et al. (2018), Tichỳ et al. (2020) to an oligopoly.

We denote $x \in [0,1]$ as the probability that a firm adopts the clean strategy. If the game is repeatedly played, the probability x can be interpreted as the share of clean firms on the market, and $1-x$ as the share of dirty firms. Denoting $k \in \{0, 1, \ldots, N-1\}$ as the number of rivals that adopt the clan strategy, the expected profit of the clean firm is:

$$\mathbb{E}(\widetilde{\pi}_c^*(x)) = \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \widetilde{\pi}_c^*(k+1)$$

where

$$\widetilde{\pi}_c^*(k+1) = [q_c^*(k+1)]^2 - [1 + \lambda\beta_c^*(k+1)](N-k-1)\delta q_d^*(k+1)$$

Analogously, the expected profit of the dirty firm is:

$$\mathbb{E}(\widetilde{\pi}_d^*(x)) = \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \widetilde{\pi}_d^*(k)$$

where

$$\widetilde{\pi}_d^*(k) = [q_d^*(k)]^2 - [1 + \lambda\beta_d^*(k)](N-k)\delta q_d^*(k)$$

The time evolution of the share x is given by the following replicator dynamics (see, for applications to insurance, Antoci et al. 2019, Galeotti et al. 2020):

$$\dot{x} = x(1-x)\left[\mathbb{E}(\widetilde{\pi}_c^*(x)) - \mathbb{E}(\widetilde{\pi}_d^*(x))\right] \tag{12}$$

The replicator equation admits three types of steady states: $\widehat{x} = 0$, in which all firms are dirty, $\widehat{x} = 1$ in which all firms are clean, and an inner state, $\widehat{x} \in (0,1)$, in which there is coexistence between firms. Only stable steady states are Nash equilibria. Therefore, denoting $\widehat{x}^*$ as a stable steady state, the corner ones $\widehat{x}^* \in \{0,1\}$ are pure Nash equilibria, while the inner $\widehat{x}^* \in (0,1)$ is a mixed-strategy Nash equilibrium (see, for further details, Hofbauer and Sigmund 2003).

We denote $\widetilde{\pi}_{cd}^*$ as the optimal random profit of a clean firm if it encounters on the market only dirty firms ($k = 0$, namely $m = 1$ in the above game), $\widetilde{\pi}_{cc}^*$ as the optimal random profit of a clean firm if it encounters on the market only clean firms ($k = N-1$, namely $m = N$). Analogously, we denote $\widetilde{\pi}_{dd}^*$ as the optimal random profit of a dirty firm if it encounters on the market only dirty firms ($k = 0$, namely $m = 0$), $\widetilde{\pi}_{dc}^*$ as the optimal random profit of a dirty firm if it encounters on the market only clean firms ($k = N-1$, namely $m = N-1$). The expected profits can be represented as affine functions of the probability x over the bounded and limited interval $[0,1]$:

$$\begin{aligned}\mathbb{E}(\widetilde{\pi}_c^*(x)) &= \widetilde{\pi}_{cd}^* + (\widetilde{\pi}_{cc}^* - \widetilde{\pi}_{cd}^*)x \\ \mathbb{E}(\widetilde{\pi}_d^*(x)) &= \widetilde{\pi}_{dd}^* + (\widetilde{\pi}_{dc}^* - \widetilde{\pi}_{dd}^*)x\end{aligned} \tag{13}$$

Therefore, if $\widetilde{\pi}_{cd}^* - \widetilde{\pi}_{dd}^* < 0$ and $\widetilde{\pi}_{cc}^* - \widetilde{\pi}_{dc}^* < 0$, then $\mathbb{E}(\widetilde{\pi}_c^*(x)) < \mathbb{E}(\widetilde{\pi}_d^*(x))$ $\forall x$ and so $\widehat{x}^* = 0$ (see Fig. 1(a)), while if $\widetilde{\pi}_{cd}^* - \widetilde{\pi}_{dd}^* > 0$ and $\widetilde{\pi}_{cc}^* - \widetilde{\pi}_{dc}^* > 0$, then $\mathbb{E}(\widetilde{\pi}_c^*(x)) > \mathbb{E}(\widetilde{\pi}_d^*(x))$ $\forall x$ and so $\widehat{x}^* = 1$ (see Fig. 1(b)). Differently, if

$$\widehat{x} = \frac{\widetilde{\pi}_{cd}^* - \widetilde{\pi}_{dd}^*}{\widetilde{\pi}_{dc}^* - \widetilde{\pi}_{dd}^* - \widetilde{\pi}_{cc}^* + \widetilde{\pi}_{cd}^*}$$

