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# How Emissions Charges affect Multiple equilibria and Stability in a Dynamic Monopoly

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## Abstract

Climate change has directed policymakers' focus to the impact of economic policies on global emissions. This paper explores the effects of emission charges within a monopoly framework where the demand function is unknown.

We investigate the case of a monopolist who learns the demand function using a rule of thumb. This approach reveals that while emission charges reduce production and emissions, they can also lead to the emergence of multiple equilibria depending on demand and cost parameters. The stability of these equilibria is influenced, among other factors, by emission charges and the firm's response to marginal profits. High taxation can destabilize production, resulting in fluctuating emissions. Also through numerical analysis, we show how learning processes significantly impact on production and emissions, highlighting the complexities in designing effective environmental policies in real-world economic contexts.

This research contributes to the theoretical literature on environmental governance by emphasizing the role of non-linearity and the unknown demand function in shaping policy outcomes.

**Keywords:** Emission charges; Climate Change; Complex Dynamics; Monopoly; Multiple Equilibria; Stability

**JEL classification:** C62, D42, Q51, Q58

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# 1 Introduction

Climate change is focusing policymakers' attention toward the role that various economic policy instruments can play in influencing global emissions. According to the IPCC report (2022), the adoption of environmental policies aimed at reducing production and emissions is urgent. At the same time, the role of non-linearity in explaining the possible non-trivial effects of environmental policies on the production system is relevant.

To the best of our knowledge, the relationship between emission charges, multiple equilibria and how their (in)stability may (or may not) affect emission reduction policy has not been studied in a monopoly framework with an unknown demand function. We believe that it certainly deserves maximum attention, as the dynamics, through the presence of positive and negative feedbacks, can influence the final outcome of a given economic policy. Non-trivial unexpected results could arise from specific initial conditions as well as from complex dynamics.

In this paper, we want to focus on the effects of emissions charges when the demand function is unknown and there is the possibility of multiple equilibria. Specifically, we consider the case where a new good enters the market and the only producer (monopolist) does not know the demand function but learns it by a rule of thumb. A monopoly, although simple, can provide us with different and clear insights. Moreover, precisely because of its simplicity, the finding of non-trivial effects within this framework leads us to be cautious in a more complicated economic environment, where the interactions between consumers and firms play an additional role.

We find, not surprisingly, that the increase in emissions charges leads to a reduction in production and hence in emissions themselves. However, in the presence of an increasing segment of the marginal revenue curve, two other stationary steady states can emerge, therefore leading to the coexistence of two maxima and a minimum. While the minimum is always unstable, the stability of the two maxima depends on the cost and demand parameters and the firm's response to marginal profits. The minimum is the separating value between two attractors. This means that the strength of the anti-emissions policy depends crucially on the initial conditions. At the same time, too high level of taxation leads to instability of the maxima, making production (and therefore emissions) unstable. Finally, the two attractors merge and production moves between a minimum and a maximum, leading also to strong fluctuations in emissions.

The rest of the paper is organized as follows. After an essential literature review (section 2), in Section 3 we present the model and discuss the existence and stability of multiple equilibria. In Section 4, through numerical simulations, we show how the learning process affect the stability of equilibria. In Section 5 we discuss and report concluding remarks.

## 2 Literature Review

Our paper lies in a wider theoretical and empirical literature in economics on environmental governance and climate action with a dynamical approach (see Costanza et al., 1993 and Bargigli and Ricchiuti, 2022). Methodologically, it relates to two strands of literature, the first on the role of non-linearity and unknown demand function and the second on environmental policies.

In the third chapter of her well-known book on Imperfect Competition, Joan Robinson discusses the case in which there are multiple equilibria in the monopoly framework (Robinson, 1969, pp.57), emphasizing that multiple equilibria can arise when demand elasticity varies along the demand function, especially in markets with heterogeneous consumer groups. The demand function

that Robinson has in mind is such that the corresponding marginal revenue has an increasing trait between two decreasing traits. Walters (1980) studies the case of an increasing marginal revenue when there is a convex kinked demand curve. While Formby et al. (1982) show that the condition for an increasing marginal revenue are not at all stringent and, therefore, the existence of multiple equilibria can be pervasive. However, they both show these conditions with numerical examples. In the same pages, Joan Robinson discussed the possibility that consumers do not know the entire demand function, highlighting that real-world businesses often lack a precise understanding of their own demand curves. She stressed that a monopolist does not need to know the exact shapes of their demand and cost curves. They only need to understand whether producing a little more or less will increase or decrease their profits. Clower (1959) was the first to emphasize the need for more complex models of price and output determination, arguing that in a monopoly setting that entrepreneurs often lack information about the demand function (for a critique on the neoclassical approach of perfect knowledge see also Keen and Standish, 2006 and Ormerod, 2006).

