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Marriage Formation with Random or Assortative "Meeting"*

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Abstract

In this paper, we study marriage formation in an optimal stopping problem where meetings can be of two types: one in which individuals meet potential partners randomly, and one ("assortative") in which the meeting occurs between individuals with similar characteristics. The presence of assortative meetings influences the expectations of the quality of potential spouses, and in turn the marriage choice. We show that individuals of high rank tend to be pickier in their marriage hunting. This does not necessarily mean that they marry later than other individuals, since the higher expected quality of their potential partners can make them marry earlier than individuals with a lower universal characteristic. In particular, individuals with medium rank tend to marry later than the other types, since they are picky but the quality of their potential partners is usually lower than for high-rank individuals.

JEL codes: C73, C78

Keywords: secretary problem, mate choice, random meeting, assortative meeting.

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1 Introduction

A common feature in models of marriage formation is the presence of "assortative matching" (Becker, 1973), which alludes to a relationship (either positive or negative) between the characteristics of partners. In the matching literature with transferable utility, assortative matching is assumed to occur according to the characteristics of the utility function (Becker, 1973, in the seminal paper and Shimer and Smith, 2000, in the search literature, *inter alia*),[1] while in the literature with nontransferable utility, partners are assortatively matched at equilibrium: individuals sharing the same social class choose each other in the marriage game (McNamara and Collins 1990, Burdett and Coles 1997 and Bloch and Ryder 2000, *inter alia*.). In both approaches, nothing is said about the way people meet. The social, educational and working environment may ultimately affect the meeting, even though individuals decide not to match afterwards.

Our aim is thus to model marriage formation by allowing for different types of meetings. We analyse an optimal stopping framework, where two characteristics determine an individual's type: a characteristic whose evaluation depends on a certain idiosyncratic preference of each potential partner ("specific" characteristic) and another characteristic ("universal" characteristic) that can be ranked commonly by all individuals, such as income, beauty, social status, and so on.

We build the model as a "two-sided secretary problem" (Eriksson, 2007): individuals meet potential partners at any period and simultaneously decide whether to propose according to the potential partner type and the expectation on the future potential partners. We assume that individuals with similar (social, educational, aesthetic) characteristics have a certain probability of meeting due to facts of life (i.e., attending similar social environments, obtaining the same level of education, and so on), even though this does not necessarily lead to marriage formation. In order to distinguish our approach, we will refer to this as "assortative meeting". Hence meeting can be *assortative* (the potential partner belongs to the same universal rank

[1] Technically, a household utility function is super-modular, if partners' characteristics are complements, leading to positive assortative matching. If the utility function is sub-modular, the partners' characteristics are substitutes (negative assortative matching).

of the individual) or *random* (the partner is randomly drawn by the population) in each period. From this perspective, the paper offers a comparison of different types of meetings and how these affect the individuals' behaviour.

The results depend on the state of the world in which an individual stands. In assortative meeting, individuals with a high universal characteristic are less demanding if the probability of having assortative meetings in the future is low, and *vice versa*. This result is due to the fact that, given a low probability of having assortative meetings in the future, the quality of the expected future partners is low for individuals with a high universal characteristic. Therefore they are less choosy with the choice of a potential partner met today of the same universal rank. In a random meeting, individuals with high universal characteristic are harder to please compared to other individuals, and they are more demanding the higher the importance of the universal characteristic in evaluating a partner. The reason is that an individual with a high universal rank knows that the chance of having future assortative meetings ensures a high expectation about the quality of future potential partners, at least from a universal-characteristic perspective. This does not necessarily mean that individuals with a high universal characteristic marry later than other individuals. In fact, assortative meeting makes them meet partners of better quality, which increases their chance of an early marriage.

Interestingly, individuals with medium-high rank tend to marry later than any other type, because they are fussy but the quality of their potential partner is lower than high rank individuals. This result is in line with some empirical evidence showing late marriages among members of the American middle class (Wilcox, 2010, Marquardt *et al.*, 2012).

Related literature. There is an extensive literature on models of spouse search. In the economic literature, the assignment problem is the framework employed to determine optimal marriage pairings (Shapley and Shubik, 1972 and Becker, 1973). In the developments of the assignment problem, the economic literature borrowed the standard Diamond-Mortensen-Pissarides search framework (see Burdett and Coles, 1997, Shimer and Smith, 2000, and Smith, 2006, *inter alia*).

While several approaches have been adopted to investigate this problem, our analysis rests on the optimal stopping framework. McNamara and Collins (1990) consider a job search game which can be modified to model marriage formation.[2] In the model, employers interview a sequence of candidates for a job and decide whether to offer the job to a candidate. Each candidate is interviewed by a sequence of employers and decide whether to accept each offer. They find that assortative matching occurs where the best employers are matched with the best candidates. Analogously in the mate choice version of the problem, both men and women of similar quality are matched together.

In recent years, this approach has been considerably extended. See, in particular, Alpern and Renyers (1999 and 2005), Malazov and Falko (2008), Ramsey (2008, 2011) and Alpern and Katrantzi (2009). As in Ramsey (2008), we analyse the problem with discrete time and finite or infinite horizon.

In particular, Ramsey (2011) considers a model where partners are ranked according to two characteristics. He assumes two stages, courting and acceptance, and in each stage it is possble to observe only one characteristic. Contrary to Ramsey (2011), we assume that individuals observe both characteristics at the same time.[3] Finally, Ramsey (2011) does not take into account the expected time to marry. Alpern *et al.* (2013) analyse marriage formation when individuals' preferences are based on age, which naturally changes over time. Finally, our approach is somewhat similar to Eriksson *et al.* (2007), who develop a model where individuals optimise the expected rank of their partner. Compared to Eriksson *et al.* (2007), we consider both two dimensions of heterogeneity and two types of meeting.

The remainder of the paper is structured as follows. Section 2 presents the model, and Section 3 shows the baseline results. Section 4 illustrates the expected time necessary to marry. Section 5 extends the analysis by assuming an infinite horizon, while Section 6 investigates the case where individuals are not aware of the type of meeting they are. Conclusions are in Section 7. All the results are formally derived in the appendix.

[2] Parker (1983) and Real (1991) formulate similar models for two-sided mate choice.

[3] A technical difference is that Ramsey (2011) assumes the distribution of characteristics as discrete.

2 The model

We study a finite, large universe of M single men and W single women. Time is discrete and the horizon is finite (Ramsey, 2008).[4] The multistage game starts at period $t = 1$ and lasts for N periods. In each period, a man from M meets a woman from W. A stage of the multistage game is called "meeting". After each meeting, a man m and a woman w simultaneously decide whether to propose. If both propose, the process ends, and the two players exit the game. If at least one of them refuses, then the game transits to the next period. We assume that being unmarried is always worse than being married. This assumption implies that, at period N, all the remaining unmatched players are willing to marry.

We assume that any pair who leaves the market is immediately replaced by clones (MacNamara and Collins, 1990, Bloch and Ryder, 1994, and Morgan, 1994, *inter alia*). Hence the formation of matchings in each period does not modify the distribution of single individuals when a marriage occurs.

Each player ranks the potential partner using two characteristics. The first characteristic, denoted by η, reflects the specific, idiosyncratic and universally unrankable trait of an individual. For example some individuals like caring and attentive partners, some others prefer independent persons. The preference is subjective and cannot be compared between different individuals. The second characteristic, denoted by I, represents a universally rankable aspect of the individual, such as income, education, social class and so forth. The universal type does not change throughout the game: clearly, this is a simplification, as characteristics may change over time, altering I.[5] For instance, income generally increases over time, whereas beauty decreases over time. Information is perfect and complete: individuals recognise the type of each potential partner and are aware of the characteristics of the distribution of individuals.[6]

We assume that an individual evaluates potential spouses according to the linear

[4]In Section 5 we extend the analysis to the case with infinite periods.

[5]See Alpern *et al.* (2013).

[6]Note that the type of potential partner is known during the meeting only. Hence players cannot anticipate whether a potential partner will accept to marry.

combination of these characteristics, which we will refer as "rank":

$$R = (1 - \alpha)\eta + \alpha I, \tag{1}$$

where $\alpha \in (0, 1)$ weights the importance of the universal characteristic compared to the individual characteristic. We assume α to be public information and identical for all players. The level of α reflects the role played in the romantic choice by universally estimable characteristics (social class, income, education) compared to personal preferences for specific aspects of a partner. For instance, it can be imagined that in a conservative society individuals put more weight on aspects such as the social status or income when they evaluate a partner.

The meeting can be of two types. We denote the set of types as $S = \{r, \bar{r}\}$, where $s = r$ is called "random" meeting while $s = \bar{r}$ is called "assortative" meeting. A random meeting occurs when an individual meets the partner by chance. This happens anytime the rankable characteristic of an individual (social status, income, education, and so forth) does not influence the occurring meeting. For example, two individuals running into each other at the grocery store, both going to the football stadium or to a public party. Therefore, with random meeting, any two people from the universe can meet. Assortative meeting occurs when an individual meets the partner in a context in which his or her rankable characteristic is relevant in determining the meeting. All the encounters at school, at the university, in a family or a private party are examples of assortative meeting. We assume that, with assortative meeting, the universal rank of the potential partner will be the same as the individual's. This assumption is made for simplicity, as considering an imperfect correlation (for instance, implemented with a noise) would complicate the analysis without altering the qualitative features of our results. Also, notice that the state of the world does not influence the specific characteristic of a potential partner. We assume that all players are aware of the meeting they participate.

In each period t, the meeting is assortative with exogenous probability $\beta \in (0, 1)$ and random with probability $1 - \beta$, β being constant, equal to all the players and known by them. The value of β depends on the customs of the society we have in mind. For instance, in a traditional society, it is more likely that individuals with common background are matched together (β high). To the best of our knowledge, this is the first contribution to the literature in which two types of meeting may alternatively take place.

Since the characteristics of potential partners are not known at the beginning of the game, suppose that an individual $i \in \{m, w\}$ in each period $t = 1, \ldots, N$ meets a partner $j \in \{m, w\}$ and $j \neq i$ in state s with the following rank:

$$R_{tj}^{s} = \begin{cases} (1-\alpha)\,\eta_{tj} + \alpha I_{tj}, & \text{if } s = r \text{ (with prob. } 1-\beta \text{)} \\ (1-\alpha)\,\eta_{tj} + \alpha I_{i}, & \text{if } s = \bar{r} \text{ (with prob. } \beta \text{)} \end{cases}$$

where

- η_{tj} is a random variable with continuous uniform distribution in $[0,1]$, reflecting the idiosyncratic preference of an individual i for a potential partner j met in $t = 1, \ldots, N$. Let η_{tj} be independent variables for $t = 1, \ldots, N$.

- I_{tj} is a random variable with continuous uniform distribution in $[0,1]$, representing the universal rank of a potential partner j met in $t = 1, \ldots, N$. Let I_{tj} be independent variables for $t = 1, \ldots, N$.

- $I_i = I \in [0,1]$ is the universal rank of the partner with assortative meeting, which is the same as individual i who evaluates the partner j.

Considering η_{tj} and I_{tj} as independent variables means that potential partners met in the past do not influence future meetings. We assume that men and women rank potential partners symmetrically, implying that they have the same probability of observing a certain η and (in the random matching state) I. This assumption is for the sake of simplicity and does not correspond exactly to what happens in the real world. For instance, in many societies beauty is more evaluated by men, whereas income is more evaluated by women.[7] The assumption that men and women

[7] See, for instance, Coles and Francesconi (2011).

rank potential partners symmetrically implies that the universal rank in assortative meeting state I is equal for man m and woman w.

