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Plenty of Fish in the Sea: Divorce Choice and the Quality of Singles*

Alessandro Tampieri† and Elena M. Parilina‡

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Abstract

We analyze divorce choice when the population distribution is non stationary. We consider a non-transferable utility, three-period model where heterogeneous individuals may divorce the partner and re-enter the marriage market. Individuals' choices are based on the change in the distribution of singles and the cost of waiting and divorcing, and take into account the individual own's eligibility in the marriage market. We show the existence of "divorce" and "no divorce" equilibria. Divorce emerges in the presence of asymmetry among spouses' types or in case of symmetry among medium-type spouses. Interestingly, lower divorce costs do not necessarily increase the probability of divorce. We then discuss how this framework can help interpreting the effects of divorce reforms on divorce rates.

Keywords: non-stationary distribution, divorce cost, waiting cost.

JEL codes: J12, C78

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1 Introduction

This paper analyzes divorce choice when the population of singles is influenced by marital choices. In the economic literature, divorce has been treated extensively, through very different perspectives.[1] In particular, the analysis of search and matching in marriage markets usually considers divorce as a random event: in steady state, the number of divorces must equate the flow creation of matches for every type (Shimer and Smith, 2000a). This condition ensures the distribution of singles remains constant over time. However, the analysis of divorce choice requires to depart from the steady state analysis: endogenous divorce decisions depend on the quality of potential partners, which in turn depends on which individuals marry and leave the marriage market. Hence the changes in the distribution of singles affect divorce decisions.

In this paper we try to fill this gap. We analyze divorce choice when the distribution of singles is non-stationary. We consider a three-period game where individuals are heterogeneous in one characteristic, equally evaluated by all, called "quality". In the first period, each individual randomly meets one potential partner, and both decide whether to marry or not. If marriage does not occur, the random match is repeated in the second period, with same rules. Conversely, an individual who married in the first period may divorce the partner in the second period and re-enter the marriage market. In the last period there is again a random matching for singles but now the match is mandatory, while couples cannot divorce. In the first two periods, individuals adopt threshold strategies, by comparing the met person (for

[1] The classic approach examined divorce as a "threat" among spouses who bargain household income (Lundberg and Pollak, 1996). More recently, the analysis of divorce legislation examined the impact of this on divorce rate (Rasul, 2006, Peters 1986), household labor supply (Chiappori *et al.*, 2002), the choice between cohabitation and marriage (Matouschek and Rasul, 2008) and investment in marriage-specific capital, such as having children, buying a house, dividing home versus market work (Stevenson, 2007, Cigno, 2012). However, modeling divorce as an exogenous threat does not allow to to analyze divorce choice and marriage decisions.

marriage decisions) or the spouse (for divorce decisions) with the expected quality of singles. The marriage and divorce choices modify the distribution of singles and thus their expected quality in the marriage market.

We investigate the existence of “divorce” and “no divorce” equilibria. Intuitively, too high divorce costs prevent divorce from happening. When this is not the case, divorce equilibria are determined by the interconnection between divorce costs and the cost of waiting for future partners. On the one hand, lower divorce costs give an incentive to “rational divorce”: this occurs when, in the first period, individuals accept a non-optimal partner to avoid loneliness, but already planning a future divorce. On the other hand, a high cost of waiting decreases the discounted value of future matches. In particular, if the cost of waiting is high, then there is a stronger incentive in agreeing to match whatever potential partner is met after divorce. A risk of “downgrading” emerges: a high quality person who divorced the partner may end up with a partner being worse than the divorced one. Accordingly, when the cost of waiting is very high, divorce does not emerge, to avoid the risk of downgrading.

Three types of divorce equilibria may emerge: a “low divorce” equilibrium, where divorce occurs between asymmetric types (one of high quality and one of low quality) and either a “medium” or “high divorce” equilibrium, where divorce occurs *also* between symmetric, medium types. Counterintuitively, the low divorce equilibrium is the only admissible outcome for very low divorce costs, implying that *lower divorce costs do not necessarily increase the probability of divorce*. The intuition is the following. A cheaper divorce gives an incentive to more eligible individuals to divorce. This increases the expected quality of singles in the marriage market. In turn, low-eligible individuals have less incentives to re-enter the marriage market: the number of them who chooses to stay married is larger than the number of high eligible individuals who chooses to divorce due to low divorce costs.

The present analysis contributes to a theoretical understanding of divorce decisions and how these affects the marriage market. In models where divorce is admissible and free of charge, individuals have an incentive in choosing whatever potential partner they met, given an implicit cost of waiting. Indeed, this non-optimal partner can then be dumped, if there is a good chance of finding a better partner in the future (Chiappori and Weiss 2006, Goldmanis *et al.*, 2012). The introduction of an explicit divorce cost changes the people behavior: an individual agrees to marry either if he/she really likes the potential partner or if the benefit of staying with the partner will offset the future divorce costs.

The theoretical model is linked to the discrete-time, three-period version of Smith (1992). In Smith (1992), divorcing does not entail an explicit divorce cost, but it makes an individual lose the turn in the marriage market. As a consequence, divorce occurs for low types, since the implicit cost of losing one turn is very high for high types, because they are "eligible" (i.e., they are a good catch). In the present analysis, divorcing does not make losing any turn, and the explicit divorce cost is identical for all individuals irrespective of their type. As a consequence, divorce is more appealing for medium/high types, since the fixed cost is relatively higher for low, not-eligible types.

The remainder of the paper is organized as follows. The next section briefly surveys some related literature. Section 3 shows some of the empirical facts that our theory aims at explaining. Section 4 introduces the theoretical model. Section 5 shows the results, while Section 6 discusses how the theory may help interpreting the effects of divorce reforms. Section 7 concludes.

2 Related literature

The present analysis is related to three strands of the economic literature, namely, the literature on non-stationary dynamic, the literature on matching with nontransferable utility and the literature on divorce.