Two different lines of research have investigated the agents' behavior when the demand function is unknown. Within the first approach, firms form conjectures analyzing the interaction between past decisions and market mechanisms. Negishi (1961) introduced imperfect competition into general equilibrium, assuming firms have subjective demand beliefs and Silvestre (1977) linked these beliefs to objective demand. However, this approach studies only the equilibrium existence, not addressing the issue of coordination. Recently, several authors (Leonard and Nishimura, 1999; Tuinstra, 2004; Naimzada and Sbragia, 2007; Naimzada and Ricchiuti, 2011) have examined decision-making situations where consumers and firms have limited rationality and, as suggested by Alchian (1950), rational choices emerge over time through processes of selection, adaptation, and learning. Onozaki and Yanagita (2003) employ a discrete-time nonlinear model to examine market structure evolution. Agents in the model have limited information and adjust production and pricing based on consumer reactions. Product differentiation and consumer inertia contribute to the transition from competition to monopoly or oligopoly. Employing a similar framework, the same authors (Yanagita and Onozaki, 2010) use a statistical approach to represent consumer share distribution. They show that firms adjust production and pricing using reinforcement learning. Three market phases emerge: uniform share, oligopolistic, and monopolistic. In the oligopolistic phase, shares follow Zipf's law, and growth rates follow Gibrat's law. The second approach assumes firms use simple rules of thumb to estimate future conditions, similar to Robinson's ideas. Baumol and Quandt (1964) first proposed a rule of thumb called the gradient rule for monopolies. Under this rule, a firm adjusts its price based on whether profits increased, decreased, or remained the same in the previous period. This approach can lead to a steady state where profits are maximized, similar to traditional economic theory. Puu (1995) and Naimzada and Ricchiuti (2008) have shown that this model can produce complex dynamics, even with simpler demand curves and higher responsiveness to profit changes (see Onozaki, 2018, for a survey). Throughout the present paper, we use the second approach, in particular, focusing on the set of parameters such that the marginal revenue (as suggested by Joan Robinson) has an increasing trait.

Non-linearity and unknown demand function allow us to better study the effects of environmental policy, both on the number of equilibria and on their stability, thus giving us some insights into the emission charges over time<sup>1</sup>. Mamada and Perrings (2020) study how emission charges affect market structure, output and emissions in Cournot competition. Assuming myopic decision making and partial matching, they find that increasing marginal emission charges stabilise duopolies, while decreasing charges destabilise them, leading to monopolies. Equilibrium output and emissions are

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<sup>1</sup>The present paper will not address other mechanisms used by governments to deal with climate change, as Emission Trading Systems (see for example Antoci et al., 2022)

higher under cost structures favouring monopolies than those favouring duopolies. Matsumoto and Szidarovszky (2022) develop an  $n$ -firm Cournot oligopoly with a hyperbolic price function and use a game-theoretic approach to examine how environmental charges affect total and individual NPS pollution levels. The main findings are: (1) individual Cournot output is affected by marginal production cost, average abatement technology, and the number of firms; (2) environmental charges effectively control total NPS pollution when the average marginal production cost is less than the average emission coefficient; (3) environmental charges control individual pollution when the tax rate and the average pollution level are high enough; (4) stronger conditions are required for the effectiveness of environmental charges under hyperbolic demand compared to linear demand.

Naimzada and Pireddu (2023) extend the dynamic Cournot duopoly model with emission charges of Mamada and Perrings (2020) to differentiated goods and analyse static and dynamic results. Firms are taxed on their own emissions using quadratic charge functions, and adjust output partly due to capacity constraints. The steady state, which is consistent with the Nash equilibrium, is allowed if marginal emission charges are positive. Their results show that environmental policy is fully effective and globally stable for independent goods or goods with low interdependence, whether substitutes or complements. In an other paper, Naimzada and Pireddu (2024) replace the linear partial adjustment rule in Mamada and Perrings (2020) with a sigmoid adaptive best response mechanism in a Cournot duopoly model with quadratic emission charges and homogeneous goods. The sigmoid nonlinearity accounts for bounded output variations due to constraints and produces stable dynamic outcomes. They analyse the stability of the unique steady state (Nash equilibrium) and the impact of parameters on stability. Two comparative dynamics studies evaluate the effectiveness of environmental policy when the Nash equilibrium is unstable. The first shows the effectiveness of the policy for both complements and substitutes at different charge levels. The second reveals higher emissions along non-stationary trajectories compared to the equilibrium path, demonstrating that adjusting the sigmoid mechanism can reduce output fluctuations, stabilize the system and limit pollution.

We focus on monopoly because, albeit its simplicity, it allows us to focus on the key role of environmental policy of existence, multiplicity and stability of equilibria.