As common in optimal stopping problems, we assume that all players adopt threshold strategies. We denote a player i's action in period t and state s as $a^s_{ti}(I)$: this is a threshold such that the player must propose in period t and state s if and only if the potential partner's rank is greater than the *threshold* in t, i.e., $R^s_{tj} > a^s_{ti}(I)$. If both players who met propose, then a marriage takes place and player i (j) gets a payoff R^s_{tj} (R^s_{ti}). Given the structure of the stage game, we may construct a multistage game defining the set of players' strategies and payoff functions. The strategy a_i of player i with universal rank I in the multistage game is a collection of stage strategies $\{a^s_{ti}(I)\}_{s\in S, t=1,\ldots,N}$ indicating whether the marriage must be proposed to a potential partner with absolute rank R^s_{tj} in period t and state s for every $t = 1, \ldots, N$. In other words, a player's strategy in the multistage game is a set of thresholds such that a player proposes a marriage in period t and state s if and only if the observed rank is greater than the *threshold* in t, i.e., $R^s_{tj} > a^s_{ti}(I)$. Therefore, a high $a^s_{ti}(I)$ implies that a player is more likely to delay marriage, since he or she needs to meet a potential partner with a high rank in order to agree to marry.

The goal of player i is to maximise the *discounted* expected rank of his or her spouse. We thus define a player's utility function in period t in state s as the partner's expected rank if the marriage takes place, i.e.:

$$U^s_{ti}(a_i(I)) = \begin{cases} E\left[R^s_{tj} | R^s_{tj} > a^s_{ti}(I), R^s_{ti} > a^s_{tj}(I_j)\right](a^s_{ti}(I)), & \text{if } R^s_{tj} > a^s_{ti}(I), R^s_{ti} > a^s_{tj}(I_j) \\ \delta U_{t+1,i}(a_i(I)), & \text{otherwise} \end{cases} \tag{2}$$

where $\delta \in (0,1]$ is the discount factor, $a^s_{tj}(I_j)$ is the potential partner j's strategy and $U_{t+1,i}(a_i(I))$ is the utility function in the subgame starting from period $t+1$:

$$U_{t+1,i}(a_i(I)) = \beta U^{\bar{r}}_{t+1,i}\left(a^{\bar{r}}_{t+1,i}(I)\right) + (1-\beta) U^{r}_{t+1,i}\left(a^{r}_{t+1,i}(I)\right).$$

Notice that the discount factor may be also interpreted as a constant search cost. The utility of an individual who remains single is assumed zero, ensuring that each player will eventually marry.

Using (2) we can write the recurrence equation for a player's utility function:

$$U_{ti}^{s}\left(a_{i}\left(I\right)\right) = \Pr_{m}^{s}\left(a_{ti}^{s}\left(I\right)\right) E\left[R_{tj}^{s} | R_{tj}^{s} > a_{ti}^{s}\left(I\right), R_{ti}^{s} > a_{tj}^{s}\left(I_{j}\right)\right]\left(a_{ti}^{s}\left(I\right)\right) \\ + \left[1 - \Pr_{m}^{s}\left(a_{ti}^{s}\left(I\right)\right)\right] \delta U_{t+1,i}\left(a_{i}\left(I\right)\right).$$

3 Baseline results

3.1 Bellman equation and strategies

The following Bellman equation represents the player i's maximal utility function. With probability $\Pr_{m}^{s}\left(a_{ti}^{s}\left(I\right)\right)$, player i marries in period t and state s, obtaining the payoff being equal to the expected rank of the partner (conditional to acceptance). With probability $(1 - \Pr_{m}^{s}\left(a_{ti}^{s}\left(I\right)\right))$, player i does not marry in period t and transits to the next time period. In this case his or her payoff will be the discounted maximal utility calculated for the next time period:

$$V_{t}^{s}(I) = \max_{a_{t}^{s}(I)} \left\{ \Pr_{m}^{s}\left(a_{ti}^{s}\left(I\right)\right) E\left[R_{tj}^{s} | R_{tj}^{s} > a_{ti}^{s}\left(I\right), R_{ti}^{s} > a_{tj}^{s}\left(I_{j}\right)\right]\left(a_{ti}^{s}\left(I\right)\right) \right. \\ \left. + \delta(1 - \Pr_{m}^{s}\left(a_{ti}^{s}\left(I\right)\right)) \left(\beta V_{t+1}^{\bar{r}}(I) + (1-\beta) V_{t+1}^{r}(I)\right) \right\} \tag{3}$$

with boundary conditions for $t = N$ and states $s = \bar{r}$ and $s = r$:

$$V_{N}^{\bar{r}}(I) = E[(1-\alpha)\eta_{Nj} + \alpha I] = \frac{1-\alpha}{2} + \alpha I, \tag{4}$$

$$V_{N}^{r} = E[(1-\alpha)\eta_{Nj} + \alpha I_{Nj}] = \frac{1}{2}. \tag{5}$$

Here the Bellman function $V_{t}^{s}(I)$ is the maximal expected rank of a partner, and $E[R_{tj}^{s} | R_{tj}^{s} > a_{i}\left(I\right), R_{ti}^{s} > a_{j}\left(I_{j}\right)]\left(a_{ti}^{s}\left(I\right)\right)$ is the expected rank of a potential partner j met in period t in state s if player i marries using strategy a. Expression $\beta V_{t+1}^{\bar{r}}(I) + (1-\beta) V_{t+1}^{r}(I)$ is the expected payoff of an individual i (or the absolute rank of j) if they chose to not marry in period t and the game transits to the next period.

Notice that player i's strategy $a_{ti}^{s}(I)$ is within the interval $[0,1]$ if $s=r$, but from the interval $[\alpha I, \alpha I+1-\alpha]$ if $s=\bar{r}$. The latter is the interval of possible values of the random variable R_{tj}^{r}. The optimal strategies for the last period N are $a_{N}^{r*}(I)=0$ for random meeting and $a_{Ni}^{\bar{r}*}(I)=\alpha I$ for assortative meeting, since a player prefers to marry than remain single.

In order to solve the Bellman equation, we begin by deriving the conditional probability of marrying according to the occurring state at time t. Then we determine the conditional expectation of the expected rank of a person if the met players both agree to marry. These steps are developed in the appendix.

3.2 Players' equilibrium strategies

From now on, we will omit the label i, j for brevity. The following proposition shows the equilibrium strategy with assortative meeting.

Proposition 1 *The equilibrium strategy* $\{a_{t}^{\bar{r}*}(I)\}_{t=1,\ldots,N-1}$ *of a player with universal rank* I *in the assortative meeting state* $\bar{r}$ *is:*

$$a_{t}^{\bar{r}*}(I)=\begin{cases}\alpha I, & \text{if } V_{t+1}(I)<\dfrac{4\alpha I+1-\alpha}{4\delta},\\ \dfrac{4\delta V_{t+1}(I)-(\alpha I+1-\alpha)}{3}, & \text{if } \dfrac{4\alpha I+1-\alpha}{4\delta}\leq V_{t+1}(I)<\dfrac{\alpha I+1-\alpha}{\delta},\\ \alpha I+1-\alpha, & \text{if } V_{t+1}(I)\geq\dfrac{\alpha I+1-\alpha}{\delta}.\end{cases} \tag{6}$$

In Proposition 1, the optimal strategy is higher the higher an individual's universal rank I in cases when $V_{t+1}(I)<\frac{\alpha I}{\delta}+\frac{1-\alpha}{4\delta}$ and $V_{t+1}(I)\geq\frac{\alpha I+1-\alpha}{\delta}$. In other words, it is less likely that an individual would accept to marry if he/she is from a high universal rank. Corollary 1 follows from Proposition 1.

Corollary 1 *In assortative meetings, a player does not marry before period* N *if and only if the expected rank* $V_{t+1}(I)$ *satisfies:*

$$V_{t+1}(I)\geq\frac{\alpha I+1-\alpha}{\delta} \tag{7}$$

for every $t = 1, \ldots, N-1$.

Condition (7) can be satisfied when the universal rank is very high, and the intensity of assortative meeting is also very high. Accordingly, players wait for potential partners with a higher rank in the following meetings. And if inequality (7) is satisfied for every $t = 1, \ldots, N-1$, then a player does not marry until period N in the assortative meeting states.

Let us turn now on the optimal strategy with random meeting for each period $t = 1, ..., N-1$. For the sake of exposition, consider first the case where $\alpha \geq \frac{1}{2}$.

Proposition 2 *The equilibrium strategy* $\{a_t^{r*}(I)\}_{t=1,\ldots,N-1}$ *of a player with universal rank* I *in the random meeting state* r *when* $\alpha \geq \frac{1}{2}$ *is:*

$$a_t^{r*}(I) = \begin{cases} 0, & \text{if } V_{t+1}(I) < \dfrac{1}{4\delta} \\ 1-\alpha, & \text{if } \dfrac{1}{4\delta} \leq V_{t+1}(I) < \dfrac{5-19\alpha+11\alpha^2}{6(1-3\alpha)\delta} \\ \dfrac{1+\alpha}{6} + \dfrac{2\delta}{3}V_{t+1}(I) - \gamma_1, & \text{if } \dfrac{5-19\alpha+11\alpha^2}{6(1-3\alpha)\delta} \leq V_{t+1}(I) < \dfrac{5\alpha+1}{6\delta} \\ \dfrac{6\delta V_{t+1}(I)-1}{5}, & \text{if } V_{t+1}(I) \geq \dfrac{5\alpha+1}{6\delta}, \end{cases} \tag{8}$$

where

$$\gamma_1 = \frac{\sqrt{16\delta^2\left(V_{t+1}(I)\right)^2 - 16\delta\left(1-\alpha\right)V_{t+1}(I) + 5\alpha^2 + 6\alpha + 5}}{6}.$$

Consider next the case where $\alpha < \frac{1}{2}$.

Proposition 3 *The equilibrium strategy* $\{a_t^{r*}(I)\}_{t=1,\ldots,N-1}$ *of a player with universal rank* I *in the random meeting state* r *when* $\alpha < \frac{1}{2}$ *is:*

$$a_t^{r*}(I) = \begin{cases} 0, & \textit{if } V_{t+1}(I) < \dfrac{1}{4\delta} \\ \alpha, & \textit{if } \dfrac{1}{4\delta} \leq V_{t+1}(I) < \dfrac{11\alpha^2 - 3\alpha - 3}{6(3\alpha - 2)\delta} \\ \dfrac{2-\alpha}{6} + \dfrac{2}{3}\delta V_{t+1}(I) - \gamma_2, & \textit{if } \dfrac{11\alpha^2 - 3\alpha - 3}{6(3\alpha - 2)\delta} \leq V_{t+1} < \dfrac{6 - 5\alpha}{6\delta} \\ \dfrac{6\delta V_{t+1}(I) - 1}{5}, & \textit{if } V_{t+1}(I) \geq \dfrac{6-5\alpha}{6\delta}, \end{cases} \tag{9}$$

where

$$\gamma_2 = \frac{\sqrt{16\delta^2(V_{t+1}(I))^2 - 16\delta(2-\alpha)V_{t+1}(I) + 5\alpha^2 - 16\alpha + 16}}{6}.$$

3.3 Existence and uniqueness of the equilibrium

Given the assumptions on the Bellman equation considered, there exists a unique subgame perfect equilibrium in the N-period meeting game. The existence of equilibrium is straightforward and follows from Selten (1975). The uniqueness of the subgame perfect equilibrium when all players use optimal strategies $a_t^{s*}(I)$, $t = 1, \ldots, N$, $s = r, \bar{r}$ yielding the Bellman equation (3) follows from the functional forms used in the right part of (3). In the case of assortative meeting $s = \bar{r}$, then (3) is a continuous function of $a_t^{\bar{r}*}(I)$ with a unique maximum on the interval of possible strategy values $[\alpha I, \alpha I + 1 - \alpha]$ for every $t = 1, \ldots, N$. Therefore, each player i has a unique optimal strategy in every period in which assortative meeting takes place. Random meetings can be considered similarly. The functions in the right part of Bellman equations (28) and (29) are continuous in $a_t^{r*}(I)$ for both $\alpha \geq \frac{1}{2}$ and $\alpha < \frac{1}{2}$ and has a unique maximum within the interval of possible strategies $[0, 1]$ for every $t = 1, \ldots, N$. Hence, a player has a unique optimal strategy in every period in which random meeting occurs.