Nonstationary dynamic. Models with nonstationary dynamic are rare, this due to their limited analytical tractability. Smith (1992) pioneered this approach: he examined a matching model where non-steady state dynamics is driven by temporary matches that are formed because of an implicit cost of waiting. Shimer and Smith (2000b) evaluate whether search and matching efficiency requires nonstationarity. Damiano *et al.* (2005) examine non-stationary dynamics and sorting efficiency in a two periods matching model. If individuals incur in a participation cost and the space of types is sufficiently large, the market unravels as almost all individuals rush to participate in the first period and match whatever potential partner they meet. This paper is an application of Smith (1992) in a modified framework where divorce is costly.

Matching models with nontransferable utility. Relevant contributions of this literature are McNamara and Collins (1990), Burdett and Coles (1997 and 1999), Eeckout (1999), Bloch and Ryder (2000), Chade (2001) and Smith (2006), *inter alia*. A common result in these works is "block segregation": at equilibrium, positive assortative matching emerges: an interval of individuals of highest type match only with each other, the next highest match only with each other, and so forth. Adopting a general payoff function, Smith (2006) finds that block segregation emerges with any multiplicatively separable payoff function. He also finds a set of conditions under which block segregation disappears.

Like in these models where divorce is banned, we find a block segregation result adjusted by the divorce option. The main difference is the fact that, in those models, the distribution of singles is stationary. In our setting, the reservation payoff of a period is given by the expected and discounted quality of singles in the next period, which changes over time according to the marriage and divorce decisions.

Goldmanis *et al.* (2012) extends Smith (2006)'s paper by allowing individuals to keep searching for better partners while they are married. Unlike the present analysis, they analyze the system in steady state, thus assuming stationarity in the distribution of singles, and no divorce costs. Steady state is obtained by assuming that agents adopt the strategy to accept everyone when single, and to divorce when a better partner is found. In the present paper, the presence of divorce costs prevents the strategy to accept any potential partner to be optimal.

Divorce. In the economic literature, divorce is usually explained through three arguments. The first is that divorce is due to an unexpected matching shock. A second argument is imperfect information about true type and learning on the match. These two arguments are often examined together or as alternative explanation (Marinescu, 2016). A third explanation of divorce is that a temporary, but not optimal, match is more convenient than being alone: individuals may marry a partner even if is not satisfactory for not being alone (Chiappori and Weiss, 2006, Barham *et al.*, 2009). Our paper is somewhat linked to the last explanation: temporary matches are more convenient than being alone, but we bind this choice by adding an explicit cost of divorce.

In Chiappori and Weiss (2006), divorce is examined in a general equilibrium model with homogeneous population: in the first period, everyone marries regardless of the quality of the match. Divorced individuals always remarry if a new partner is met, and there is no divorce cost. By contrast, we consider heterogeneous population and a further choice on remarriage.

3 Empirical motivation

In this section we illustrate the empirical relevance of our theoretical analysis, through two simple exercises. First, we determine which couples divorce according to spouses' individual characteristics. To do so, we use data from the Marital Instability Over the Life Course study (Booth, Amato, Johnson, Edwards, 1993). This is a 20-year panel survey of a nationally representative sample of married individuals in the United States. The goal of this dataset is to investigate the role played by a large variety of factors that may affect divorce. It started in 1980, based on telephone interviews conducted with a sample of 2,034 heterosexual married persons younger than 55 years of age. Respondents were then re-interviewed in 1983, 1988, 1992, 1997 and 2000.

We have initially considered three possible spouses' characteristics: years of schooling, income and self-rated health status. We then excluded both income and health status. The first measure was excluded because in a large proportion of couples, only one partner was working, showing a large asymmetry in terms of income. The latter was discarded as responses to health status were scant. We thus focused on years of schooling.

For years after 1980, we drop an observation if the individual did not divorce and did not change the years of schooling acquired in the period, since the observation after 1980 would be only a repetition of the same couple in 1980. We however keep observations after 1980 if divorce occurred, or if one of the spouses have acquired further education. Indeed, the new level of education changes the spouse characteristic, and thus it is considered as a new observation compared to the couple in the 1980.

In Figure 1, we scatter the years of schooling of respondents and spouses, by marking differently whether the couple divorced (red thick cross) or not (black dot). Divorce occurs in the presence of asymmetry among the spouses' years of education or in case of symmetry if spouses acquired a medium range of years of schooling.

The second exercise regards the relationship between the probability of divorce and average age at marriage across countries. This information may proxy the time to which, on average, individuals from a certain population might feel it is time to marry, thus representing some kind of cultural trait. The European Demography Report by Eurostat provides an overview of recent demographic trends in the European Union for years 1990, 2000, 2010 and 2013. Figure 2 summarizes the information about average age at marriage and divorce rate by available countries and years. The information is then plotted to seek whether some relationship can be found. Figure 3 shows a non-monotone relationship between average age at marriage and divorce rate: this is negative when age at marriage is low, and positive otherwise. The theoretical framework that follows aims at explaining these stylized facts.

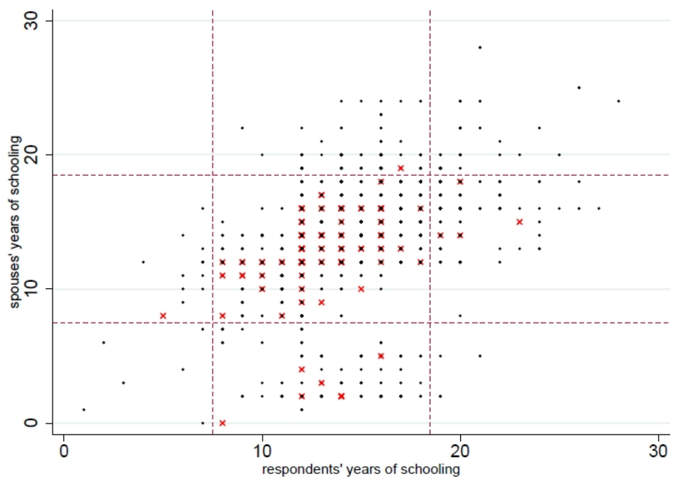

Figure 1: Divorce choice according to spouses' education: red cross = divorced couple, back dot= married couple.