### 3 The model

Following the intuition of Robinson (1928) discussed above, we assume that the demand function is cubic (see Puu, 1990 and Naimzada and Ricchiuti, 2008), and it comes in the form:

$$P(q) = a - \frac{b}{2}q + \frac{c}{3}q^2 - \frac{d}{4}q^3 \quad (1)$$

where  $q \geq 0$  is the output and  $a, b, c, d$  are positive constants. The corresponding revenue function is  $R(q) = q \cdot P(q)$ . It is easily seen that  $P$  is a strictly decreasing function on the positive axis provided  $c^2 < (27/8)bd$  and that the marginal revenue function  $R'$  has a local minimum and a local maximum, that is its graph displays an increasing trait, provided  $c^2 > 3bd$  (see Figure 1). Since  $3 < 27/8$ , this means that we may ensure both features by selecting  $c$  in the interval:

$$\sqrt{3bd} < c < \sqrt{\frac{27}{8}bd}. \quad (2)$$

We next assume that production costs are quadratic in  $q$  and given by the function

$$C^p(q) = \frac{m}{2}q^2 \quad (3)$$

for a positive constant  $m$ .

Finally, the monopolist is subject to emission charges that are quadratic in the output (as in Mamada and Perrings, 2020; Naimzada and Pireddu, 2023), and given by the function

$$C^e(q) = ku + \frac{ks}{2}u^2 \quad u = \varepsilon q \quad (4)$$

where the positive constants  $k, \varepsilon, s$  are the key parameters of the environmental policy. Indeed, here:  $k$  is the intensity of environmental policy,  $s$  determines the shape of  $C^e$  and  $\varepsilon$  gives the emission per unit output.

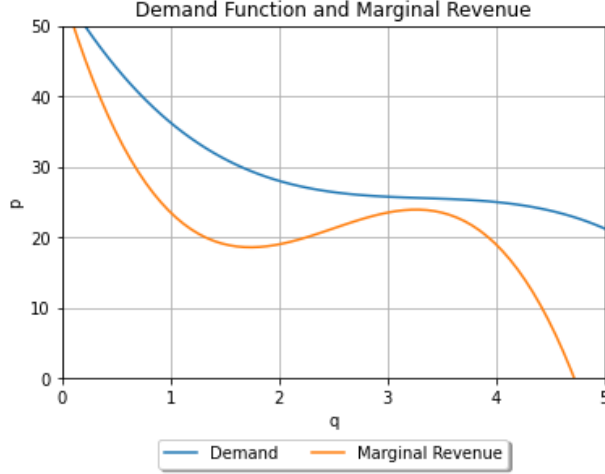


Figure 1: Downward demand function and marginal revenue with positive slope

The profit is then given by the quartic function

$$\pi(q) = q \cdot P(q) - C^p(q) - C^e(q) = (a - \varepsilon k)q - \frac{b + m + s\varepsilon^2 k}{2}q^2 + \frac{c}{3}q^3 - \frac{d}{4}q^4 \quad (5)$$

and the marginal profit  $\pi'$  is a cubic function, thus presenting the possibility of multiple equilibria.

We assume that the market is new and it needs to be explored, therefore the monopolist does not know the entire demand function. She uses a simple rule of thumb - a gradient rule - choosing the quantity to be produced (Baumol and Quandt, 1964; Puu, 1990; Bischi and Lamantia, 2002; Naimzada and Ricchiuti 2008). Over time, the production increases (decreases) if the marginal profit increases (decreases) and it is stable when the marginal profit is zero. This mechanism is captured by the following dynamics:

$$q_{t+1} = q_t + \gamma \pi'(q_t) \quad (6)$$

where the positive constant  $\gamma$  describes how the output reacts to marginal profit's variations. We immediately see that the steady states of the dynamics (6) are precisely the critical points of the profit function  $\pi$ , that is the points where the marginal profit  $\pi'$  vanishes.

### 3.1 Steady states and equilibria

In what follows, we discuss the existence and the number of equilibria for the dynamics (6), that are the positive zeros of the cubic function

$$\pi'(q) = a - \varepsilon k - (b + m + s\varepsilon^2 k)q + cq^2 - dq^3 \quad (7)$$

We may safely assume that when production is zero the marginal profit is positive, that is we posit  $a > \varepsilon k$ . In order to ease notation, we set

$$A = a - \varepsilon k \quad B = b + m + s\varepsilon^2 k \quad C = c \quad D = d \quad (8)$$

so that

$$\pi'(q) = A - Bq + Cq^2 - Dq^3 \quad (9)$$

We observe that all four constants in (8) are positive, by the stated assumptions on the parameters. Therefore, by Descartes' Rule of Signs, the number of positive roots of the cubic function  $\pi'$  in (9), with properly counted multiplicities, is either 1 or 3. In other words, there exists at least one and at most three equilibria for the dynamics (6). Moreover, recalling that the discriminant of the cubic equation  $\pi' = 0$  is defined as

$$\Delta = 18ABCD - 4AC^3 + B^2C^2 - 4B^3D - 27A^2D^2, \quad (10)$$

a classical result in Algebra (see Irving, 2004) states that the case  $\Delta > 0$  corresponds to three distinct positive roots, while the case  $\Delta < 0$  corresponds to just one positive root.<sup>2</sup>