3.4 Analysis of equilibrium

In this section, we consider some comparative statics of optimal strategies in both random and assortative meetings.

3.4.1 Variation of I

Begin by showing how a variation of the universal rank influences the payoff and the optimal strategy in equilibrium. The following propositions summarise the results.

Proposition 4 *The equilibrium payoff in the assortative (random) meeting state is an increasing (non decreasing) function of a player's universal rank I.*

Proposition 4 intuitively says that players with higher universal rank obtain a higher payoff in equilibrium.

Proposition 5 ***Assortative meeting.*** *For:*

$$\beta > \frac{1}{4\delta^{N-1}}, \tag{10}$$

the optimal strategy $a_t^{\bar{r}*}(I)$ is a non-decreasing function of a player's universal rank I for any $t = 1, \ldots, N$.

Random meeting. *The optimal strategy $a_t^{r*}(I)$ is a non-decreasing function of I.*

Condition (10) is sufficient but not necessary. Indeed the necessary condition for $a_t^{\bar{r}*}(I)$ to be non-decreasing cannot be obtained explicitly, this due to the recurrent form of optimal strategies. For example, for $N = 2$ the condition $\beta > \frac{1}{4\delta}$ is also necessary. For $N = 3$ the necessary condition is:

$$\beta > \frac{1 - \alpha + \alpha I - \frac{1-\alpha}{4}\sqrt{\frac{3(1-\delta)}{\delta}}}{\delta\alpha},$$

and so on.

Proposition 5 shows that, in the assortative meeting state, the optimal strategy changes with a player's universal rank according to the intensity of assortative meeting. If β is high, then players with high universal rank are more "demanding", because the future chance of being in the assortative meeting state (and thus to meet high ranked partners) will be higher. Therefore they can wait for a better idiosyncratic match. Conversely, if β is low, then players are pickier if they have a low I, since a low β implies a relatively higher future expectation for low-I types. Indeed, low universal rank players expect a relatively higher payoff from a random meeting. In the random meeting state, the high-I types generally are more patient, as their future potential partners generally have a higher expected rank, due to the chance of being in the assortative meeting state.

3.4.2 Variation of β

Consider next the effects of a variation of the intensity of assortative meeting β on the optimal strategies.

Proposition 6 ***Assortative meeting.*** *If*

$$\frac{4\alpha I + 1 - \alpha}{4\delta} \leq V_{t+1}(I) < \frac{\alpha I + 1 - \alpha}{\delta},$$

then

$$\frac{\partial a_t^{\bar{r}*}(I)}{\partial \beta} \propto \frac{\partial V_{t+1}(I)}{\partial \beta} \lesseqgtr 0 \textit{ for } V_{t+1}^{\bar{r}}(I) \lesseqgtr V_{t+1}^{r}(I).$$

If

$$V_{t+1}(I) < \frac{4\alpha I + 1 - \alpha}{4\delta} \vee V_{t+1}(I) \geq \frac{\alpha I + 1 - \alpha}{\delta},$$

then a variation of β does not affect the optimal strategy.

Random meeting. *If*

$$V_{t+1}(I) \geq \frac{5 - 19\alpha + 11\alpha^2}{6(1 - 3\alpha)\delta},$$

then

$$\frac{\partial a_t^{r*}(I)}{\partial \beta} \propto \frac{\partial V_{t+1}(I)}{\partial \beta} \lesseqgtr 0 \text{ for } V_{t+1}^{\bar{r}}(I) \lesseqgtr V_{t+1}^{r}(I).$$

If

$$V_{t+1}(I) < \frac{5 - 19\alpha + 11\alpha^2}{6(1 - 3\alpha)\delta},$$

then a variation of β does not affect the optimal strategy.

Therefore, the effects of β on the optimal strategy depend on which future conditional expectation is higher. Since the value of $V_{t+1}^{\bar{r}}(I)$ strictly depends on the individual's universal rank, in turn it is more likely that $V_{t+1}^{\bar{r}}(I) > V_{t+1}^{r}(I)$ (and in turn $\frac{\partial V_{t+1}(I)}{\partial \beta} > 0$) for higher levels of I.

For completeness, we examine the game when $\beta = 0$ and $\beta = 1$. When $\beta = 0$, players participate only to random meetings, and $V_t(I) = V_t^r(I)$ satisfies the Bellman equation (28) for $\alpha \geq \frac{1}{2}$ and (29) for $\alpha < \frac{1}{2}$, and the optimal strategies take the forms (8) and (9), respectively. If $\beta = 1$, the Bellman equation $V_t(I) = V_t^{\bar{r}}(I)$ satisfies equation (27) and the optimal strategy takes the form of equation (6).

4 Expected time to marry

In this section, we examine how long an individual expects to remain unmarried. We denote T as a discrete random variable representing the number of the periods in which a player expects to marry, where $T = 1, 2, ..., N$. In optimal stopping literature, this variable is called stopping time. In order to calculate the mathematical expectation of the number of periods needed to marry we need to find the probability that a player marries in each particular period t. Denote this probability as P_t, $\forall\ t = 1, \ldots, N$. For period 1, this probability can be defined by the following expression:

$$P_1 = (1 - \beta) \Pr_m^r \left(a_1^r (I)\right) + \beta \Pr_m^{\bar{r}} \left(a_1^{\bar{r}} (I)\right) \equiv M_1.$$

For period 2, the probability of marrying is as follows:

$$P_2 = (1 - M_1) \left((1 - \beta) \Pr_m^r \left(a_2^r (I)\right) + \beta \Pr_m^{\bar{r}} \left(a_2^{\bar{r}} (I)\right)\right) = (1 - M_1) M_2.$$

Hence for period k, the probability can be obtained by the expression:

$$\begin{aligned} P_k =& (1 - M_1) \dots (1 - M_{k-1}) \left((1 - \beta) \Pr_m^r \left(a_k^r (I) \right) \right) \\ &+ \beta \Pr_m^{\bar{r}} \left(a_k^{\bar{r}} (I) \right)) = (1 - M_1) \dots (1 - M_{k-1}) M_k. \end{aligned}$$

If a player does not marry in the first $N-1$ periods of the game and participates in the last N^{th} period he marries in this period with probability 1 (given the assumption that a player always prefers to marry than to remain single), i.e.,

$$P_N = (1 - M_1) \dots (1 - M_{N-1}).$$

The expectation of T is determined as follows.

Proposition 7 *The expected time to marry is given by:*

$$ET = P_1 + 2P_2 + \dots + NP_N = \sum_{i=1}^{N} i \left\{ \prod_{k=1}^{i-1} (1 - M_k) \right\} M_i.$$

The expected number of periods before to marry is a function of the player's strategy a and all parameters α, β, I.

Unfortunately, it is not possible to elicit analytical results, thus we examine the properties of Proposition 7 using a numerical simulation. First, we consider it for different universal ranks. We appoint the following parameters values: $\beta = 0.7$, $N = 100$, $\delta = 1$, $I = 0.01$, 0.33, 0.66, and 0.99. The results are summarised in Table 1 and shown graphically in Figure 1.

The relationship between universal rank and time to marry is non-monotone. The time to marry is low for individuals with low levels of universal rank, it increases for medium levels of universal rank, and it decreases again for high universal rank. In particular, the peak in the expected time to marry is for low-medium types when $\alpha = 0.25$ and for high-medium types when $\alpha = 0.80$.[8] Two factors contribute to this result. First, the higher importance of α makes individuals with a high universal

[8] For $\alpha = 0.25$, the peak is around $I = 0.12$, so that the non-monotonicity cannot be detected from Table 3.

I	$\alpha = 0.25$	$\alpha = 0.80$
0.01	46.38	42.36
0.33	44.60	42.82
0.66	42.48	50.53
0.99	42.27	41.55

Table 1: Expected number of periods needed to marry for different I.

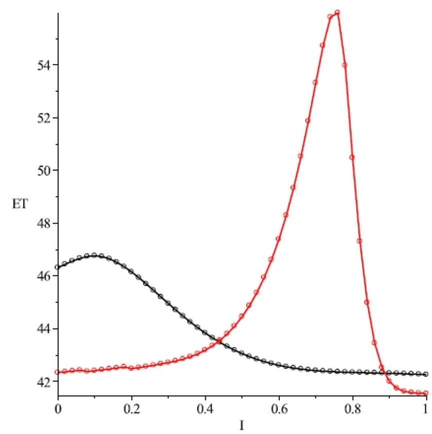

Figure 1: Expected time needed to marry as a function of I ("black": $\alpha = 0.25$, "red": $\alpha = 0.8$).

characteristic more picky in their partner choice, thus delaying the marriage. The first effect prevails on the second effect when the universal characteristic is not so high but, for very high universal characteristics, the second effect more than offsets the first effect, so that the time expected of marrying is lower. Thus individuals with a very high universal characteristic tend to marry sooner than other individuals. Alternatively, medium-rank individuals tend to marry later because they are fussy and the quality of their potential partners is more likely to be lower. This result is in line with some stylised facts, showing that individuals coming from middle class tend to marry later (Wilcox, 2010, Marquardt *et al.*, 2012). Among other reasons, the dynamic that led to this evidence can be explained with the increase in the ways of meeting people (cheaper transportation costs, chats, social networks, and so forth), which allows more information about the characteristics of individuals and induces more meetings among potential partners with similar characteristics (assortative meeting). Individuals expect this type of meetings in the future, which affects their marriage decisions.

We then consider the change in the expected number of periods before marrying for different β. We appoint the following parameters values: $I = 0.9$, $N = 100$, $\delta = 1$, $\beta = 0.01$, 0.33, 0.66, and 0.99. As in the previous example, we assume either $\alpha = 0.25$ or $\alpha = 0.80$. See Table 2 and Figure 2. The results are interpreted with a

β	$\alpha = 0.25$	$\alpha = 0.80$
0.01	44.70	43.24
0.33	43.66	41.90
0.66	42.42	42.01
0.99	41.81	41.80

Table 2: Expected number of periods needed to marry for different β.

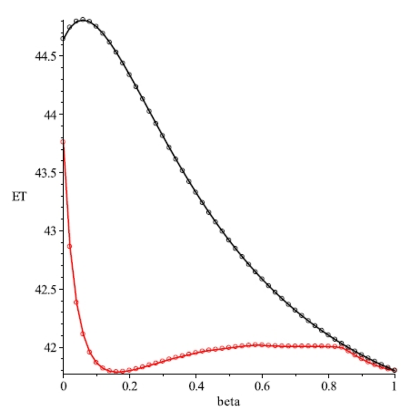

Figure 2: Expected time needed to marry as a function of β ("black": $\alpha = 0.25$, "red": $\alpha = 0.8$).

high-type player ($I = 0.9$) as reference. The results are opposite if we consider a low-type player, this because, while a high-type player is happy to be in the assortative meeting state, a low-type player prefers to be in the random matching state in order to avoid meetings with low types.[9] The function is non-monotone, since two effects interplay: on the one hand, given $I = 0.9$, the probability of meeting someone of high type increases with β; on the other hand, an increase in β increases the future chance of being in assortative meeting state, and in turn the expected future matches. When the weight of the universal characteristic is low, the first effect is dominated by the second effect for low levels of β, which leads to a peak in the expected time to marry for low β. Conversely, when α is high, the longest expected time to marry emerges for values of $\beta \lesssim 0.1$ and between 0.6 and 0.85. Moreover, consistent with the previous simulation, a high α ensures shorter waiting to marry due to the high universal type considered.