	Average age at marriage				Divorce rate			
	1990	2000	2010	2013	1990	2000	2010	2013
Belgium	25.5	28.0	30.6	.	2.0	2.6	2.7	.
Bulgaria	23.1	26.6	28.2	28.4	1.3	1.3	1.5	1.5
Czech Republic	23.0	26.1	29.4	29.9	3.1	2.9	2.9	2.7
Denmark	29.2	31.2	32.4	33.2	2.7	2.7	2.6	3.4
Germany	26.9	29.1	.	31.9	1.9	2.4	.	2.1
Estonia	.	26.7	29.3	30.1	3.7	3.0	2.2	2.5
Ireland	.	.	32.2	.	.	.	0.7	.
Greece	27.0	29.2	31.0	31.3	0.6	1.0	1.2	1.5
Spain	26.7	29.2	32.0	33.3	0.6	0.9	2.2	2.0
France	.	29.6	31.8	.	.	1.9	2.1	.
Croatia	.	.	28.6	29.1	.	.	1.2	1.4
Italy	27.4	29.4	31.8	32.6	0.5	0.7	0.9	0.9
Cyprus	26.4	28.1	.	.	0.6	1.7	.	.
Latvia	.	.	28.3	29.8	.	.	2.4	3.5
Lithuania	23.7	25.0	27.6	28.3	3.4	3.1	3.2	3.4
Luxembourg	26.7	28.9	31.5	31.2	2.0	2.4	2.1	2.1
Hungary	23.4	26.2	29.8	30.5	2.4	2.3	2.4	2.0
Netherlands	27.3	29.4	31.1	31.6	1.9	2.2	2.0	2.0
Austria	26.5	28.7	.	.	2.1	2.4	.	.
Poland	.	25.3	27.3	27.8	.	1.1	1.6	1.7
Portugal	25.6	26.3	28.8	30.3	0.9	1.9	2.6	2.2
Romania	24.0	25.5	.	28.0	1.4	1.4	.	1.4
Slovenia	25.4	28.5	30.4	31.0	0.9	1.1	1.2	1.1
Slovakia	.	25.5	28.6	29.2	.	1.7	2.2	2.0
Finland	27.4	29.4	31.4	31.6	2.6	2.7	2.5	2.5
Sweden	29.0	31.7	34.2	34.4	2.3	2.4	2.5	2.8

Figure 2: Average age at marriage and divorce rate in Europe. Source: Eurostat

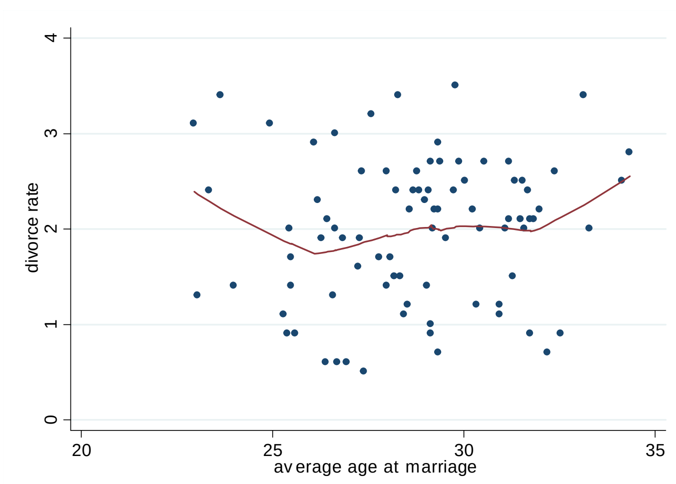

Figure 3: Relationship between divorce rate and average age at marriage by European countries

4 The model

4.1 A 3-periods game with random match

There are two populations of the same size, one of individuals i and one of partners p. Each population is a continuum with types θ, equally distributed uniformly in $[0, 1]$, and living in three periods. In every period, each individual i (he) is randomly matched with a partner p (she). In what follows, we will keep the point of view of individual i, but the behavior of a partner is symmetric. Either party can refuse the match in the first 2 periods, while in the third period the match is compulsory. If both an individual and a partner agree to match, the couple obtains a flow utility of $2\theta_i\theta_p$, where θ_i is the individual's type and θ_p is the partner's type. If at least one refuses the match, each gets a zero flow utility. Spouses equally share the flow utility, so that individual i's flow utility is

$$u_i(\theta_i, \theta_p) = \theta_i\theta_p, \tag{1}$$

per period. Given that everyone benefits from larger θ, the type represents an individual's characteristic that is ranked in the same way by everyone. Hence, every individual prefers to be matched with a partner of type $\theta'_p = \theta'$ rather than with a partner of type $\theta''_p = \theta''$ if $\theta' > \theta''$. Thus a type may represent, for instance, an individual's income, education, health status or, more generally, his "quality". Notice that, since the utility function is multiplicatively separable, a person's parameter has no effect on preferences, so that all individuals share the same decision criterion.