Let us next discuss how the intensity of the environmental policy  $k$  influences the number of equilibria. To this end, we first observe that the marginal profit  $\pi'$  is the difference between the marginal revenue (MR) and the marginal cost (MC). The former does not depend on  $k$ , and this fact eases the analysis of the effect of taxation on the existence of multiple equilibria when the MR graph has an increasing trait. For all values of  $k$ , the graph of MC is a half-line starting from the value  $\varepsilon k$  and having slope  $m + s\varepsilon^2 k$ . Figure 3, which is based on a choice of the parameters to be specified later, depicts a typical situation. When  $k = 0$  the half-line starts from the origin and crosses the graph of MR just once, for a high value of the output. By increasing  $k$ , the half-line representing MC moves upward to the left and it crosses the MR curve at a lower level. This means that when the environmental policy is enforced, marginal costs increase and the output maximizing profits is lower, the equilibrium being still unique. However, there is a threshold  $\underline{k}$  for which the half-line is tangent to the MR graph at a lower level than the existing equilibrium. Therefore, when  $k > \underline{k}$  two new equilibria emerge. The number of equilibria is three until  $k$  reaches a second, higher threshold  $\bar{k}$  for which the half-line is again tangent to the MR graph. Then, for all values of  $k$  above  $\bar{k}$ , we come back to a unique equilibrium.<sup>3</sup>

In order to obtain a more general picture, we insert (8) into (10) and write  $\Delta$  as a cubic function of  $k$ , thus getting

$$\Delta(k) = \eta_3 k^3 + \eta_2 k^2 + \eta_1 k + \eta_0$$

where the constants  $\eta_i$  depend on all the other coefficients of the system. It turns out that, under the standing assumption  $c^2 < 27bd/8$ , both  $\eta_3$  and  $\eta_2$  are negative. This fact has two relevant consequences. First, it is always the case that, for all  $k$  high enough,  $\Delta(k) < 0$ , i.e. there is a unique steady state. Second, by Descartes' Rule of Signs,  $\Delta(k) = 0$  may have 0, 1 or 2 positive roots, depending on the choice of the coefficients. In the first case, there is a unique steady state for all  $k$ . In the second case, there are three equilibria when  $k$  stays below a given positive threshold and a unique equilibrium otherwise. Finally, in the last case we are in the setting of Figure 3, described above (and the two roots are precisely  $\underline{k}$  and  $\bar{k}$ ).

The above description can be seen as a static type result which is an intersection between an exercise of comparative statics and one of multiplicity of equilibria. However, it helps us to highlight two important features of the model. Firstly, from a static point of view, an increase

<sup>2</sup>The boundary case  $\Delta = 0$  corresponds to either 2 positive roots, one simple and one double, or a unique triple positive root.

<sup>3</sup>A similar effect can be due to a change in  $\varepsilon$  and  $s$ . We focus on  $k$  because this parameter does not depend on technology applied but on the environmental policy.

in emission charges leads to a decrease in the quantity produced and, consequently, a decrease in emissions. Secondly, an increase of the parameter  $k$  may have a dramatic effect on the existence of multiple equilibria. Therefore, in order to better understand the effects of the environmental policy on emissions, the stability of equilibria may give us further information (see Benhabib et al., 1999 and Menuet et al, 2023). In the next section, we are going to study how the parameter  $k$  (but also  $\varepsilon$  and  $s$ ) influences the stability of the equilibria.

### 3.2 Stability of the equilibria

It is well known that a steady state  $q_*$  of the dynamics (6), i.e. a positive root of  $\pi'(q) = 0$ , is asymptotically stable provided  $|1 + \gamma\pi''(q_*)| < 1$ , that is, provided

$$-\frac{2}{\gamma} < \pi''(q_*) < 0$$

and it is unstable provided one of the two previous inequalities holds (strictly) in the opposite direction. So, we have the following cases for a steady state:

- if  $\pi''(q_*) < -2/\gamma$ , then  $q_*$  locally maximizes the profit, but  $q_*$  is unstable
- if  $-2/\gamma < \pi''(q_*) < 0$ , then  $q_*$  locally maximizes the profit and  $q_*$  is asymptotically stable
- if  $\pi''(q_*) > 0$ , then  $q_*$  locally minimizes the profit and  $q_*$  is unstable

The two boundary cases  $\pi''(q_*) = 0$  or  $-2/\gamma$  are inconclusive.

From (8) and (9), the conditions for stability of a steady state  $q_*$  read

$$-\frac{2}{\gamma} < -(b + m + s\varepsilon^2 k) + 2cq_* - 3dq_*^2 < 0 \quad (11)$$

and we see that they crucially depend on the constant  $\gamma$  (though, they not depend on  $a$ ).