5 Infinite horizon

In this section, we extend the analysis to the case with infinite periods. We show that the findings are qualitatively similar to the results obtained in the finite case. Suppose that the universes M and W are infinite, and the game lasts an infinite number of periods. In this case, the backward Bellman approach considered above cannot be applied. In order to have boundary conditions to solve the Bellman equation, given the infinite periods, we look for equilibria in stationary strategies. In other words, a player's strategy $a = a^s(I)$ does not depend on the period. In this case, we are not able to show the equilibrium strategies in an explicit form. The functional equation for the player's payoff which is the expected partner's rank in the game beginning from state s is now:

$$\begin{aligned} V^s\left(a^s\left(I\right)\right) = {} & \Pr_m^s\left(a^s\left(I\right)\right) E\left[R_j^s | R_j^s > a^s\left(I\right), R_i^s > a^s\left(I_j\right)\right] \\ & + \delta(1 - \Pr_m^s\left(a^s\left(I\right)\right))\left(\beta V^{\bar{r}}\left(a^{\bar{r}}\left(I\right)\right) + (1-\beta) V^r\left(a^r\left(I\right)\right)\right). \end{aligned} \tag{11}$$

[9] Upon request, the results for low-universal type players can be provided.

Equation (11) is the equivalent of equation (3) for the case with infinite periods. Denote the vector $(V^{\bar{r}}(a^{\bar{r}}(I)), V^{r}(a^{r}(I)))^{T}$ as $\mathbb{V}(a^{\bar{r}}(I), a^{r}(I))$ and rewrite equation (11) in the vectorial form:

$$\mathbb{V}(a^{\bar{r}}(I), a^{r}(I)) = \mathbb{A}_1 + \delta\mathbb{A}_2(\beta, 1-\beta)\mathbb{V}(a^{\bar{r}}(I), a^{r}(I)), \tag{12}$$

where

$$\begin{gathered}
\mathbb{A}_1 = (\mathbb{A}_{11}, \mathbb{A}_{12}) = \left(\Pr_m^{\bar{r}}(a^{\bar{r}}(I))\, E\left[R_j^{\bar{r}} | R_j^{\bar{r}} > a^{\bar{r}}(I), R_i^{\bar{r}} > a^{\bar{r}}(I_j)\right],\right. \\
\left.\Pr_m^{r}(a^{r}(I))\, E\left[R_j^{r} | R_j^{r} > a^{r}(I), R_i^{r} > a^{r}(I_j)\right]\right)^T \\
\mathbb{A}_2 = (\mathbb{A}_{21}, \mathbb{A}_{22}) = \left(1-\Pr_m^{\bar{r}}(a^{\bar{r}}(I))\, E\left[R_j^{\bar{r}} | R_j^{\bar{r}} > a^{\bar{r}}(I), R_i^{\bar{r}} > a^{\bar{r}}(I_j)\right],\right. \\
\left.1-\Pr_m^{r}(a^{r}(I))\, E\left[R_j^{r} | R_j^{r} > a^{r}(I), R_i^{r} > a^{r}(I_j)\right]\right)^T.
\end{gathered}$$

Given equation (12) we obtain the following result.

Proposition 8 *Assume $\delta \neq 1$ and that players do not use their highest strategies ($a^{\bar{r}}(I) = \alpha I + 1 - \alpha$ and $a^{r}(I) = 1$). Then a player's payoff is:*

$$\mathbb{V}(a^{\bar{r}}(I), a^{r}(I)) = \frac{1}{1-\delta(\beta\mathbb{A}_{21} + (1-\beta)\mathbb{A}_{22})}\begin{pmatrix} 1-\delta(1-\beta)\mathbb{A}_{22} & \delta(1-\beta)\mathbb{A}_{21} \\ \delta\beta\mathbb{A}_{22} & 1-\delta\beta\mathbb{A}_{21}\end{pmatrix}\mathbb{A}_1. \tag{13}$$

When $\delta = 1$ and players use their highest strategies in any period and for any kind of meeting we obtain the following system of Bellman equations:

$$\begin{gathered}
V^{\bar{r}}(a_{\max}^{\bar{r}}(I), a_{\max}^{r}(I)) = \beta V^{\bar{r}}(a_{\max}^{\bar{r}}(I), a_{\max}^{r}(I)) + (1-\beta)V^{r}(a_{\max}^{\bar{r}}(I), a_{\max}^{r}(I)), \\
V^{r}(a_{\max}^{\bar{r}}(I), a_{\max}^{r}(I)) = \beta V^{\bar{r}}(a_{\max}^{\bar{r}}(I), a_{\max}^{r}(I)) + (1-\beta)V^{r}(a_{\max}^{\bar{r}}(I), a_{\max}^{r}(I)),
\end{gathered}$$

which amounts to $V^{\bar{r}}(a_{\max}^{\bar{r}}(I), a_{\max}^{r}(I)) = V^{r}(a_{\max}^{\bar{r}}(I), a_{\max}^{r}(I))$. In this case a player never proposes a marriage in the game, and the game never stops.

As in the finite case, the player's optimal strategy maximises the expected rank of the expected partner:

$$\beta V^{\bar{r}}(a^{\bar{r}}(I)) + (1-\beta)V^{r}(a^{r}(I)) = \frac{\beta\mathbb{A}_{11} + (1-\beta)\mathbb{A}_{12}}{1-\delta(\beta\mathbb{A}_{21} + (1-\beta)\mathbb{A}_{22})}, \tag{14}$$

$$\text{s.t. } a^{\bar{r}}(I) \in [\alpha I, \alpha I + 1 - \alpha] \text{ and } a^{r}(I) \in [0,1].$$

We then analyse the equilibrium payoffs in a numerical example. In particular, we compare the equilibrium payoff with infinite horizon with the results in the finite case when $N = 100$. We appoint the following parameters values: $\delta = 0.95$, and either $\alpha = 0.25$ or $\alpha = 0.80$. In Table 3, we keep $\beta = 0.7$ and we examine the equilibrium payoff for $I = 0.01$, $I = 0.33$, $I = 0.66$ and $I = 0.99$. In Table 4, we keep $I = 0.9$ and we show the results for $\beta = 0.01$, $\beta = 0.33$, $\beta = 0.66$ and $\beta = 0.99$.

	$\alpha = 0.25$		$\alpha = 0.80$	
I	Infinite game	Finite game	Infinite game	Finite game
0.01	0.5359	0.5359	0.4838	0.4838
0.33	0.5762	0.5762	0.4838	0.4838
0.66	0.6272	0.6272	0.6283	0.6283
0.99	0.6864	0.6864	0.8735	0.8735

Table 3: Equilibrium payoff for finite ($N = 100$) and infinite game with $\beta = 0.7$ and different I.

	$\alpha = 0.25$		$\alpha = 0.80$	
β	Infinite game	Finite game	Infinite game	Finite game
0.01	0.6065	0.6066	0.6199	0.6198
0.33	0.6392	0.6392	0.7559	0.7559
0.66	0.6666	0.6666	0.8010	0.8009
0.99	0.6890	0.6890	0.8202	0.8201

Table 4: Equilibrium payoff for finite ($N = 100$) and infinite game with $I = 0.9$ and different β.

The tables show that the equilibrium payoffs in the finite and infinite case are very close. Thus the analytical results obtained in the finite case are robust by assuming an infinite horizon.[10]

6 Incompete information

In this section, we consider the case with incomplete information about the states, i.e., the situation in which players do not know which kind of meeting they participate. Hence the strategies adopted by players are state-independent. In order to model this, we modify the N-period meeting game as follows. Suppose that, for every $t = 1, \ldots, N$, a player uses the same strategy $a_t(I)$ in assortative and random meetings, so that $a_t(I) = a_t^{\bar{r}}(I) = a_t^r(I)$. This situation reflects the situations in which an individual does not know exactly which kind of meeting (state) takes place in every period.

In this modified meeting game we consider the payoff of a player in the N-period game, as the linear combination of the player's expected payoffs in the games beginning with particular meetings (assortative and random):

$$V_1(a_t(I)) = \beta V_1^{\bar{r}}(a_t(I)) + (1-\beta) V_1^r(a_t(I)).$$

The Bellman equation for the maximal expected rank $V_t(a_t(I))$ in period t takes the form of:

$$\begin{aligned} V_t(I) = \max_{a_t(I)} & \left\{\beta \Pr_m^{\bar{r}}(a_t(I)) E\left[R_{tj}^{\bar{r}} | R_{tj}^{\bar{r}} > a_t(I), R_{ti}^{\bar{r}} > a_t(I_j)\right]\right. \\ & + (1-\beta)\Pr_m^r(a_t(I)) E\left[R_{tj}^r | R_{tj}^r > a_t(I), R_{ti}^r > a_t(I_j)\right] \\ & \left. + \,\delta\left\{\beta(1-\Pr_m^{\bar{r}}(a_t(I))) + (1-\beta)(1-\Pr_m^r(a_t(I))\right\} V_{t+1}(I)\right\}, \end{aligned} \tag{15}$$

with boundary condition:

$$V_N(I) = \beta\left(\frac{1-\alpha}{2} + \alpha I\right) + \frac{1-\beta}{2}. \tag{16}$$

[10] Upon request, we can provide additional numerical examples in which the similarities of results between the finite and the infinite case are confirmed.

With state-independent strategies, a player uses the same strategies for random and assortative meetings in the same period. Then, the set of possible strategies are in the set $[0,1]$ for all states. The probability of marrying is given by:

$$\Pr_m^{\bar{r}}\left(a_t\left(I\right)\right)=\begin{cases}1, & \text{if } a_t\left(I\right)\in\left[0,\alpha I\right),\\ \left(1-\dfrac{a_t\left(I\right)-\alpha I}{1-\alpha}\right)^2, & \text{if } a_t\left(I\right)\in\left[\alpha I,\alpha I+1-\alpha\right),\\ 0, & \text{if } a_t\left(I\right)\in\left[\alpha I+1-\alpha,1\right].\end{cases}$$

Moreover in the assortative meeting state, the conditional expectation of the absolute rank of the chosen j under the condition that the marriage takes place in period t is:

$$E\left[R_{tj}^{\bar{r}}|R_{tj}^{\bar{r}}>a_t\left(I\right),R_{ti}^{\bar{r}}>a_t\left(I_j\right)\right]$$
$$=\begin{cases}\alpha I+\dfrac{1-\alpha}{2}, & \text{if } a_t\left(I\right)\in\left[0,\alpha I\right),\\ \dfrac{\alpha I+1-\alpha+a_t\left(I\right)}{2}, & \text{if } a_t\left(I\right)\in\left[\alpha I,\alpha I+1-\alpha\right),\\ 0, & \text{if } a_t\left(I\right)\in\left[\alpha I+1-\alpha,1\right].\end{cases}$$

With state-independent strategies, the player's optimal strategy is implicitly defined. Notice that the player's payoff in the N-period meeting game with state-independent strategies, i.e. the expected rank of the potential partner, is not larger than the payoff in the game with state-dependent strategies.

7 Concluding remarks

We have studied marriage formation through an optimal stopping problem approach, where individuals have two different dimensions of heterogeneity, and two possible kinds of meetings, a random and an assortative one, may occur over time. We show that individuals with a high universal characteristic tend to be pickier in their marriage hunting. This does not necessarily mean that they marry later than other individuals, since the higher expected quality of their potential partners in the assortative meeting state can make them marry earlier than individuals with a lower universal characteristic. Interestingly, individuals with medium-high rank tend to marry later than the other types, since they are picky, but the quality of the individuals they meet tends to be lower than high-rank individuals.