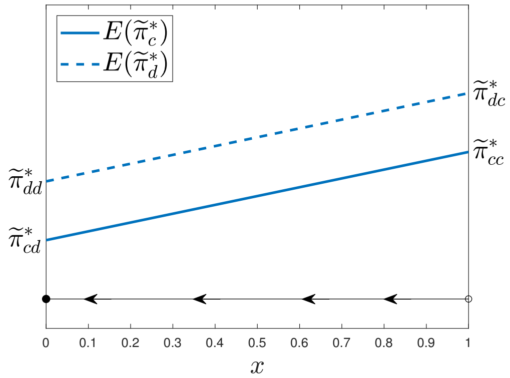
then $\mathbb{E}(\widetilde{\pi}_c^*(x)) = \mathbb{E}(\widetilde{\pi}_d^*(x))$. This inner steady state is unstable if $\widetilde{\pi}_{cd}^* - \widetilde{\pi}_{dd}^* < 0$ and $\widetilde{\pi}_{cc}^* - \widetilde{\pi}_{dc}^* > 0$ (see Fig. 1(c)), while it is stable if $\widetilde{\pi}_{cd}^* - \widetilde{\pi}_{dd}^* > 0$ and $\widetilde{\pi}_{cc}^* - \widetilde{\pi}_{dc}^* < 0$ (see Fig. 1(d)). The following proposition holds.

Proposition 10 *Suppose the existence of $\widehat{x} \in (0,1)$ such that $\mathbb{E}(\widetilde{\pi}_c^*(x)) = \mathbb{E}(\widetilde{\pi}_d^*(x))$. The inner steady state is unstable if $(\widehat{\tau}_1^{x0}, \widehat{\tau}_2^{x0}) \cap (\widehat{\tau}_1^{x1}, \widehat{\tau}_2^{x1}) \neq \varnothing$. Conversely, it is stable if $\Big((\underline{\tau}, \widehat{\tau}_1^{x0}) \cup (\widehat{\tau}_2^{x0}, \overline{\tau})\Big) \cap \Big((\underline{\tau}, \widehat{\tau}_1^{x1}) \cup (\widehat{\tau}_2^{x1}, \overline{\tau})\Big) \neq \varnothing$. Where $\widehat{\tau}_1^{x_0}$ and $\widehat{\tau}_2^{x_0}$ are the solutions of the equation $\widetilde{\pi}_{cd}^* - \widetilde{\pi}_{dd}^* = 0$, with $\underline{\tau} \leqslant \widehat{\tau}_1^{x_0} < \widehat{\tau}_2^{x_0} \leqslant \overline{\tau}$, while $\widehat{\tau}_1^{x_1}$ and $\widehat{\tau}_2^{x_1}$ are the solutions of the equation $\widetilde{\pi}_{cc}^* - \widetilde{\pi}_{dc}^* = 0$, with $\underline{\tau} \leqslant \widehat{\tau}_1^{x_1} < \widehat{\tau}_2^{x_1} \leqslant \overline{\tau}$.*

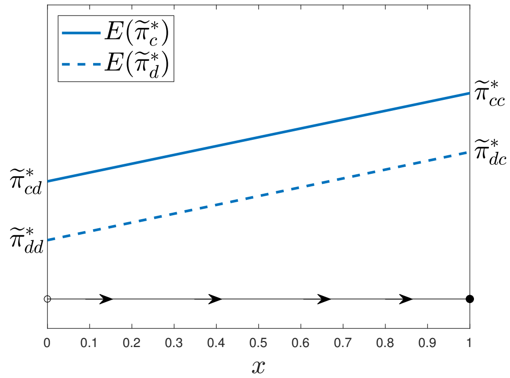
Proof. The inequality $\widetilde{\pi}_{cd}^* - \widetilde{\pi}_{dd}^* > 0$ can be expressed as a quadratic function of the emission tax: $A\tau^2 + B\tau + C$, where $A = (1-\eta)N^2 + (1-2\eta)\lambda N^2 - (1-\eta)2N - (1-2\eta)2\lambda N$. Since $\lambda \in [0,1)$, $\eta \in (0, \frac{1}{2})$ and $N \geqslant 2$, then $A > 0$ and so the parabola is concave up. Assuming $\Delta > 0$, it holds $\widetilde{\pi}_{cd}^* - \widetilde{\pi}_{dd}^* > 0$ $\forall \tau \in (\underline{\tau}, \widehat{\tau}_1^{x0}) \cup (\widehat{\tau}_2^{x0}, \overline{\tau})$, with $\underline{\tau} \leqslant \widehat{\tau}_1^{x_0} < \widehat{\tau}_2^{x_0} \leqslant \overline{\tau}$.