## 4 Simulations

In this section we analyse, through simulations, the role of the environmental policy on the number and stability of equilibria. The aim is to highlight relevant elements that, in our view, need to be taken into account in the implementation of the environmental policy. We will focus on the role of  $k$  and  $\varepsilon$ , in particular analyzing the case where the demand function admits up to three steady states. Throughout all simulations we will consider the following simple choice for the parameters  $d, m, s$ :

$$d = 1 \quad m = 1 \quad s = 1. \quad (12)$$

### 4.1 The role of $k$

Let us consider the following set of parameters for the demand function:

$$a = 70 \quad b = 50 \quad c = 12.80 \quad (13)$$

They satisfy condition (2), and this guarantees that the graph of the marginal revenue has an increasing trait. The graph of the profit function corresponding to  $k = 10$  and  $\varepsilon = 0.3$  is reported in Figure 2 on the left: we clearly see the existence of three critical points, two local maxima and a local minimum. This is consistent with Figure 2 on the right, where the graph of the marginal revenue (black) is crossed three times by the half-line of the marginal cost (red).



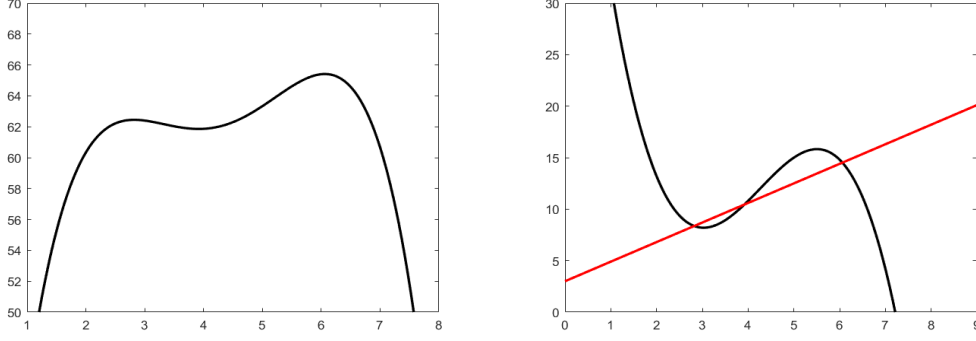


Figure 2: Left: profit vs  $q$ ; right: Marginal Revenue (black) and Marginal Cost (red) vs  $q$ . Both graphs: model as in (12) and (13) with  $k = 10$  and  $\varepsilon = 0.3$ .

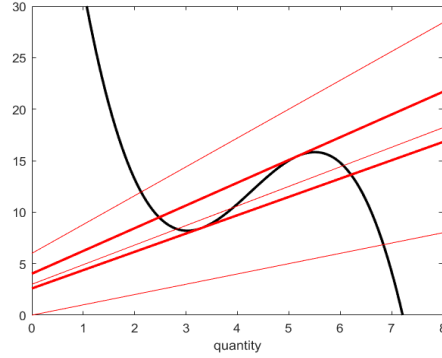


Figure 3: Marginal Revenue (black) and Marginal Cost (red) vs  $q$  for model as in (12) and (13) with  $\varepsilon = 0.3$  and different values of  $k$  (from the bottom to the top half-line:  $k = 0, 8.64, 10, 13.42, 20$ ).

Figure 3 illustrates the critical role of the parameter  $k$  in determining the number of steady states. For  $k = 0$ , a unique steady state exists. As  $k$  increases, the marginal costs rise, leading to a decrease in the profit-maximizing quantity, yet the equilibrium remains unique provided  $k$  remains below the threshold  $\underline{k} \approx 8.64$ . Within the interval  $k \in (\underline{k}, \bar{k})$ , where  $\bar{k} \approx 13.42$ , three equilibria emerge. For all  $k > \bar{k}$ , the system reverts to a single equilibrium. Thus, increasing emission charges result in reduced production and, consequently, lower emissions.

## 4.2 The role of $\gamma$ on stability

As always in this literature, a crucial role in characterizing stability of equilibria is played by the parameter  $\gamma$ , which describes the responsiveness of the monopolist. Indeed, a steady state may become unstable if the monopolist overreacts to changes in marginal profit. Note, however, that  $\gamma$  plays no role in determining the number and location of equilibria.

In Figure 4 we plot, for three different values of  $\gamma$ , the steady states as a function of  $k$ , specifying whether the state is stable (black) or unstable (red). We can observe that the areas of instability increase as  $\gamma$  increases. It is also confirmed that the central steady state, the minimum of the profit function, is always unstable, as discussed above.