In flow utility (1), we do not take into account of an idiosyncratic benefit that may represent, say, "love" or some subjective preferences. A way to consider this aspect is by adding a second, additive component of flow utility, and by assuming that it is IID among individuals with zero mean. If implemented along tese lines, this feature would not influence the individuals'

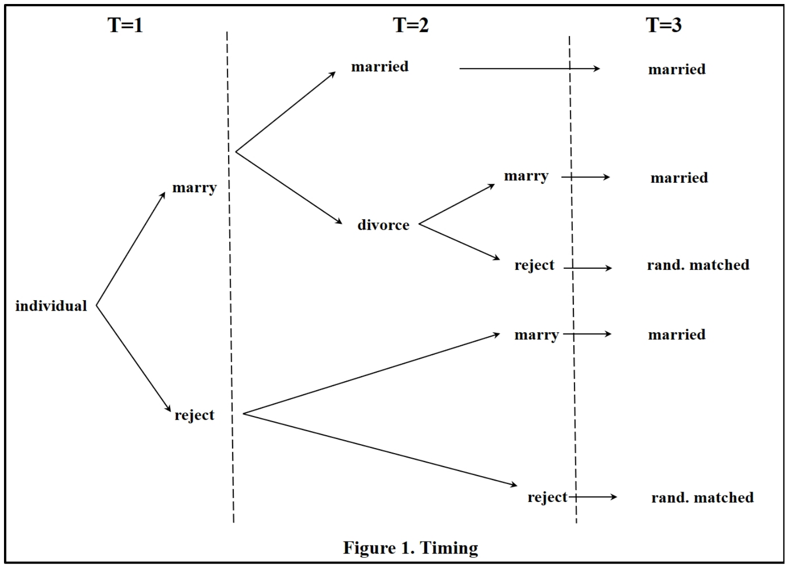

Figure 4: Timing

decisions and may be omitted.

In addition, we assume that individual types are constant over time. In reality, this may be the case if a type is determined by the level of education, as usually marriage starts after schooling. About health status, we may expect that, on average, this will decrease for the entire cohort over time, by keeping a similar ranking. It is less justifiable if heterogeneity represents labor income. In any case, changes in income are hard to be predicted: individuals might assume them as, again, zero mean noises that do not affect future expectations.

Figure 4 shows the timing of the game. We begin from the case in which the individual rejects his potential partner in $T = 1$. If so, then he goes to the marriage market in $T = 2$. Accepting the marriage in $T = 2$ implies remaining married forever, while rejecting in $T = 2$ leads to the random match with mandatory acceptance in $T = 3$.

Consider next the more complex situation where an individual accepts the match in $T = 1$. After agreeing to marry in $T = 1$, the individual may divorce the partner in $T = 2$ at a cost $c \in (0, 1)$ and go back to the marriage market in the same period. The assumption of fixed cost of divorce among types is acceptable for some interpretation of heterogeneity: for instance, there is no reason why individuals with different educational level of health status should bear different divorce costs. By contrast, individuals differing in income pay substantially different costs of divorce. In this case, there are however two contrasting effects. First, divorce costs increase with income, so that higher types would pay more for divorce. Second, the relative cost with respect to income decreases, so that the overall divorce cost has a stronger impact on lower incomes. The outcome of these two opposite effects is not obvious.[2] Thus even if heterogeneity represents individual income, the assumption of fixed cost may be reasonable.

Note that divorce is possible only in $T = 2$. Once that the divorced individual met his new potential partner, then he may agree or not to the new match. Finally, we assume that divorced individuals do not know the new potential partner met in $T = 2$, when they decide to divorce their spouses (i.e., no "affairs" are allowed).

In his marriage and divorce decisions, an individual i takes into account two factors:

1. the potential partner type θ_p: each individual would like to increase the partner's quality as much as possible.

2. his own type θ_i: an individual type determines his "eligibility" in the future, that is, the chances to be chosen in the marriage market. For instance, if the individual is of low quality, then he knows he will be refused by high-quality partners. Thus he is likely not to divorce his partner even if he preferred a better one.

[2] To the best of our knowledge, empirical evidence investigating the relationship among divorce cost based and household income are not available.

We assume that individuals adopt threshold strategies: when they are singles and must choose whether to marry, they observe the discounted expected quality of singles in the next period, and compare it with the quality of their potential partner. Let the per period discount rate be $\delta < 1$. In this context, δ can be interpreted as an inverse measure of "cost of waiting": if δ is low, it is more likely that a potential partner met today has a higher quality than the expected quality of singles in the next period. When they are married and must choose whether to divorce (in $T = 2$), they observe the expected quality of singles in the same period net of the divorce cost. In both cases, they take into account their future eligibility by comparing *their own* quality with the discounted expected quality of singles in the next period. Note that the analysis of a three-period game is necessary precisely to take into account the role of eligibility in the divorce choice. If the game lasted two periods and the mandatory match was at $T = 2$ instead, still allowing for ending the relationship started in $T = 1$, then the divorce choice would only be based on the quality of the partner compared to the average quality in the market.

We solve the problem by backward induction, starting from Period 2, since in Period 3 no choice is allowed.

4.2 Period 2

In $T = 2$, individuals are singles if (i) they rejected/were rejected by the potential partner in $T = 1$, or (ii) they divorced/were divorced by their partners in $T = 2$. With this information, individuals are able to determine the *discounted* expected partner's quality in $T = 3$, denoted by θ_2.[3] Hence θ_2 is the threshold strategy for the second period: a single accepts a proposed partner when her index θ_p is at least $\theta_2 > 0$. For singles in $T = 2$, eligibility

[3] We denote θ_2 as discounted expected payoff (that is, instead of using "$\delta\theta_2$") to ease the exposition of the analysis, and because this threshold never plays a role when "undiscounted" (i.e., at time $T = 3$).

does not affect the strategy, since in $T = 3$ the match is compulsory.

Conversely, individuals are married in $T = 2$ if they accepted and were accepted by their partners in $T = 1$. At the beginning of $T = 2$, married individuals compare their partner's quality with the expected quality of singles in $T = 2$. This is denoted as θ_1 and is derived similarly as θ_2. Note that expectation θ_1 takes into account divorce decisions: a married individual foresees that certain individuals divorce in $T = 2$.

Hence a married individual in $T = 2$ divorces his spouse for two reasons:

[i] **unhappiness**: the partner's quality is lower than the expected quality in $T = 2$, minus the divorce cost, $\theta_p < \theta_1 - c$;

[ii] **eligibility**: the individual is eligible in the marriage market in $T = 2$, $\theta_i \geq \theta_2$. No one divorce, even if "unhappy" of his partner ($\theta_p < \theta_1 - c$), if he thinks he has no chance of being accepted in the marriage market.