The analysis carried out did not consider divorce explicitly, but this indeed can be easily implemented. First notice that, given a large universe of men and women, the number of divorces occurring in each period would not change the distribution of single individuals. Second, once assumed that divorce occurs with exogenous probability, then there is no reason to expect that this probability may change according to whether two individuals decide to marry or not in a certain period. Of course, the probability of divorcing may change with the length of a relationship.

Further extensions may take into account different universal characteristics for men and women. According to the customs considered, these may be different for the two genders. For example in Western societies, men appoint a higher value to beauty compared to women, whereas women appoint a higher value to financial security (See Coles and Francesconi, 2011). These developments of the current model are left for future work.

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8 Appendix

8.1 Bellman equation

In what follows we solve the Bellman equation by steps.

8.1.1 Conditional probability

The first step is to find the probability of marrying according to the occurring state at time t. Begin by the assortative meeting state.

Proposition 9 *The conditional probability to marry in the assortative meeting state for any period $t = 1, \ldots, N-1$ is given by*

$$\Pr_m^{\bar{r}}\left(a_t^{\bar{r}}(I)\right) = \left(1 - \frac{a_t^{\bar{r}}(I) - \alpha I}{1-\alpha}\right)^2, \tag{17}$$

where $a_t^{\bar{r}}(I) \in [\alpha I, \alpha I + 1 - \alpha]$.

Proof. The probability that both ranks satisfy condition $R_{tj}^s > a_{ti}^s(I), R_{ti}^s > a_{tj}^s(I_j)$ is the probability that a marriage takes place in period t and state s, i.e.

$$\Pr_m^s\left(a_{ti}^s(I)\right) \equiv \Pr^s\left[R_{tj}^s > a_{ti}^s(I), R_{ti}^s > a_{tj}^s(I_j)\right].$$

Since men and women are symmetric, then their strategy is symmetric too, so that

$$\Pr^s\left[R_{tj}^s > a_{ti}^s(I), R_{ti}^s > a_{tj}^s(I_j)\right] \equiv \Pr^s\left[R_{tj}^s > a_{ti}^s(I)\right]^2.$$

We find the probability density distribution function $f_{R_{tj}^{\bar{r}}}(x)$ of $R_{tj}^{\bar{r}} = (1-\alpha)\eta_{tj} + \alpha I$ by using the consolidation formula of independent random variables:

$$f_{R_{tj}^{\bar{r}}}(x) = \frac{1}{1-\alpha} f_{\eta_{tj}}\left(\frac{x - \alpha I}{1-\alpha}\right) = \begin{cases} \dfrac{1}{1-\alpha}, & \text{if } x \in [\alpha I, \alpha I + 1 - \alpha] \\ 0, & \text{if } x \notin [\alpha I, \alpha I + 1 - \alpha], \end{cases}$$

where $f_{\eta_{tj}}(x)$ is a probability density function of the variable η_{tj}. Thus the cumulative distribution function $F_{R_{tj}^{\bar{r}}}(x) = \Pr\{R_{tj}^{\bar{r}} \leq x\} = \int_{-\infty}^{x} f_{R_{tj}^{\bar{r}}}(u)du$ of the random variable $R_{tj}^{\bar{r}}$ is as follows:

$$F_{R_{tj}^{\bar{r}}}(x) = \begin{cases} 0, & \text{if } x \in (-\infty, \alpha I) \\ \dfrac{x - \alpha I}{1 - \alpha}, & \text{if } x \in [\alpha I, \alpha I + 1 - \alpha) \\ 1, & \text{if } x \in [\alpha I + 1 - \alpha, \infty) \end{cases}$$

Therefore, the linear transformation of η_{tj} keeps the same distribution type but changes the interval of possible values, i.e. the distribution of rank $R_{tj}^{\bar{r}}$ is a continuous uniform in the interval $[\alpha I, \alpha I + 1 - \alpha]$.

Given the probability density and the cumulative distribution functions, we are now able to determine the conditional probabilities to marry. This is the probability that both players i and j who met in period t accept to marry under the condition that their choices are independent and they both use the same type of strategies. If the meeting is assortative ($s = \bar{r}$), the conditional probability to marry is as follows:

$$\Pr_m^{\bar{r}}\left(a_t^{\bar{r}}(I)\right) = \Pr\left\{\left(R_{tj}^{\bar{r}} > a_t^{\bar{r}}(I)\right) \cap R_{ti}^{\bar{r}} > a_t^{\bar{r}}(I)\right\} = \left(\Pr\left\{R_{tj}^{\bar{r}} > a_t^{\bar{r}}(I)\right\}\right)^2,$$

where the events $R_{tj}^{\bar{r}} > a_t^{\bar{r}}(I)$ and $R_{ti}^{\bar{r}} > a_t^{\bar{r}}(I)$ are independent and symmetric. Substituting and rearranging, we obtain the proposition. ■

The conditional probability in the random meeting state is summarised by the following proposition.

Proposition 10 *The conditional probability to marry in the random meeting state is given by*

1. For $\alpha \geq \dfrac{1}{2}$:

$$\Pr_m^r\left(a_t^r(I)\right) = \begin{cases} \left(1 - \dfrac{\left(a_t^r(I)\right)^2}{2\alpha(1-\alpha)}\right)^2, & \textit{if } a_t^r(I) \in [0, 1-\alpha) \\ \left(1 - \dfrac{2a_t^r(I) - (1-\alpha)}{2\alpha}\right)^2, & \textit{if } a_t^r(I) \in [1-\alpha, \alpha) \\ \left(\dfrac{\left(1 - a_t^r(I)\right)^2}{2\alpha(1-\alpha)}\right)^2, & \textit{if } a_t^r(I) \in [\alpha, 1] \end{cases} \tag{18}$$

2. For $\alpha < \dfrac{1}{2}$:

$$\Pr_m^r\left(a_t^r(I)\right) = \begin{cases} \left(1 - \dfrac{\left(a_t^r(I)\right)^2}{2\alpha(1-\alpha)}\right)^2, & \textit{if } a_t^r(I) \in [0, \alpha) \\ \left(1 - \dfrac{2a_t^r(I) - \alpha}{2(1-\alpha)}\right)^2, & \textit{if } a_t^r(I) \in [\alpha, 1-\alpha) \\ \left(\dfrac{\left(1 - a_t^r(I)\right)^2}{2\alpha(1-\alpha)}\right)^2, & \textit{if } a_t^r(I) \in [1-\alpha, 1] \end{cases} \tag{19}$$

Proof. A player i ranks a potential partner j as follows: $R_{tj}^r = (1-\alpha)\eta_{tj} + \alpha I_{tj}$. Here the random variables η_{tj} and I_{tj}, $t = 1, \ldots, N$ are independent and have the same uniform continuous distribution on the interval $[0, 1]$. The expression for the probability density distribution function $f_{R_{tj}^r}(x)$ of a random variable R_{tj}^r can be found using the formula of consolidation of two continuous independent variables:

- Case $\alpha \geq \frac{1}{2}$:

$$f_{R_{tj}^r}(x) = \int_{-\infty}^{\infty} f_{(1-\alpha)\eta_{tj}}(u) f_{\alpha I_{tj}}(x-u)du = \tag{20}$$

$$= \begin{cases} \dfrac{x}{\alpha(1-\alpha)}, & \text{if } x \in [0, 1-\alpha) \\ \dfrac{1}{\alpha}, & \text{if } x \in [1-\alpha, \alpha) \\ \dfrac{1-x}{\alpha(1-\alpha)}, & \text{if } x \in [\alpha, 1] \\ 0, & \text{if } x \notin [0, 1] \end{cases}$$

- Case $\alpha < \frac{1}{2}$:

$$f_{R^r_{tj}}(x) = \begin{cases} \dfrac{x}{\alpha(1-\alpha)}, & \text{if } x \in [0, \alpha) \\ \dfrac{1}{1-\alpha}, & \text{if } x \in [\alpha, 1-\alpha) \\ \dfrac{1-x}{\alpha(1-\alpha)}, & \text{if } x \in [1-\alpha, 1] \\ 0, & \text{if } x \notin [0, 1] \end{cases} \tag{21}$$

The cumulative distribution function $F_{R^r_{tj}}(x)$ of the random variable R^r_{tj} according to α is:

- Case $\alpha \geq \frac{1}{2}$:

$$F_{R^r_{tj}}(x) = \begin{cases} 0, & \text{if } x \in (-\infty, 0) \\ \dfrac{x^2}{2\alpha(1-\alpha)}, & \text{if } x \in [0, 1-\alpha) \\ \dfrac{2x-(1-\alpha)}{2\alpha}, & \text{if } x \in [1-\alpha, \alpha) \\ 1 - \dfrac{(1-x)^2}{2\alpha(1-\alpha)}, & \text{if } x \in [\alpha, 1) \\ 1, & \text{if } x \in [1, \infty) \end{cases} \tag{22}$$

- Case $\alpha < \frac{1}{2}$:

$$F_{R^r_{tj}}(x) = \begin{cases} 0, & \text{if } x \in (-\infty, 0) \\ \dfrac{x^2}{2\alpha(1-\alpha)}, & \text{if } x \in [0, \alpha) \\ \dfrac{2x-\alpha}{2(1-\alpha)}, & \text{if } x \in [\alpha, 1-\alpha) \\ 1 - \dfrac{(1-x)^2}{2\alpha(1-\alpha)}, & \text{if } x \in [1-\alpha, 1) \\ 1, & \text{if } x \in [1, \infty) \end{cases} \tag{23}$$

  Notice that in the case of random meeting $s = r$ the distribution of rank R^r_{tj} is not uniform.

Given the probability density and the cumulative distribution functions, we are now able to determine the conditional probabilities to marry. We consider the two cases according to $\alpha \geq \frac{1}{2}$, $\alpha < \frac{1}{2}$, and we find the expressions of probability to marry $\Pr^r_m(a^r_t(I))$ under the condition that the state is $s = r$ and a player i of universal type I uses strategy $a^r_t(I)$. This is the probability that both players i and j who met in period t accept to marry under the condition that their choices are independent and they both use the same type of strategies.

In the case of random meeting ($s = r$), this probability is given by:

$$\Pr^r_m(a^r_t(I)) = \left(\Pr\left\{R^r_{tj} > a^r_t(I)\right\}\right)^2 = \left(1 - F_{R^r_{ti}}(a^r_t(I))\right)^2.$$

Substituting and rearranging, we obtain the proposition. ■

8.1.2 Conditional expectations

The last step for deriving the Bellman equation is to determione the conditional expectation of the expected rank of a person if the met players both agree to marriage. This is summarised in the following proposition for the assortative meeting state.