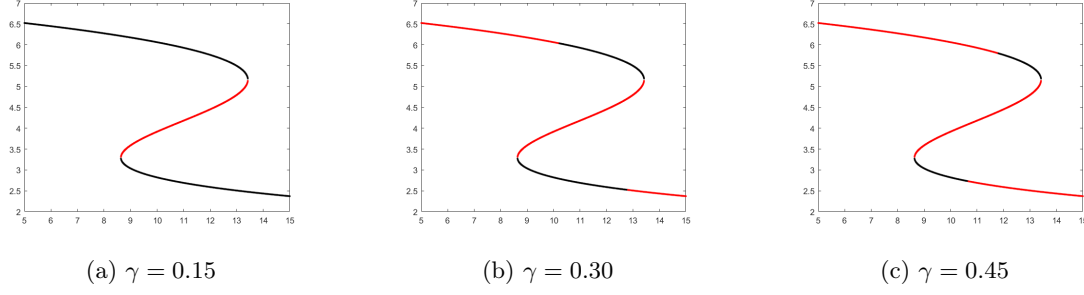


Figure 4: Steady states vs  $k$  for dynamics (6) for the model as (12) and (13) with  $\varepsilon = 0.3$  (black: stable; red: unstable)

### 4.3 Two equal maximum levels and the role of $\varepsilon$ and $k$

Let us now consider the following set of parameters, still ensuring an increasing trait in the graph of the marginal revenue:

$$a = 70 \quad b = 52 \quad c = 13.19 \quad (14)$$

Figure 5 shows the effect that the parameters  $k$  and  $\varepsilon$  have on the number and stability of the equilibria. The left graph confirms the existence of two thresholds  $\underline{k}$  and  $\bar{k}$  such that three equilibria co-exist whenever  $k \in (\underline{k}, \bar{k})$ . Remarkably, a similar pattern can be observed when considering the dependence on  $\varepsilon$ , as it is evident from the right graph. We can also observe that, as both the intensity of environmental policy ( $k$ ) and the emission per unit of output ( $\varepsilon$ ) increase, the profit-maximising output decreases. From either graphs we can notice that the choice  $k = 10$  and  $\varepsilon = 0.3$  yields a "symmetric" configuration, in which the two outer equilibria are roughly at a same distance from the inner one. This symmetry is broken as we move apart from those values of the two parameters. Next, sufficiently low or high values of the two parameters make the unique equilibrium unstable. Finally, in both cases, it is possible to choose the parameters ( $k$  just below 5 on the left graph,  $\varepsilon$  just below 0.2 in the right one) under which a stable maximum coexists with an unstable one.

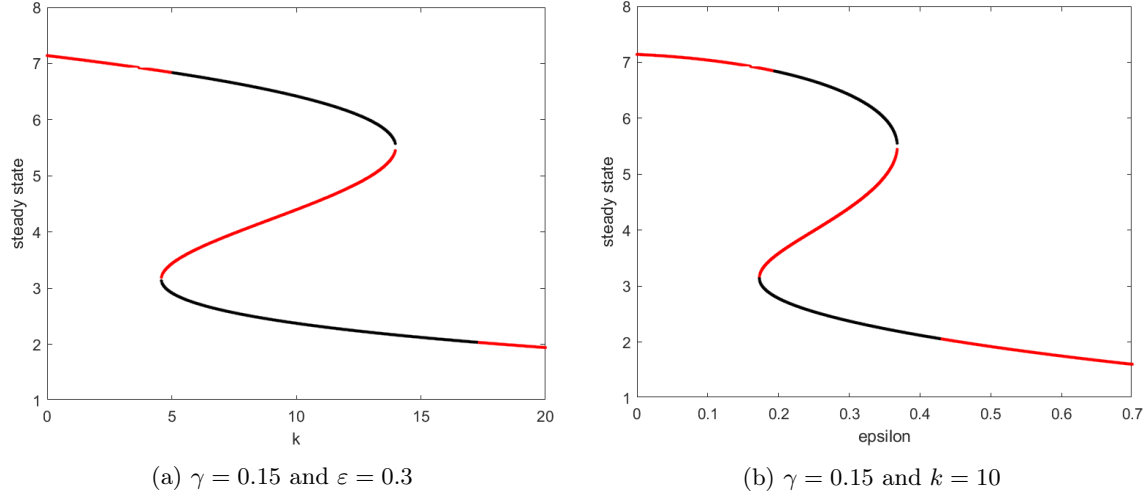


Figure 5: Location of steady states vs  $k$  (left) and  $\varepsilon$  (right) for the model as in (12) and (14). Black: stable; red: unstable.

We now fix

$$k = 10 \quad \varepsilon = 0.3 \quad \gamma = 0.15 \quad (15)$$

Figure 6 shows that in this case there are two stable local maxima, located at 2.4 and 6.4, respectively, whose profit is roughly at the same level (58.1). We can then easily simulate the dynamics (6), starting from a grids of initial points between  $q_0 = 0$  and  $q_0 = 9$ . The numerical results are not surprising: the long-run average<sup>4</sup> of the quantity coincides with 2.4 when  $q_0$  is below the unstable equilibrium (located at 4.4) and with 6.4 otherwise. As far as the long-run profit is concerned, this is always the common maximum level 58.1, independently from the initial quantity  $q_0$ .