4.3 Period 1

The first period behavior is related to the level of divorce costs. An individual i agrees to marry if

[I] he is eligible in $T = 2$, $\theta_i \geq \theta_2$, and

- [I.a] **partner's quality**: the potential partner's index is at least as high as the expected quality of types in $T = 2$, $\theta_p \geq \delta\theta_1$, or
- [I.b] **divorce option**: the discounted divorce cost is lower than the current payoff of marrying her, $\theta_p > \delta c$.

[II] he is not eligible in $T = 2$, $\theta_i < \theta_2$.

The way that the divorce option in period $T = 2$ affects the marriage choice in $T = 1$ deserves some discussion. A potential partner who is unacceptable without possibility of divorce ($\theta_p < \delta\theta_1$) may be accepted if her quality is not lower than the discounted divorce cost, $\theta_p \geq \delta c$. This can be defined as "rational divorce": if $\theta_p \in (\delta c, \delta\theta_1)$, an individual can marry a potential partner even planning a future divorce. This occurs since the cost of divorce is offset by the payoff given from the partner type. At the limit, for $c = 0$, it is optimal to marry whatever potential partner met in $T = 1$, since it is costless to divorce her in the second period to find someone better.

5 Results

5.1 No divorce

To begin with, we consider the scenario with no divorce. This occurs either if the cost of divorce is too high (see Section 5.2 for details) or if unhappiness or eligibility does not hold in $T = 2$.

Since divorce disappears from the scenes, an individual in $T = 1$ compares the partner's quality with $\delta\theta_1$ and disregards δc. On the other hand, eligibility still applies: the individual compares his type with θ_2. The following proposition summarises the equilibrium with no divorce:

Proposition 1 *Suppose that divorce does not occur. Then a no divorce equilibrium is characterized by* (θ_1^n, θ_2^n), *where* $\theta_2^n < \delta\theta_1^n$.

Proof. See Appendix A. ■

Figure 5 shows the marriage choices in $T = 1$ in the no divorce equilibrium. In every region, each couple of letters represents the behavior of agents i and p, with the first letter indicating the partner's behavior and the second letter indicating the individual's behavior. Marriage takes place in $T = 1$ and lasts for the entire game if both letters are m (married), while r indicates that one agent rejects the person met in $T = 1$. So, for example, an individual i of type $\theta_i > \delta\theta_1$ agrees to marry a potential partner of type $\theta_p > \delta\theta_1$ (letters mm) but refuses a type $\theta_p < \delta\theta_1$, who instead would be happy to marry him (letters mr).

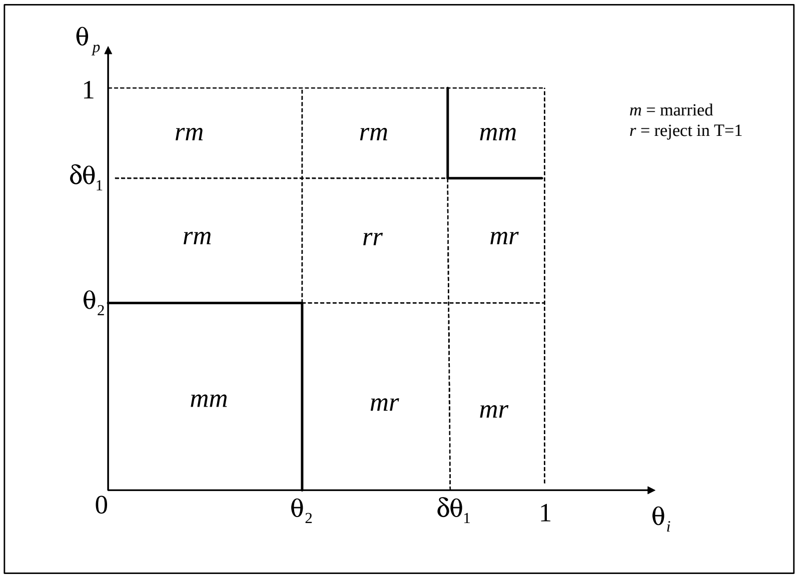

Figure 5: No divorce equilibrium: marriage choice in $T = 1$.

The straight lines separate the regions in which different outcomes occur. Note that the strategies in the first stage are the same as in Smith (1992), but they differ in the second period since here divorce does not occur. This is also a "block segregation" result, which is typical of matching models with non-transferable utility (McNamara and Collins 1990, Burdett and Coles, 1997, 1999, Eeckout 1999, Bloch and Rider 2000, Chade 2001 and Smith 2006, *inter alia*). At equilibrium, positive assortative matching emerges as only similar individuals marry each other: individuals agree to marry only with potential partners of the same "block". Compared to the standard block segregation result, agents from the block in the middle $(\theta_2, \delta\theta_1)$ remain single in $T = 1$, and wait for a better match in $T = 2$.[4]

[4]Smith (1992) calls this a "Groucho Marx result": no one wants to match with anyone who wants to match with him.

5.2 Divorce

In this section we analyse the equilibria where individuals may choose to divorce. This occurs if the divorce cost is not too high and if the two conditions "unhappiness" and "eligibility" hold in $T = 2$. The necessary condition of cost according to which divorce may occur is the following.

Proposition 2 *For $c < \frac{\theta_1}{1+\delta}$, a divorce equilibrium may take place.*

Proposition 2 can be explained as follows. The condition for the decision of divorce ("unhappiness") is $\theta_p < \theta_1 - c$, while a condition to marry with the "divorce option" (condition [I.b]) is $\theta_p > \delta c$. Comparing the two thresholds, we obtain that divorce could happen for $\theta_1 - c > \delta c$ which, rearranged, yields the proposition.