Similarly, the inequality $\widetilde{\pi}_{cc}^* - \widetilde{\pi}_{dc}^* < 0$, can be expressed as a quadratic function of the emission tax: $D\tau^2 + E\tau + F$, where $D = \frac{-\eta\lambda^2 N^2}{[1-(1+\lambda)\eta](1+\lambda)(N+1)^2} - \frac{N^2}{(N+1)^2}$. Since $D < 0$, then the parabola is concave down. Assuming $\Delta > 0$, it holds $\widetilde{\pi}_{cc}^* - \widetilde{\pi}_{dc}^* < 0$ $\forall \tau \in (\underline{\tau}, \widehat{\tau}_1^{x1}) \cup (\widehat{\tau}_2^{x1}, \overline{\tau})$, with $\underline{\tau} \leqslant \widehat{\tau}_1^{x_1} < \widehat{\tau}_2^{x_1} \leqslant \overline{\tau}$.

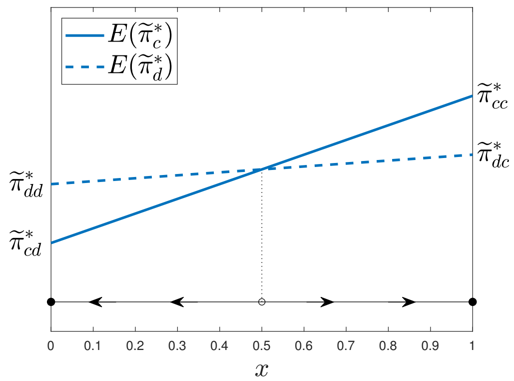
Therefore, we derive that $\widehat{x} \in (0,1)$ is unstable if $(\widehat{\tau}_1^{x0}, \widehat{\tau}_2^{x0}) \cap (\widehat{\tau}_1^{x1}, \widehat{\tau}_2^{x1}) \neq \varnothing$, while it is stable if $\Big((\underline{\tau}, \widehat{\tau}_1^{x0}) \cup (\widehat{\tau}_2^{x0}, \overline{\tau})\Big) \cap \Big((\underline{\tau}, \widehat{\tau}_1^{x1}) \cup (\widehat{\tau}_2^{x1}, \overline{\tau})\Big) \neq \varnothing$. $\square$



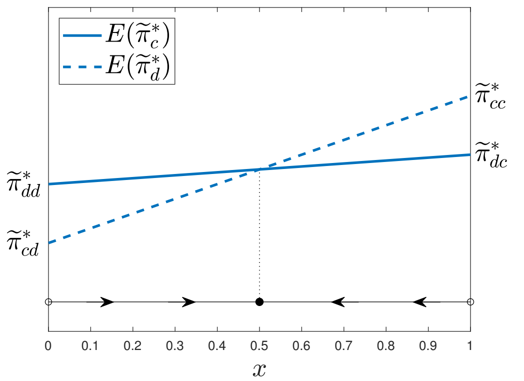
(a) The dirty strategy dominates the clean strategy.



(b) The clean strategy dominates the dirty strategy.



(c) No dominance of strategy, the inner equilibrium is unstable.