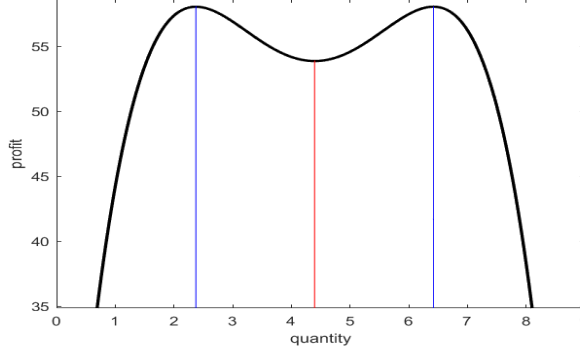


Figure 6: Profit vs quantity for the model as in (12), (14) and (15). Blue: stable equilibrium, red: unstable.

#### 4.4 A lower maximum may lead to higher profits

The most interesting case is when the two local maxima are characterized by a different level of profit. In the following we consider the parameters

$$a = 100 \quad b = 71.17 \quad c = 15.42 \quad (16)$$

satisfying condition (2).

Figure 7 (left) shows that the profit function has two maxima located at  $q_l^*$  and  $q_h^*$ , with  $q_l^* < q_h^*$  and  $\pi(q_l^*) > \pi(q_h^*)$ . Furthermore, the same Figure (right) shows that these two maxima are unstable for values of  $k$  below a threshold (roughly equal to 23), while for  $k$  roughly in the interval (23, 47),  $q_l^*$  is unstable while  $q_h^*$  is stable.<sup>5</sup>

We then ran the dynamics (6) for the considered model, with varying initial quantity  $q_0$ , and computed the average and the standard deviation of the obtained time series of quantities and profits.<sup>6</sup> Results are displayed in Figure 8 for the case  $k = 8$ , corresponding to both steady states being unstable. Remarkably, starting from an initial quantity  $q_0$  around  $q_h^*$  results in a higher long-run average profit than starting around  $q_l^*$ , notwithstanding the fact that  $\pi(q_l^*) > \pi(q_h^*)$ .

Thus, there is a clear trade-off between policymakers, who would like to reduce the quantity produced, and the monopolist, who would suffer a reduction in profits (on average). At the same time, it is important to stress that this trade-off only emerges through the analysis of equilibrium stability. By comparing the two steady states, it would have been clear (also for the firm) to choose the one with a lower quantity, corresponding to a higher profit (at the steady state).

<sup>4</sup>The average is computed over 2000 steps of the dynamics, with the first 100 steps considered as "transient", hence discarded.

<sup>5</sup>This is true also for specific values of  $\varepsilon$ .

<sup>6</sup>As before, we consider 2000 steps and discard the first 100

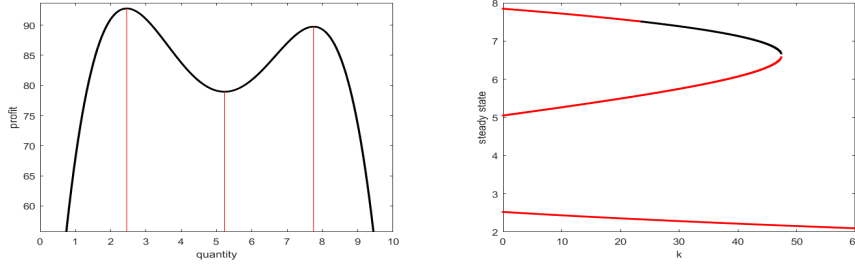


Figure 7: Left: profit vs  $q$  for model as in (12) and (16) with  $\varepsilon = 0.10$  and  $k = 8$ ; vertical bars: steady states (blue: stable; red: unstable). Right: steady states vs  $k$  (black: stable; red: unstable). Both graphs:  $\gamma = 0.20$