Proposition 2 implies that $\theta_1 > \delta c$ whenever divorce occurs. It follows that, unlike in the no divorce situation, an individual decides to marry by comparing his partner θ_p with δc instead of θ_1. Hence the expected discounted quality is not relevant in $T = 1$, since divorce is sufficiently cheap so as the individual can leave the potential partner in the next period and get a net benefit $\theta_p - \delta c > 0$. Therefore, θ_1 here plays a role only to determine the divorce decision in the comparison between θ_p and $\theta_1 - c$ in $T = 2$.

We next define the possible equilibria where divorce may emerge. The configuration of divorce equilibria is affected by the ranking between $\theta_1 - c$, θ_2 and δc. By Proposition 2, we know that divorce may occur only if $\theta_1 - c > \delta c$. Thus we have three difference configurations, according to whether $\theta_2 > \theta_1 - c$, $\theta_2 \in (\delta c, \theta_1 - c)$ and $\theta_2 < \delta c$. Appendix B provides a formal derivation of them.

Definition 3 *Low, medium and high divorce equilibria are characterized by configurations $\left(\theta_1^l, \theta_2^l\right)$, $\left(\theta_1^m, \theta_2^m\right)$ and $\left(\theta_1^h, \theta_2^h\right)$, respectively, where*

- $\theta_2^l > \theta_1^l - c$;

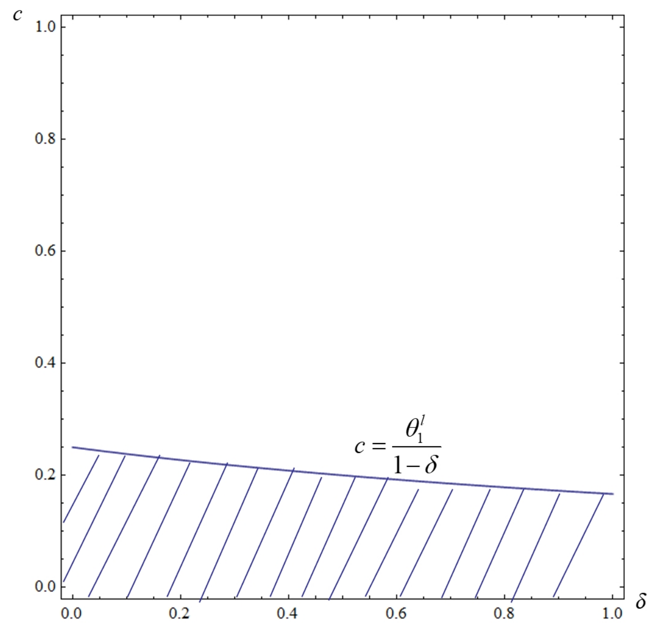

Figure 6: Divorce region for low divorce equilibrium

- $\theta_2^m \in [\delta c, \theta_1^m - c]$.
- $\theta_2^h < \delta c$.

Firstly, Definition 3 allows us to identify the region when a divorce may occur in the plane (δ, c). Figure 6 shows this for low divorce, while the regions for medium and high divorce are very similar. Intuitively, a very high divorce cost prevents divorce from happening. Note that the opposite is not necessarily true: divorce could be cheap, but not appealing, if individuals met the "right" partner (namely, either unhappiness or eligibility does not apply).

Secondly, through Definition 3 we can examine the features of the divorce equilibria, illustrated in Figure 7. In each figure, the area with diagonal lines denote the cases where individual i will divorce, either because he wants to end the relationship (second letter: d) or because the partner wants to end it (first letter: d). The first figure depicts the low divorce equilibrium, where divorce involves only individuals with asymmetric quality and the standard block segregation result emerges. The second picture shows the medium divorce equilibrium, where divorce is widespread and involves both matches of asymmetric and symmetric types. The third figure describes the high divorce equilibrium, similar to the medium divorce equilibrium but with more probability of divorce among symmetric types. The order of the figure follows, as in Proposition 3, the position of θ_2 with respect of $\theta_1 - c$ and δc (with $\theta_1 - c > \delta c$ by Proposition 2). The equilibria show that divorce occurs when spouses are asymmetric in their types or if they are symmetric but at a medium level. Note that the interplay between θ_1 and θ_2, and hence the changes in the distribution of singles, is what determines the features of the equilibrium and who divorces whom. Finally, medium and high divorce equilibria in Figure 7 seem consistent with the empirical findings in Figure 1, where years of schooling represent spouses types. In particular, divorced couples in the data have medium/symmetric or asymmetric levels of education.[5]

5.3 Existence of divorce equilibria

In this section we analyze the condition allowing a divorce equilibrium to exist. Existence requires that:

- the assumed ranking between θ_2, δc and $\theta_1 - c$ holds;
- the divorce cost is not too high according to Proposition 2.

Unfortunately, it is not possible to determine the existence of divorce equilibria analytically. We can, however, determine the existence for some values of (δ, c). To fix ideas, we consider the following values of δ: 0.001,

[5] Clearly, the dataset does not give any information on rejections, which limits the comparison with the theory.