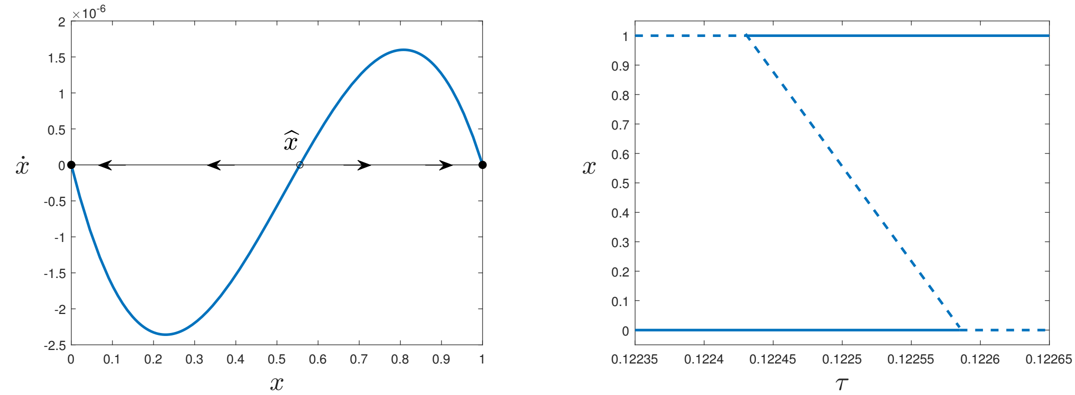


(d) No dominance of strategy, the inner equilibrium is stable.

Fig. 1. Dominance of strategies.

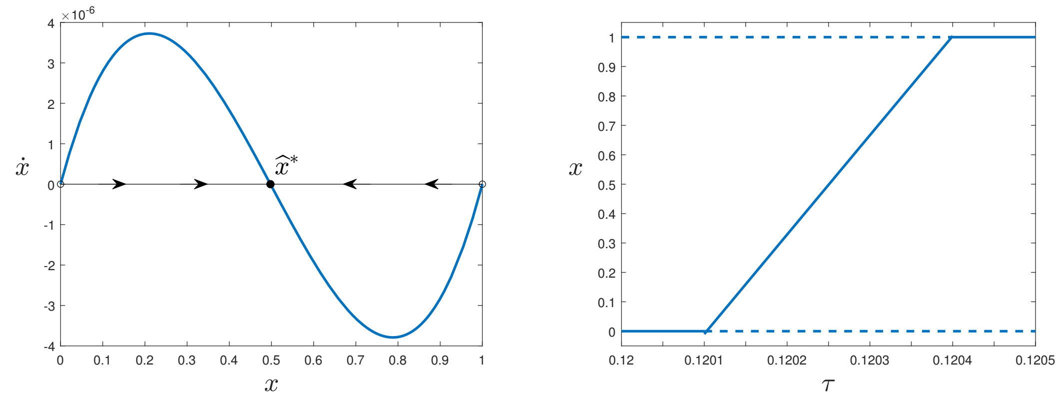
5 Comparative dynamics

In this section we analyze numerically the dynamics (12) at changes values of the emission tax. Fig. 2(a) shows how the inner unstable steady state separates the two basins of attraction. Indeed, denoting x_0 as the initial conditions, if $x_0 \in (0, \widehat{x})$, with $\widehat{x} < 1$, then all the trajectories approach the equilibrium in which all firms are dirty. Conversely, if $x_0 \in (\widehat{x}, 1)$, with $\widehat{x} > 0$, then all the trajectories approach the equilibrium in which all firms are clean. This means that, except the case $x_0 = \widehat{x} \in (0, 1)$, the market will be composed of only one type of firms. Moreover, if the inner steady state is unstable, lower is the value of $\widehat{x} \in (0, 1)$ and greater is the basin of attraction of the zero-emission economy ($\widehat{x}^* = 1$). Notice in Fig. 2(a) that if the emission tax



(a) The unstable inner steady state separates two basins of attraction.

(b) Bifurcation diagram if the inner inner steady state is unstable.



(c) All trajectories converge to the stable inner steady state.

(d) Bifurcation diagram if the inner inner steady state is stable.

Fig. 2. Dynamic regimes. *Legend*: in **(b)** and **(d)** continuous line represents stable states, while dotted line unstable ones. *Parameter values*: $N = 3$, $\gamma = 1$, $c_c = 0.3$, $c_d = 0.15$, $\delta = 0.1$, $\eta = 0.15$, $\lambda = 0.1$ in **(a)** while $\lambda = 0.2$ in **(c)**, $\tau = 0.1225$ in **(a)** while $\tau = 0.12025$ in **(c)**.

increases, then $\widehat{x} \in (0, 1)$ decreases. This means that the basin of attraction $\widehat{x}^* = 1$ increases, favoring a market composition in which only clean firms exist.