Proposition 11 *The expected rank of a potential partner in the assortative meeting state for any period $t = 1, \ldots, N-1$ is given by*

$$E\left[R_{tj}^{\bar{r}} | R_{tj}^{\bar{r}} > a_{ti}^{\bar{r}}(I), R_{ti}^{\bar{r}} > a_{tj}^{\bar{r}}(I_j)\right](a_{ti}^{\bar{r}}(I)) = \frac{\alpha I + 1 - \alpha + a_{ti}^{\bar{r}}(I)}{2},$$

Proof. We denote as $E\left[R_{tj}^{\bar{r}} | R_{tj}^{\bar{r}} > a_t^{\bar{r}}(I), R_{ti}^{\bar{r}} > a_t^{\bar{r}}(I_j)\right]$ the expectation of absolute rank of the potential partner j chosen by a player' i, under the condition that the marriage takes place in period t and $E\left[R_{tj}^{\bar{r}} | R_{tj}^{\bar{r}} > a_t^{\bar{r}}(I), R_{ti}^{\bar{r}} > a_t^{\bar{r}}(I_j)\right]$ is a function of a player i's strategy $a_t^{\bar{r}}(I)$. For $s = \bar{r}$, the conditional expectation is given by:

$$\begin{aligned}
E\left[R_{tj}^{\bar{r}} | R_{tj}^{\bar{r}} > a_t^{\bar{r}}(I), R_{ti}^{\bar{r}} > a_t^{\bar{r}}(I_j)\right] &= \frac{E\left[R_{tj}^{\bar{r}} | R_{tj}^{\bar{r}} > a_t^{\bar{r}}(I)\right] \Pr\left\{R_{ti}^{\bar{r}} > a_t^{\bar{r}}(I_j)\right\}}{\Pr_m^{\bar{r}}(a_t^{\bar{r}}(I))} \\
&= \frac{E\left[R_{tj}^{\bar{r}} | R_{tj}^{\bar{r}} > a_t^{\bar{r}}(I)\right] \Pr\left\{R_{ti}^{\bar{r}} > a_t^{\bar{r}}(I_j)\right\}}{\Pr\left\{R_{tj}^{\bar{r}} > a_t^{\bar{r}}(I)\right\} \Pr\left\{R_{ti}^{\bar{r}} > a_t^{\bar{r}}(I_j)\right\}} \\
&= \frac{E\left[R_{tj}^{\bar{r}} | R_{tj}^{\bar{r}} > a_t^{\bar{r}}(I)\right]}{\Pr\left\{R_{tj}^{\bar{r}} > a_t^{\bar{r}}(I)\right\}} \\
&= \frac{\int_{a_t^{\bar{r}}(I)}^{\infty} u f_{R_{tj}^{\bar{r}}}(u) du}{\int_{a_t^{\bar{r}}(I)}^{\infty} f_{R_{tj}^{\bar{r}}}(u) du} = \frac{\alpha I + 1 - \alpha + a_t^{\bar{r}}(I)}{2},
\end{aligned}$$

where $a_t^{\bar{r}}(I) \in [\alpha I, \alpha I + 1 - \alpha]$. ■

For random meeting, the conditional expectation is summarised as follows.

Proposition 12 *The expected rank of a potential partner in the random meeting state for any period $t = 1, \ldots, N-1$ is given by*

1. For $\alpha \geq \dfrac{1}{2}$*:*

$$E\left[R_{tj}^{r}|R_{tj}^{r} > a_{ti}^{r}(I), R_{ti}^{r} > a_{tj}^{r}(I_j)\right](a_{ti}^{r}(I))$$
$$= \begin{cases} \dfrac{2(a_{ti}^{r}(I))^3 - 3\alpha(1-\alpha)}{3(a_{ti}^{r}(I))^2 - 6\alpha(1-\alpha)}, & \textit{if } a_{ti}^{r}(I) \in [0, 1-\alpha) \\ \dfrac{3(a_{ti}^{r}(I))^2 - (1+\alpha+\alpha^2)}{6a_{ti}^{r}(I) - 3(1+\alpha)}, & \textit{if } a_{ti}^{r}(I) \in [1-\alpha, \alpha) \\ \dfrac{2a_{ti}^{r}(I) + 1}{3}, & \textit{if } a_{ti}^{r}(I) \in [\alpha, 1] \end{cases}$$

2. For $\alpha < \dfrac{1}{2}$*:*

$$E\left[R_{tj}^{r}|R_{tj}^{r} > a_{ti}^{r}(I), R_{ti}^{r} > a_{tj}^{r}(I_j)\right](a_{ti}^{r}(I))$$
$$= \begin{cases} \dfrac{2(a_{ti}^{r}(I))^3 - 3\alpha(1-\alpha)}{3(a_{ti}^{r}(I))^2 - 6\alpha(1-\alpha)}, & \textit{if } a_{ti}^{r}(I) \in [0, \alpha) \\ \dfrac{3(a_{ti}^{r}(I))^2 - (3-3\alpha+\alpha^2)}{6a_{ti}^{r}(I) - 3(2-\alpha)}, & \textit{if } a_{ti}^{r}(I) \in [\alpha, 1-\alpha) \\ \dfrac{2a_{ti}^{r}(I) + 1}{3}, & \textit{if } a_{ti}^{r}(I) \in [1-\alpha, 1] \end{cases} .$$

Proof. We denote as $E\left[R_{tj}^{r}|R_{tj}^{r} > a_{t}^{r}(I), R_{ti}^{r} > a_{t}^{r}(I_j)\right]$ the expectation of absolute rank of the potential partner j chosen by a player' i, under the condition that the marriage takes place in period t and $E\left[R_{tj}^{r}|R_{tj}^{r} > a_{t}^{r}(I), R_{ti}^{r} > a_{t}^{r}(I_j)\right]$ is a function of a player i's strategy $a_{t}^{r}(I)$. We make use of the analysis carried out for determining the conditional expectation for $s = \bar{r}$ using equations (20), (21), (18), (19). The conditional expectation is given by:

$$E\left[R_{tj}^{r}|R_{tj}^{r} > a_{t}^{r}(I), R_{ti}^{r} > a_{t}^{r}(I_j)\right] = \frac{E\left[R_{ti}^{r}|R_{ti}^{r} > a_{t}^{r}(I)\right]\Pr\left\{R_{ti}^{r} > a_{t}^{r}(I_j)\right\}}{\Pr_{m}^{r}(a_{t}^{r}(I))}$$
$$= \frac{E\left[R_{ti}^{r}|R_{ti}^{r} > a_{t}^{r}(I)\right]}{\Pr\{R_{tj}^{r} > a_{t}^{r}(I)\}} = \frac{\int\limits_{a_{t}^{r}(I)}^{\infty} u f_{R_{tj}^{r}}(u)du}{\int\limits_{a_{t}^{r}(I)}^{\infty} f_{R_{tj}^{r}}(u)du}. \quad (24)$$

For $\alpha \geq \frac{1}{2}$, equation (24) becomes:

$$E\left[R^r_{tj}|R^r_{tj} > a^r_t\left(I\right), R^r_{ti} > a^r_t\left(I_j\right)\right] = \begin{cases} \dfrac{2a^r_t\left(I\right)^3 - 3\alpha(1-\alpha)}{3\left(a^r_t\left(I_j\right)\right)^2 - 6\alpha(1-\alpha)}, & \text{if } a^r_t\left(I_j\right) \in [0, 1-\alpha) \\ \dfrac{3\left(a^r_t\left(I_j\right)\right)^2 - (1+\alpha+\alpha^2)}{6a^r_t\left(I_j\right) - 3(1+\alpha)}, & \text{if } a^r_t\left(I_j\right) \in [1-\alpha, \alpha) \\ \dfrac{2a^r_t\left(I_j\right)+1}{3}, & \text{if } a^r_t\left(I_j\right) \in [\alpha, 1] \end{cases} \tag{25}$$

whereas for $\alpha < \frac{1}{2}$, equation (24) becomes:

$$E\left[R^r_{tj}|R^r_{tj} > a^r_t\left(I\right), R^r_{ti} > a^r_t\left(I_j\right)\right] = \begin{cases} \dfrac{2\left(a^r_t\left(I_j\right)\right)^3 - 3\alpha(1-\alpha)}{3\left(a^r_t\left(I_j\right)\right)^2 - 6\alpha(1-\alpha)}, & \text{if } a^r_t\left(I_j\right) \in [0, \alpha) \\ \dfrac{3\left(a^r_t\left(I_j\right)\right)^2 - (3-3\alpha+\alpha^2)}{6a^r_t\left(I_j\right) - 3(2-\alpha)}, & \text{if } a^r_t\left(I_j\right) \in [\alpha, 1-\alpha) \\ \dfrac{2a^r_t\left(I_j\right)+1}{3}, & \text{if } a^r_t\left(I_j\right) \in [1-\alpha, 1] \end{cases} \tag{26}$$

■

8.1.3 Bellman Equations according to state

We are now in a position to determine a player's optimal strategy through the analysis of the Bellman equation (3). First, we examine separately the two states of the world $\bar{r}$ and r for each period $t = 1, ..., N-1$. First, consider the assortative meeting state $s = \bar{r}$. The Bellman equation (3) is:

$$V^{\bar{r}}_t(I) = \max_{a^{\bar{r}}_t(I)} \left\{ \left(1 - \frac{a^{\bar{r}}_t\left(I\right) - \alpha I}{1-\alpha}\right)^2 \frac{\alpha I + 1 - \alpha + a^{\bar{r}}_t\left(I\right)}{2} + \left(1 - \left(1 - \frac{a^{\bar{r}}_t\left(I\right) - \alpha I}{1-\alpha}\right)^2\right) \delta V_{t+1}(I) \right\}, \tag{27}$$

where $V_{t+1}(I) = \beta V^{\bar{r}}_{t+1}(I) + (1-\beta)V^r_{t+1}(I)$ and with boundary conditions (4) and (5). Remember that men and women are symmetric, implying that opposites play the same strategies. All multipliers in the right hand side part of (27) are nonnegative, so, for each period t from 1 to $N-1$ we investigate $a_t^{\bar{r}}(I)$ that yields $V_t^{\bar{r}}(I)$.

We turn now to the case of random meeting. In the case in which $\alpha \geq \frac{1}{2}$, the Bellman equation (3) is:

$$V_t^r(I) = \begin{cases} \max\limits_{a_t^r(I)} \left\{ \left(1 - \frac{(a_t^r(I))^2}{2\alpha(1-\alpha)}\right)^2 \frac{2(a_t^r(I))^3 - 3\alpha(1-\alpha)}{3(a_t^r(I))^2 - 6\alpha(1-\alpha)} \right. \\ \quad \left. + \left(1 - \left(1 - \frac{(a_t^r(I))^2}{2\alpha(1-\alpha)}\right)^2\right) \delta V_{t+1}(I) \right\}, & \text{if } a_t^r(I) \in [0, 1-\alpha), \\ \max\limits_{a_t^r(I)} \left\{ \left(1 - \frac{2a_t^r(I) - 1 + \alpha}{2\alpha}\right)^2 \frac{3(a_t^r(I))^2 - (1+\alpha+\alpha^2)}{6a_t^r(I) - 3(1+\alpha)} \right. \\ \quad \left. + \left(1 - \left(1 - \frac{2a_t^r(I) - (1-\alpha)}{2\alpha}\right)^2\right) \delta V_{t+1}(I) \right\}, & \text{if } a_t^r(I) \in [1-\alpha, \alpha), \\ \max\limits_{a_t^r(I)} \left\{ \left(\frac{(1 - a_t^r(I))^2}{2\alpha(1-\alpha)}\right)^2 \frac{2a_t^r(I) + 1}{3} \right. \\ \quad \left. + \left(1 - \left(\frac{(1 - a_t^r(I))^2}{2\alpha(1-\alpha)}\right)^2\right) \delta V_{t+1}(I) \right\}, & \text{if } a_t^r(I) \in [\alpha, 1] \end{cases} \tag{28}$$

Conversely if $\alpha < \frac{1}{2}$, then the Bellman equation (3) becomes:

$$V_t^r(I) = \begin{cases} \max\limits_{a_t^r(I)} \left\{ \left(1 - \frac{(a_t^r(I))^2}{2\alpha(1-\alpha)}\right)^2 \frac{2(a_t^r(I))^3 - 3\alpha(1-\alpha)}{3(a_t^r(I))^2 - 6\alpha(1-\alpha)} \right. \\ \quad \left. + \left(1 - \left(1 - \frac{(a_t^r(I))^2}{2\alpha(1-\alpha)}\right)^2\right) \delta V_{t+1}(I) \right\}, & \text{if } a_t^r(I) \in [0, \alpha), \\ \max\limits_{a_t^r(I)} \left\{ \left(1 - \frac{2a_t^r(I) - \alpha}{2(1-\alpha)}\right)^2 \frac{3(a_t^r(I))^2 - (3 - 3\alpha + \alpha^2)}{6a_t^r(I) - 3(2-\alpha)} \right. \\ \quad \left. + \left(1 - \left(1 - \frac{2a_t^r(I) - \alpha}{2(1-\alpha)}\right)^2\right) \delta V_{t+1}(I) \right\}, & \text{if } a_t^r(I) \in [\alpha, 1-\alpha), \\ \max\limits_{a_t^r(I)} \left\{ \left(\frac{(1 - a_t^r(I))^2}{2\alpha(1-\alpha)}\right)^2 \frac{2a_t^r(I) + 1}{3} \right. \\ \quad \left. + \left(1 - \left(\frac{(1 - a_t^r(I))^2}{2\alpha(1-\alpha)}\right)^2\right) \delta V_{t+1}(I) \right\}, & \text{if } a_t^r(I) \in [1-\alpha, 1]. \end{cases} \tag{29}$$

with boundary conditions (4) and (5).