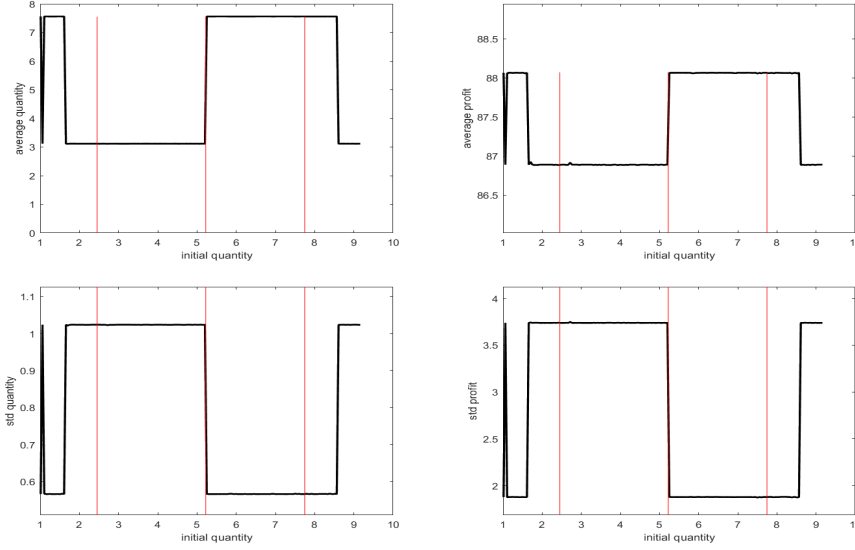


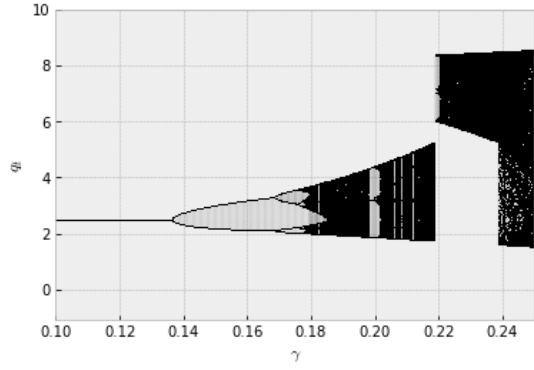
Figure 8: Left: dynamic average (top) and standard deviation (bottom) of a time series of quantities according to dynamics (6) vs initial quantity  $q_0$ . Right: same graphs for profit. All graph refer to model as in (12) and (16) with  $\varepsilon = 0.10$ ,  $k = 8$  and  $\gamma = 0.20$

This result crucially depends on the value of the parameter  $\gamma$ . Indeed, as clearly shown in Figure (9a), an increase in  $\gamma$ , i.e. an higher monopolist's responsiveness, results in an increased production quantity. Moreover, given that  $q_h^*$  is stable, this leads to an overall increase in production and emissions.

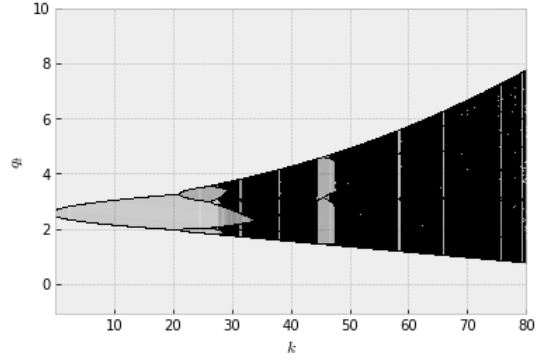
On the other hand, as Figure (9a) shows, if the reactivity of the monopolist is high ( $\gamma \in (0.219, 0.239)$ ), even though it starts from a low production value, the monopolist falls back to the attractor with the highest production. Moreover, a further increase in the monopolist's reaction leads to the merge of the two attractors and production (as well as emissions) fluctuates between a low and a high value.

Figure (9b) complements this reading by showing that even if  $\gamma$  is set at a 'low' value, an increase in  $k$  - a tightening of environmental policy - leads to an increase in output fluctuations with a consequent reduction in emission control.

The policy implication is clear: in addition to emission charges, the Governments' focus should be on the behaviour of firms. Therefore, a study of the dynamics provides insights that go beyond the static analysis.



(a)  $k = 8$  and  $\varepsilon = 0.10$



(b)  $\gamma = 0.15$  and  $\varepsilon = 0.10$

Figure 9: Bifurcation Diagram for  $\gamma$  (left) and  $k$  (right). Both graphs refer to dynamics (6) and model as in (12) and (16) and  $q_0 = 4$

## 5 Discussion and Conclusions

The ongoing climate crisis calls for robust economic policies to mitigate the damage, with taxation playing a central role. Our results provide several key insights into the effects of emissions charges.

Increasing emissions charges reduce production and hence emissions. With a bend in the inverse demand function and an increasing segment of the marginal revenue curve, two additional steady states can emerge, leading to the coexistence of two maxima and a minimum. The minimum is always unstable, while the stability of the two maxima depends on cost and demand parameters, as well as the firm's response to marginal returns. The minimum serves as a separating value between two attractors, implying that the effectiveness of emissions policies is highly sensitive to initial conditions. Excessive increases in emissions taxes can destabilise the maxima, leading to unstable production and emissions. Beyond a certain point, the two attractors converge, causing production to fluctuate between a minimum and a maximum and leading to significant emissions volatility.

These findings highlight the complex interplay between emissions charges and market dynamics, and underline the need for carefully calibrated environmental policies to achieve stable and effective outcomes.

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