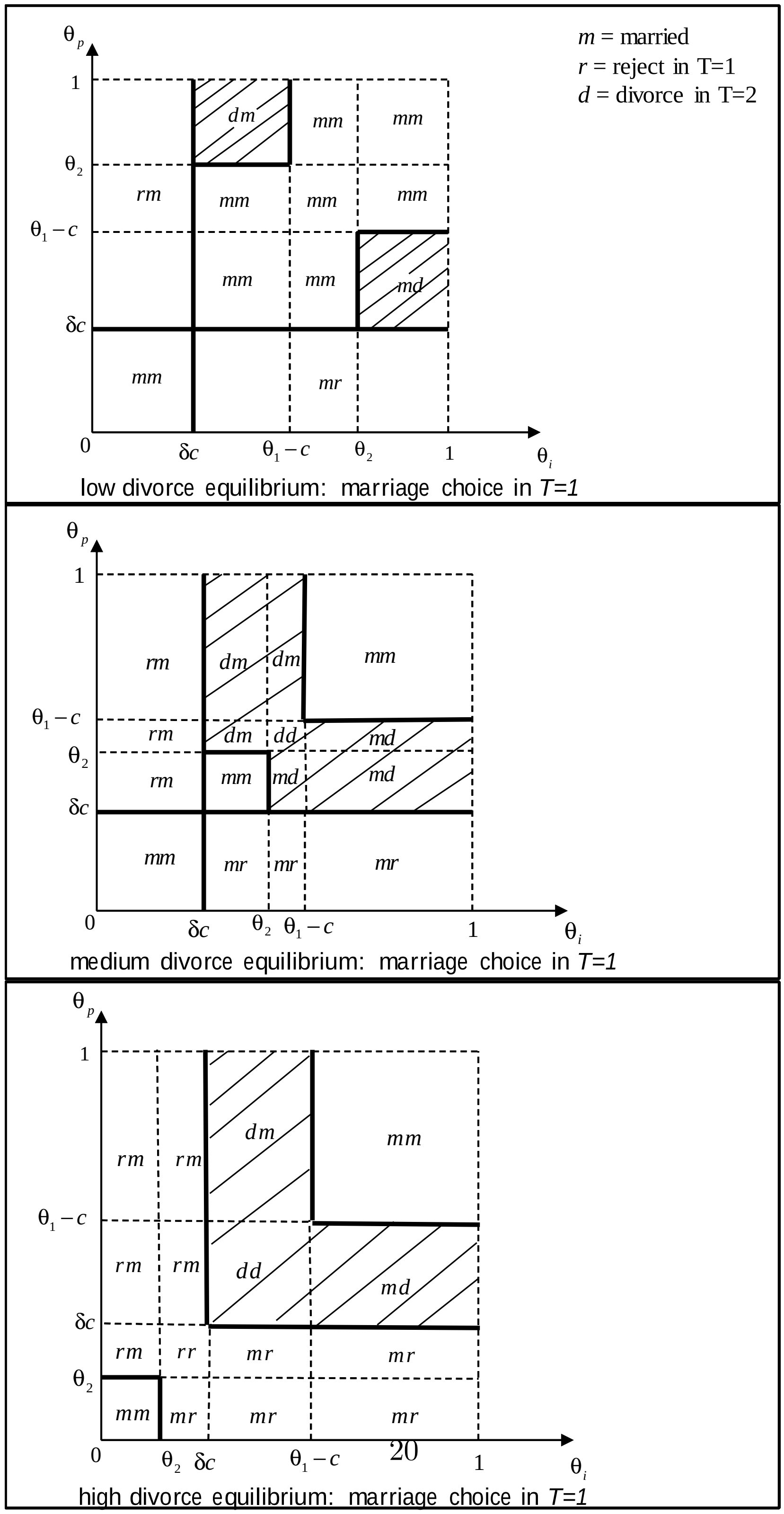

Figure 7: Divorce equilibria

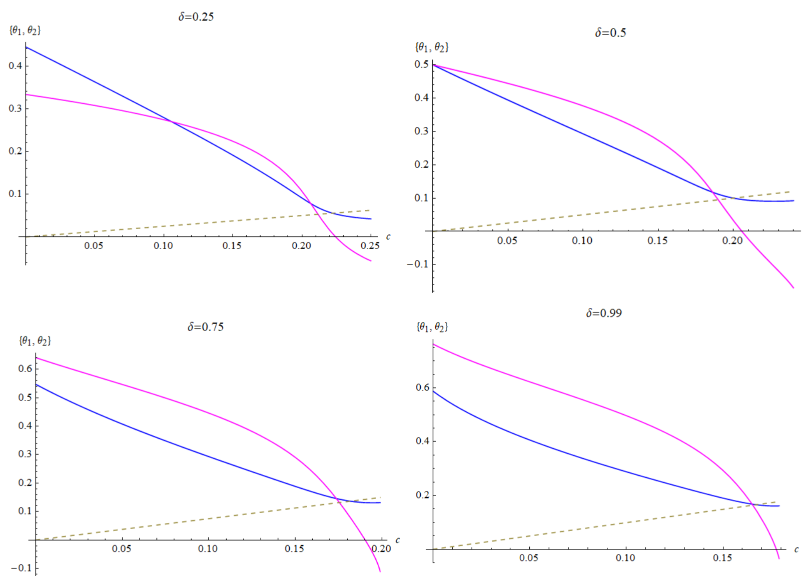

Figure 8: Existence of low divorce equilibrium: $\theta_2 > \theta_1 - c$.

0.25, 0.5, 0.75 and 0.99. Figures 8, 9 and 10 show the existence conditions with respect to c. The blue, magenta and dashed lines are, respectively $\theta_1^j(\delta, c) - c$, $\theta_2^j(\delta, c)$, $j \in \{l, h, m\}$ and δc.

δ	low	medium	high
0.01	/	/	/
0.25	$0.11 \lesssim c \lesssim 0.21$	$0.06 \lesssim c \lesssim 0.11$	$0.11 \lesssim c \lesssim 0.21$
0.5	$c \lesssim 0.19$	$0.07 \lesssim c \lesssim 0.12$	$0.12 \lesssim c \lesssim 0.19$
0.75	$c \lesssim 0.175$	$0.08 \lesssim c \lesssim 0.13$	$0.13 \lesssim c \lesssim 0.175$
0.99	$c \lesssim 0.165$	$0.09 \lesssim c \lesssim 0.15$	$0.15 \lesssim c \lesssim 0.165$

Table 1. Existence conditions

Table 1 summarizes the existence conditions for divorce equilibria. All values lie below the necessary conditions required by Proposition 2. Some remarks can be drawn from the analysis of existence. First, when the cost of waiting is very high (very low δ), then divorce does not emerge. Indeed,

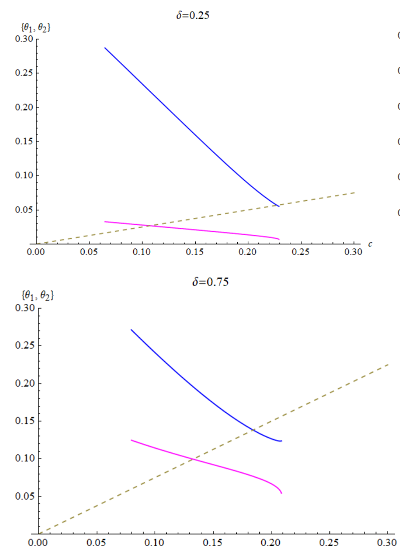

Figure 9: Existence of medium divorce equilibrium: $\theta_2 \in [\delta c, \theta_1 - c]$.