8.2 Proof of Proposition 1

To find the optimal strategy for period t and state $s = \bar{r}$ we first differentiate the expression in the right part of (27) with respect to a, then we equate the differential with zero and solve it for a. We denote the solution as $b_t^{\bar{r}}$. There are two solutions:

$$b_{t1}^{\bar{r}} = \tfrac{4}{3}\delta V_{t+1}(I) - \tfrac{1}{3}[\alpha I + 1 - \alpha],$$
$$b_{t2}^{\bar{r}} = \alpha I + 1 - \alpha.$$

Consider two possible cases for the value of expected rank $V_{t+1}(I)$: $V_{t+1}(I) < \frac{\alpha I + 1 - \alpha}{\delta}$ and $V_{t+1}(I) \geq \frac{\alpha I + 1 - \alpha}{\delta}$:

1. Let $V_{t+1}(I) < \frac{\alpha I + 1 - \alpha}{\delta}$, so that $b_{t1}^{\bar{r}} < b_{t2}^{\bar{r}}$. In this case the second derivative of the right part of (27) with respect to a calculated in $b_{t1}^{\bar{r}}$ ($b_{t2}^{\bar{r}}$) equals to $\frac{-2(\alpha I + 1 - \alpha - \delta V_{t+1}(I))}{(1-\alpha)^2}$

$\left(\frac{2(\alpha I+1-\alpha-\delta V_{t+1}(I))}{(1-\alpha)^2}\right)$. Thus the strategy $a_t^{\bar{r}}(I) = b_{t1}^{\bar{r}}$ maximises the right part of (27) whereas $a_t^{\bar{r}}(I) = b_{t2}^{\bar{r}}$ minimizes it. Hence the function in the right part of (27) decreases in $[b_{t1}^{\bar{r}}, b_{t2}^{\bar{r}}]$. If additionally $b_{t1}^{\bar{r}} < \alpha I$, then the optimal strategy is the minimum possible value for the strategy, i.e. $a_N^{\bar{r}*}(I) = \alpha I$. For $b_{t1}^{\bar{r}} \geq \alpha I$, the strategy $a_N^{\bar{r}*}(I) = b_{t2}^{\bar{r}}$ maximises the right part of (27).

2. Let $V_{t+1}(I) \geq \frac{\alpha I+1-\alpha}{\delta}$. In this case $b_{t2}^{\bar{r}} < b_{t1}^{\bar{r}}$ and $a = b_{t1}^{\bar{r}}$ minimizes the right part of (27) while $a = b_{t2}^{\bar{r}}$ maximises it. Function in the right part of (27) increases from $a = \alpha I$ to $a = b_{t2}^{\bar{r}}$ where obtains the maximum value.

8.3 Proof of Proposition 2

For brevity, we will consider the case $\alpha \geq \frac{1}{2}$ and omit the case $\alpha < \frac{1}{2}$ as it is very similar.[11] The problem is to find the maximum of the piecewise function in the right part of (28) with respect to the strategy $a = a_t^r(I)$. This function is continuous with respect to $a_t^r(I)$. When $a_t^r(I) \in [0, 1-\alpha)$, then it has a unique maximum at $a_t^r(I) = 0$. The second derivative of the function in the right part of (28) calculated in $a_t^r(I) = 0$ equals $\frac{4\delta V_{t+1}(I)-1}{2\alpha(1-\alpha)}$. If $V_{t+1}(I) < \frac{1}{4\delta}$, then the strategy $a_N^{r*}(I) = 0$ maximises the right part of (28). Also, the right part of (28) is a decreasing function with respect to $a_t^r(I)$ in the interval of possible strategy values $[0, 1]$. This implies that the optimal strategy is $a_N^{r*}(I) = 0$.

For $V_{t+1}(I) \geq \frac{1}{4\delta}$, the right part of (28) increases in the interval $a_t^r(I) \in [0, 1-\alpha)$. Consider the case in which $a_t^r(I) \in [1-\alpha, \alpha)$. Differentiation of the function in the right part of (28) yields:

$$b_{t1}^r = \tfrac{1}{6}(1+\alpha) + \tfrac{2}{3}\delta V_{t+1}(I)$$
$$-\tfrac{1}{6}\sqrt{16\delta^2(V_{t+1}(I))^2 - 16(1+\alpha)\delta V_{t+1}(I) + 5\alpha^2 + 6\alpha + 5},$$

$$b_{t2}^r = \tfrac{1}{6}(1+\alpha) + \tfrac{2}{3}\delta V_{t+1}(I)$$
$$+\tfrac{1}{6}\sqrt{16\delta^2(V_{t+1}(I))^2 - 16(1+\alpha)\delta V_{t+1}(I) + 5\alpha^2 + 6\alpha + 5},$$

[11] The complete proof can be provided upon request.

where $b^r_{t1} < b^r_{t2}$. The second derivative of the function in the right part of (28) in b^r_{t1} is negative, while the second derivative of it in b^r_{t2} is positive. Hence b^r_{t1} maximises the function in the right part of (28) and b^r_{t2} minimizes it, and function in the right part of (28) decreases from b^r_{t1} to b^r_{t2}. Here we should consider three cases:

1. For $b^r_{t1} < 1-\alpha$ $\left(\Leftrightarrow \frac{1}{4\delta} \leq V_{t+1}(I) < \frac{5-19\alpha+11\alpha^2}{6(1-3\alpha)\delta}\right)$, the function in the right part of (28) decreases on the interval $[1-\alpha, 1]$. Thus, the optimal strategy is $a^{r*}_N(I) = 1-\alpha$.

2. For $1-\alpha \leq b^r_{t1} < \alpha$ $\left(\Leftrightarrow \frac{5-19\alpha+11\alpha^2}{6(1-3\alpha)\delta} \leq V_{t+1}(I) < \frac{5\alpha+1}{6\delta}\right)$, the function in the right part of (28) increases on $[0, b^r_{t1})$ and decreases on $(b^r_{t1}, 1]$, so that the optimal strategy is $a^r_t(I) = b^r_{t1}$.

3. For $b^r_{t1} \geq \alpha$, the function in the right part of (28) increases on $[0, \alpha)$. For $[\alpha, 1)$, it has one extreme point $a^r_t(I) = b^r_{t3}$, where

$$b^r_{t3} = -\frac{1}{5} + \frac{6}{5}\delta V_{t+1}(I),$$

   and the second derivative shows that it maximises the function in the right part of (28). Hence it increases on $[0, b^{t,r}_3)$ and decreases on $[b^{t,r}_3, 1]$, so that the optimal strategy is $a^{r*}_N(I) = b^r_{t3}$ if and only if $b^r_{t3} \in [\alpha, 1)$, i.e. $V_{t+1}(I) \geq \frac{5\alpha+1}{6\delta}$.

8.4 Proof of Proposition 4

First consider the case with assortative meeting, $s = \bar{r}$. For $a^{\bar{r}*}_t(I) = \alpha I$, the payoff in equilibrium $V^{\bar{r}}_t(I)$ is an increasing function of the universal rank I as

$$\frac{\partial V^{\bar{r}}_t(I)}{\partial I} = \alpha.$$

For $a^{\bar{r}*}_t(I) = \alpha I + 1 - \alpha$, differentiation of $\partial V^{\bar{r}}_t(I)$ with respect to I yields:

$$\frac{\partial V^{\bar{r}}_t(I)}{\partial I} = \frac{\partial V_{t+1}(I)}{\partial I}.$$

Given $\frac{\partial V_N^{\bar{r}}(I)}{\partial I} = \alpha\beta > 0$, we can easily prove the positiveness of $\frac{\partial V_N^{\bar{r}}(I)}{\partial I}$ for any $t = 1, \ldots, N-1$. Consider next the case $a^*(t, \bar{r}) = \frac{4\delta V_{t+1}(I) - (\alpha I + 1 - \alpha)}{3}$ when $\frac{\alpha I}{\delta} + \frac{1-\alpha}{4\delta} \leq V_{t+1}(I) < \frac{\alpha I + 1 - \alpha}{\delta}$, in which:

$$\begin{aligned}\frac{\partial V_t^{\bar{r}}(I)}{\partial I} =& \frac{16\alpha}{9(1-\alpha)^2}\left(\delta V_{t+1}(I) - (1-\alpha+\alpha I)\right)^2 \\ & - \frac{16\delta\left(\delta V_{t+1}(I) - (1-\alpha+\alpha I)\right)^2 - 9\delta(1-\alpha)^2}{9(1-\alpha)^2}\frac{\partial V_{t+1}(I)}{\partial I}.\end{aligned}$$

The right hand side is positive for any $t = 1, \ldots, N$, because $\frac{\alpha I}{\delta} + \frac{1-\alpha}{4\delta} \leq V_{t+1}(I) < \frac{\alpha I + 1 - \alpha}{\delta}$ and the fact that $\frac{\partial V_N^{\bar{r}}(I)}{\partial I} = \alpha\beta > 0$. Therefore, in the assortative meeting case, a player's payoff in equilibrium is an increasing function of the universal rank I.

Finally consider the case with random meeting, $s = r$, and suppose $\alpha \geq \frac{1}{2}$ (the case where $\alpha < \frac{1}{2}$ can be considered in the same way and leads to the same results). We show the proof when $a_t^{r*}(I) \in [0, 1-\alpha)$, and omit the cases in which $a_t^{r*}(I) \in [1-\alpha, \alpha)$ and $a_t^{r*}(I) \in [\alpha, 1]$, as the algebra is very similar and leads to the same results. Differentiating $V_t^r(I)$ w.r.t. I yields:

$$\begin{aligned}\frac{\partial V_t^r(I)}{\partial I} =& 2\left(1 - \frac{(a_t^{r*}(I))^2}{2\alpha(1-\alpha)}\right)\left(-\frac{a_t^{r*}(I)\frac{\partial a_t^{r*}(I)}{\partial I}}{\alpha(1-\alpha)}\right)\times \\ & \left(\frac{2(a_t^{r*}(I))^3 - 3\alpha(1-\alpha)}{3(a_t^{r*}(I))^2 - 6\alpha(1-\alpha)} - \delta V_{t+1}(I)\right) \\ & + \left(1 - \frac{(a_t^{r*}(I))^2}{2\alpha(1-\alpha)}\right)^2 \times \\ & \left(\frac{18a_t^{r*}(I)\frac{\partial a_t^{r*}(I)}{\partial I}\alpha(1-\alpha)(1-2a_t^{r*}(I))}{(3(a_t^{r*}(I))^2 - 6\alpha(1-\alpha))^2} - \delta\frac{\partial V_{t+1}(I)}{\partial I}\right) \\ & + \delta\frac{\partial V_{t+1}(I)}{\partial I}.\end{aligned} \tag{30}$$

For $a_t^{r*}(I) = 0$ and $V_{t+1}(I) < \frac{1}{4\delta}$, we obtain $V_t^r(I) = \frac{1}{2}$, so that $V_t^r(I)$ is a non-decreasing function of I. The similar result can be obtained for the case $a_t^{r*}(I) = 1-\alpha$ and $V_{t+1}(I) \in [\frac{1}{4\delta}, \frac{5-19\alpha+11\alpha^2}{6(1-3\alpha)\delta})$. For $\frac{5-19\alpha+11\alpha^2}{6(1-3\alpha)\delta} \leq V_{t+1}(I) < \frac{5\alpha+1}{6\delta}$ and $V_{t+1}(I) \geq \frac{5\alpha+1}{6\delta}$

we obtain $\frac{\partial V_t^r(I)}{\partial I} \gtrless 0$ if $\frac{\partial V_{t+1}(I)}{\partial I} \gtrless 0$. Given $\frac{\partial V_N^r}{\partial I} = \alpha\beta > 0$, we prove that for any $t = 1, \ldots, N$, the player's payoff in random meeting is an increasing function of I. Therefore, in any possible case, a player's optimal payoff in random meetings is a non-decreasing function of the universal rank.