Otherwise, Fig. 2(c) shows what happens if the inner steady state is stable. Regardless of the initial conditions x_0, all the trajectories approach $\widehat{x}^* \in (0, 1)$. This means that the market will be composed of both clean and dirty firms. Moreover, from Fig. 2(d) it emerges that an increase of the emissions increases $\widehat{x}^* \in (0, 1)$, namely the number of clean firms goes up, fostering a low-carbon transition. The numerical results of Fig. 2(b) and Fig. 2(d) confirm the analytical results of Proposition 9, namely an increase of τ favors the clean strategy, so the number of clean

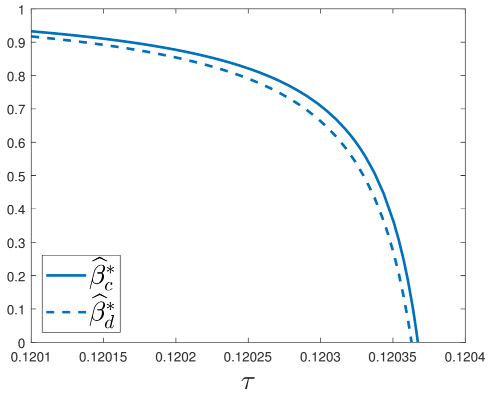


Fig. 3. Comparative dynamics of the optimal insurance coverage at the stable steady state.

firms increases.

We study the behavior of the demand for insurance at the inner stable steady state (see Fig. 3). Notice that if the emission tax increases then the optimal insurance coverage of both types of firms increases, and clean firms demand more insurance than dirty ones. From Proposition 6, it emerges that if τ increases then the optimal coinsurance rate of the clean firms decreases, while the optimal coinsurance rate of the dirty firms increases. However, an increase of τ causes an increase of the number of clean firms (see Proposition 9), and an increase of the number of clean firms causes a decrease of the optimal insurance coverage of both types of firms (see Proposition 7). The final effect is that $\widehat{\beta}_i^*$ goes down. Finally, $\widehat{\beta}_c^* > \widehat{\beta}_d^*$ because $\tau < \breve{\tau}$ (in the simulations $\breve{\tau} = 0.1320$), as noted in Proposition 8.

6 Conclusions

The model studies the behavior of a market in which polluting and zero-emission companies operate. Polluting firms instead contribute to pollute and so must bear an emission tax on the amount of their production. Both categories of companies have to support a potential environmental loss and have an insurance climate change coverage available. Due to the risk aversion, they can decide how much to insure themselves choosing a coinsurance rate.

Since the insurance alone is unable to lead the market towards green behavior, we have studied the interaction that it can have with policy interventions. The emission tax and the insurance coverage are in this paper two key factors influencing the behavior of the companies for their profit maximization. The model shows a substitution effect between the two instruments. The work consists of two main parts. The first one presents a comparative analysis from which the relationships among taxation and market structure is algebraically investigate. We highlight some important features. The potential damage and the loading factor influence the choice of the coinsurance rate and the model provides instructions for the regulator on the setting of the emission tax, independently from the composition in term of polluting and zero-emission firms on the market. A higher emission tax provokes a decrease of the polluting production and favor the decrease of the potential climate change loss. The model shows a substitution effect for the clean firms that can reduce their coinsurance rate. On the contrary dirty firms increase the coverage because of the reduction of their profit caused by a lower production. For this income effect dirty firms are unable to cover the potential loss and the high cost of the tax and so increase the demand for insurance. In presence of a relatively low emission tax, polluting companies will have an incentive to increase the quantity of goods produced compared to clean ones. For the cleans is convenient to protect themselves by increasing the coinsurance rate. This is an interesting result that shows how the support of the insurance market is fundamental for the survival of clean firms on the market. The opposite will occur when the emission tax is relatively high. A very important interaction between the regulator and the insurance sector emerges. Only a combined action between these policy tools can nudge firms to play in compliant with an environmental transition.

In the second part of the paper, the evolutionary dynamics shows how the model is able to understand the changing in the business strategies on the base of different market conditions. The probability to be clean or dirty firm operating on the market has been endogenized and two dynamic regimes, in which both types of firms exist at the equilibrium, can be observed. In the case of unstable inner steady state, the market will be composed of only one type of firms and the transition dynamics depends on the initial conditions. In the case of stable inner steady state, the market will be composed of both type of firms and all trajectories will approach to it.

Finally, the numerically comparative dynamics performed at the stable inner steady state confirms the main analytic results: the substitution effect between insurance and taxation and that clean firms demand more insurance than dirty one if the emission tax is relatively low.

References

Antoci, A., Fiori Maccioni, A., Galeotti, M., Russu, P., 2019. Defensive medicine, liability insurance and malpractice litigation in an evolutionary model. Nonlinear Analysis: Real World Applications 47, 414–435.