8.5 Proof of Proposition 5

Begin by examining the assortative meeting state. It is straightforward that for $V_{t+1}(I) < \frac{\alpha I}{\delta} + \frac{1-\alpha}{4\delta}$ and $V_{t+1}(I) \geq \frac{\alpha I + 1 - \alpha}{\delta}$, the optimal strategy is a constant, hence $\frac{\partial a_t^{\bar{r}*}(I)}{dI} = \alpha$ is a non-decreasing function of the universal rank I.

Consider next $\frac{\alpha I}{\delta} + \frac{1-\alpha}{4\delta} \leq V_{t+1}(I) < \frac{\alpha I+1-\alpha}{\delta}$, for which the optimal strategy is $a_t^{\bar{r}*}(I) = \frac{4\delta V_{t+1}(I) - (\alpha I + 1 - \alpha)}{3}$. Here we obtain $\frac{\partial a_t^{\bar{r}*}(I)}{dI} = \frac{4\delta}{3}\frac{\partial V_{t+1}(I)}{\partial I} - \frac{\alpha}{3}$, where $\frac{\partial V_{t+1}(I)}{\partial I} = \beta \frac{\partial V_{t+1}^{\bar{r}}(I)}{\partial I} + (1-\beta)\frac{\partial V_{t+1}^{r}(I)}{\partial I}$. Thus $\frac{\partial a_t^{\bar{r}*}(I)}{dI}$ is non-negative during the whole game if and only if $\frac{\partial V_{t+1}(I)}{\partial I} > \frac{\alpha}{4\delta}$.

For $t = N-1$, the player's payoff $\frac{\partial V_N^{\bar{r}}(I)}{\partial I} = \alpha\beta$, hence the condition of non-negativity is $\beta > \frac{1}{4\delta}$. Now consider $t = N-2$. Substituting the optimal strategy $a_{N-2}^{\bar{r}*}(I)$ into expression (30) and writing down the condition $\frac{\partial V_{N-1}^{\bar{r}}(I)}{\partial I} > \frac{\alpha}{4\delta}$ yields:

$$[\delta V_{t+1}(I) - (1 - \alpha + \alpha I)]^2 \frac{16}{9(1-\alpha)^2}(1 - \delta\beta)\delta + \left(\delta^2\beta - \frac{1}{4}\right) > 0.$$

Given $\beta > \frac{1}{4\delta^2}$ we can easily prove that $\frac{\partial V_{N-1}^{\bar{r}}(I)}{\partial I} > \frac{\alpha}{4\delta}$. Therefore $a_{N-2}^{\bar{r}*}(I)$ is a non-decreasing function of universal rank I. By repeating the procedure recurrently for all t we prove the result of the proposition.

Consider next the random meeting when $\alpha \geq \frac{1}{2}$.[12] The non-negativity of $\frac{\partial a_t^{r*}(I)}{\partial I}$ is straightforward for $V_{t+1}(I) < \frac{5-19\alpha+11\alpha^2}{6(1-3\alpha)\delta}$. The optimal strategy $a_t^{r*}(I)$ is a non-decreasing function for $V_{t+1}(I) \geq \frac{5\alpha+1}{6\delta}$ iff $\frac{\partial V_{t+1}(I)}{\partial I}$ is non-negative, which is proved by Proposition 4.

[12] The case with $\alpha < 1/2$ yields the same results and it is omitted.

Now consider $\frac{5-19\alpha+11\alpha^2}{6(1-3\alpha)\delta} \leq V_{t+1}(I) < \frac{5\alpha+1}{6\delta}$:

$$\frac{\partial a_t^{r*}(I)}{\partial I} = \frac{2\delta}{3}\frac{\partial V_{t+1}(I)}{\partial I} \times$$

$$\left(1 - \frac{4\delta V_{t+1}(I) - 2(1-\alpha)}{\sqrt{16\delta^2 (V_{t+1}(I))^2 - 16(1-\alpha)\delta V_{t+1}(I) + 5\alpha^2 + 6\alpha + 5}}\right).$$

We can easily obtain $\frac{\partial a_t^{r*}(I)}{\partial I} \gtreqless 0$ when $\frac{\partial V_{t+1}(I)}{\partial I} \gtreqless 0$ since the right hand side is always positive when $\frac{5-19\alpha+11\alpha^2}{6(1-3\alpha)\delta} \leq V_{t+1}(I) < \frac{5\alpha+1}{6\delta}$. Therefore, given Propostion 4 we prove Proposition 5.

8.6 Proof of Proposition 6

Begin by noticing that V_{t+1} is a function of β, in particular

$$\frac{\partial V_{t+1}(I)}{\partial \beta} \lesseqgtr 0 \text{ as } V_{t+1}^{\bar{r}}(I) \lesseqgtr V_{t+1}^{r}(I). \tag{31}$$

Therefore the effect of β on the optimal strategy in both assortative and random meeting state depends on which future conditional expectation is higher.

Consider the conditions on parameters that gives the sign of the derivative of the optimal strategy with respect to β. The derivative of $a_t^{\bar{r}*}(I)$ with respect to β is zero for $V_{t+1}(I) < \frac{4\alpha I+1-\alpha}{4\delta}$ and $V_{t+1}(I) \geq \frac{\alpha I+1-\alpha}{\delta}$. Conversely, for $\frac{4\alpha I+1-\alpha}{4\delta} \leq V_{t+1}(I) < \frac{\alpha I+1-\alpha}{\delta}$, we get

$$\frac{\partial a_t^{\bar{r}*}(I)}{\partial \beta} = \frac{4\delta}{3}\frac{\partial V_{t+1}(I)}{\partial \beta} \lesseqgtr 0 \text{ for } \frac{\partial V_{t+1}(I)}{\partial \beta} \lesseqgtr 0,$$

which holds if and only if (31) holds. Let us turn on the optimal strategy in the random meeting state. The derivative of $a_t^{r*}(I)$ with respect to β when $\alpha \geq \frac{1}{2}$ is:

$$\frac{\partial a_t^{r*}(I)}{\partial \beta} = \begin{cases} 0, & \text{if } V_{t+1}(I) < \dfrac{5-19\alpha+11\alpha^2}{6(1-3\alpha)\delta} \\ \dfrac{2\delta}{3}\tau\dfrac{\partial V_{t+1}(I)}{\partial \beta} & \text{if } \dfrac{5-19\alpha+11\alpha^2}{6(1-3\alpha)\delta} \leq V_{t+1}(I) < \dfrac{5\alpha+1}{6\delta} \\ \dfrac{6\delta}{5}\dfrac{\partial V_{t+1}(I)}{\partial \beta}, & \text{if } V_{t+1}(I) \geq \dfrac{5\alpha+1}{6\delta} \end{cases}$$

where:

$$\tau = 1 - \frac{4\delta V_{t+1} - 2(1-\alpha)}{\sqrt{([4\delta V_{t+1} - 2(1-\alpha)]^2 + (\alpha^2 + 14\alpha + 1)}} > 0.$$

Therefore

$$\frac{\partial a_t^{r*}(I)}{\partial \beta} \lesseqqgtr 0 \Leftrightarrow \frac{\partial V_{t+1}(I)}{\partial \beta} \lesseqqgtr 0 \Leftrightarrow V_{t+1}^{\bar{r}}(I) \lesseqqgtr V_{t+1}^{r}(I).$$

The same result applies when $\alpha < \frac{1}{2}$:

$$\frac{\partial a_t^{r*}(I)}{\partial \beta} = \begin{cases} 0, & \text{if } V_{t+1}(I) < \dfrac{11\alpha^2-3\alpha-3}{6(3\alpha-2)\delta} \\ \dfrac{2\delta}{3}\tau'\dfrac{\partial V_{t+1}(I)}{\partial \beta} & \text{if } \dfrac{11\alpha^2-3\alpha-3}{6(3\alpha-2)\delta} \leq V_{t+1} < \dfrac{6-5\alpha}{6\delta} \\ \dfrac{6\delta}{5}\dfrac{\partial V_{t+1}(I)}{\partial \beta}, & \text{if } V_{t+1}(I) \geq \dfrac{6-5\alpha}{6\delta} \end{cases}$$

where:

$$\tau' = 1 - \frac{4\delta V_{t+1} - 2(2-\alpha)}{\sqrt{(4\delta V_{t+1} - 2(2-\alpha))^2 + (9\alpha^2 - 16\alpha + 32)}} > 0,$$

8.7 Proof of Proposition 8

The Bellman equation in the vectorial form for the game with infinite horizon is:

$$\mathbb{V}\left(a^{\bar{r}}\left(I\right), a^{r}\left(I\right)\right) = \mathbb{A}_1 + \delta\mathbb{A}_2(\beta, 1-\beta)\mathbb{V}\left(a^{\bar{r}}\left(I\right), a^{r}\left(I\right)\right), \tag{32}$$

By transforming equation (32) to obtain the explicit form of $\mathbb{V}\left(a^{\bar{r}}\left(I\right), a^{r}\left(I\right)\right)$, we get

$$\left(\mathbb{I} - \delta\mathbb{A}_2(\beta, 1-\beta)\right)\mathbb{V}\left(a^{\bar{r}}\left(I\right), a^{r}\left(I\right)\right) = \mathbb{A}_1.$$

If the determinant of matrix $\left(\mathbb{I} - \delta\mathbb{A}_2(\beta, 1-\beta)\right)$ does not equal to zero, then the solution of the last equation is vector $\mathbb{V}\left(a^{\bar{r}}\left(I\right), a^{r}\left(I\right)\right)$ that is determined by the following expression:

$$\mathbb{V}\left(a^{\bar{r}}\left(I\right), a^{r}\left(I\right)\right) = \left(\mathbb{I} - \delta\mathbb{A}_2(\beta, 1-\beta)\right)^{-1}\mathbb{A}_1.$$

The determinant of matrix $\left(\mathbb{I} - \delta\mathbb{A}_2(\beta, 1-\beta)\right)$ equals to zero when $\delta = 1$ and, at the same time, both elements of matrix $\mathbb{A}_2$ equal to zero. The elements $\mathbb{A}_{21}$, $\mathbb{A}_{22}$ equal to zero if and only if a player uses his/her highest possible strategy. Finally, we compute the matrix $\left(\mathbb{I} - \delta\mathbb{A}_2(\beta, 1-\beta)\right)^{-1}$ and obtain:

$$\mathbb{V}\left(a^{\bar{r}}\left(I\right), a^{r}\left(I\right)\right) = \frac{\begin{pmatrix} 1-\delta(1-\beta)\mathbb{A}_{22} & \delta(1-\beta)\mathbb{A}_{21} \\ \delta\beta\mathbb{A}_{22} & 1-\delta\beta\mathbb{A}_{21} \end{pmatrix}\mathbb{A}_1}{1-\delta(\beta\mathbb{A}_{21}+(1-\beta)\mathbb{A}_{22})}.$$