Bian, J., Guo, X., 2022. Policy analysis for emission-reduction with green technology investment in manufacturing. Annals of Operations Research 316 (1), 5–32.

Botzen, W. W., Deschenes, O., Sanders, M., 2019. The economic impacts of natural disasters: a review of models and empirical studies. Review of Environmental Economics and Policy 13 (2), 167–188.

Castelli, F., Galeotti, M., Rabitti, G., 2019. Financial instruments for mitigation of flood risks: the case of Florence. Risk Analysis 39 (2), 462–472.

Chen, X., Yang, H., Wang, X., Choi, T.-M., 2020. Optimal carbon tax design for achieving low carbon supply chains. Annals of Operations Research, 1-28.

Colivicchi, I., Iannucci, G., 2022. The environmental responsibility of firms and insurance coverage in an evolutionary game. Dynamic Games and Applications, 1-18.

Cressman, R., 2003. Evolutionary dynamics and extensive form games. MIT Press, Cambridge, MA, USA.

De Giovanni, D., Lamantia, F., 2016. Control delegation, information and beliefs in evolutionary oligopolies. Journal of Evolutionary Economics 26 (5), 1089–1116.

Droste, E., Hommes, C., Tuinstra, J., 2002. Endogenous fluctuations under evolutionary pressure in Cournot competition. Games and Economic Behavior 40 (2), 232–269.

Feess, E., Muehlheusser, G., 2002. Strategic environmental policy, clean technologies and the learning curve. Environmental and Resource Economics 23, 149–166.

Feichtinger, G., Lambertini, L., Leitmann, G., Wrzaczek, S., 2016. R&D for green technologies in a dynamic oligopoly: Schumpeter, Arrow and inverted-U's. European Journal of Operational Research 249 (3), 1131–1138.

Galeotti, M., Rabitti, G., Vannucci, E., 2020. An evolutionary approach to fraud management. European Journal of Operational Research 284 (3), 1167–1177.

Hofbauer, J., Sigmund, K., 2003. Evolutionary game dynamics. Bulletin of the American Mathematical Society 40 (4), 479–519.

Hommes, C. H., Ochea, M. I., Tuinstra, J., 2018. Evolutionary competition between adjustment processes in Cournot oligopoly: instability and complex dynamics. Dynamic Games and Applications 8 (4), 822–843.

IPCC, 2022. Climate change 2022: impacts, adaptation and vulnerability. Cambridge University Press, Cambridge, UK.

Kolstad, C. D., Moore, F. C., 2020. Estimating the economic impacts of climate change using weather observations. Review of Environmental Economics and Policy 14 (1), 1–24.

Kopel, M., Lamantia, F., 2018. The persistence of social strategies under increasing competitive pressure. Journal of Economic Dynamics and Control 91, 71–83.

Lamantia, F., Negriu, A., Tuinstra, J., 2018. Technology choice in an evolutionary oligopoly game. Decisions in Economics and Finance 41 (2), 335–356.

Mol, J. M., Botzen, W. W., Blasch, J. E., 2020. Risk reduction in compulsory disaster insurance: Experimental evidence on moral hazard and financial incentives. Journal of Behavioral and Experimental Economics 84, 101500.

Mossin, J., 1968. Aspects of rational insurance purchasing. Journal of Political Economy 76 (4), 553–568.

Sandholm, W., 2010. Population games and evolutionary dynamics. MIT Press, Cambridge, MA, USA.

Schlesinger, H., 2013. The theory of insurance demand. In: Dionne, G. (eds) Handbook of Insurance. Springer, New York, NY, USA, pp. 167–184.

Seog, S. H., 2006. Strategic demand for insurance. Journal of Risk and Insurance 73 (2), 279–295.

Tichỳ, T., Radi, D., Lamantia, F., 2020. Hybrid evolutionary oligopolies and the dynamics of corporate social responsibility. Journal of Economic Interaction and Coordination 17 (1), 87–114.

Tol, R. S., 2018. The economic impacts of climate change. Review of Environmental Economics and Policy 12 (1), 4–25.

Weibull, J., 1995. Evolutionary game theory. MIT Press, Cambridge, MA, USA.

Yi, Y., Wei, Z., Fu, C., 2021. An optimal combination of emissions tax and green innovation subsidies for polluting oligopolies. Journal of Cleaner Production 284, 124693.