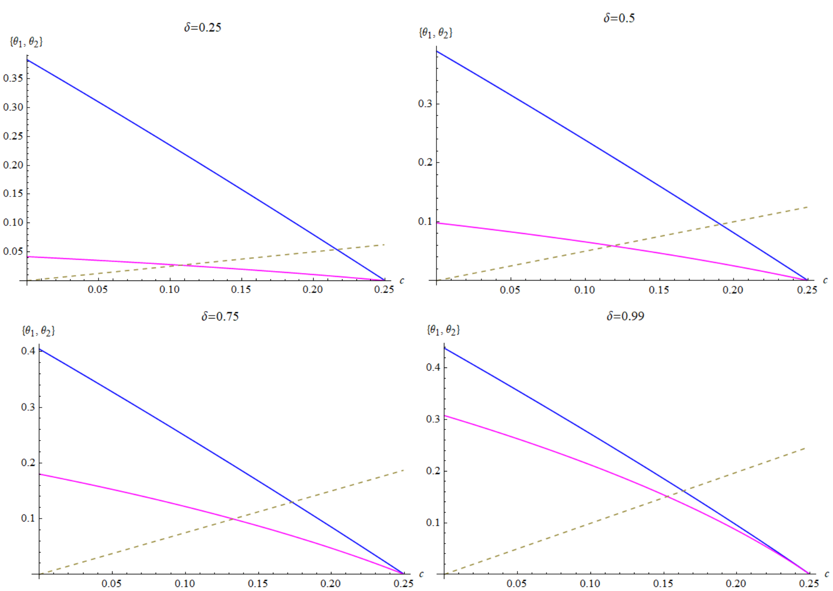

Figure 10: Existence of high divorce equilibrium: $\theta_2 < \delta c$.

the risk of divorcing to wind up, in $T = 2$, with a partner that is worse than the divorced one is very strong. If δ is low, there is a stronger incentive in agreeing to match a partner in $T = 2$, hence this risk of "downgrading" increases with lower δ.

The low divorce equilibrium is unique in some parameter range, while multiple equilibria may emerge in the type of divorce, either low and medium, or low and high. The medium and high divorce equilibria never emerge for the same parameter values. Taken the entire parameter region where the divorce equilibria occur, one may see that an increase in δ (i.e., a decrease in the cost of waiting) increases the probability of divorce until $\delta = 0.5$, then the probability decreases. This result is thus consistent with the empirical findings in Figure 3, where the cost of waiting is represented by average age at marriage across countries. Indeed, if the average age at marriage is particularly low (high) in a certain country, one may expect that the cost of waiting to marry is particularly high (low) for that specific population.

For very low divorce costs, the low divorce equilibrium is unique. This is very counterintuitive, as one would expect an increase in divorce with a fall in divorce costs, while the low divorce equilibrium exhibits a lower probability of divorcing. This reasoning, however, fails to consider the fact that the equilibrium thresholds θ_2^l, θ_2^h and θ_2^m are affected by c. In particular, a lower c (divorce is cheap) entails that more eligible individuals choose to divorce. This increases the expected quality θ_2, which gets higher than $\theta_1 - c$, thus the low divorce equilibrium emerges. The intuition is the following: on the one hand, a lower divorce cost gives an incentive to more eligible individuals to re-enter the marriage market in the second period. On the other hand, this makes married individuals with lower eligibility have less chances to find a partner if they divorce. Therefore a larger number of them prefer to stay married. Overall, their number more than offsets the proportion of eligible individuals who choose to divorce because it is cheap.

Finally, we may discuss the more nuanced case where asymmetry of types occurs between men and women. Suppose that being in the marriage market in the last period is somewhat "harder" for women than men. This can be interpreted in several ways: in many societies, men appoint a higher value to beauty compared to women (Coles and Francesconi, 2011), so that the loss in attractiveness is relatively more harmful for them. We can model this asymmetry like a cost κ that single men pay to marry a woman in $T = 3$, so that they compare the partner met $T = 2$ with a threshold $\theta_2 + \kappa$. Although tractability prevents us to elicit analytical results, we may still elaborate some reasonable conjectures. First, equilibria with differing divorce type may emerge between men and women. For instance, we may expect equilibria where divorce is "low" for women, but "medium" or "high" for men, or an equilibrium where divorce is "medium" for women and "high" for men. Second, the higher threshold for women in $T = 2$ lowers their eligibility, and affects also the marriage choice in the first period. As a result, intuitively, the divorce option is less likely for a woman: only high types choose to divorce, which in turn entails a higher quality of single women in the last period.

6 Divorce legislation

In this section we discuss the role of divorce legislation in the divorce decisions. Two important reforms in the divorce legislation regarded the introduction of unilateral and no fault divorce. These types of reforms have been applied at the end of the 1960s in the United States, where the reform allowed for both unilateral and no fault divorce. In European countries, these reforms have been applied all over the 20^{th} century, and usually introduced separately (González-Val and Marcén, 2012).

No-fault divorce requests can be filed for incompatibility, irreconcilable differences and irretrievable breakdown. No proof of fault is necessary, and can be completed more quickly than fault divorces. In addition, no fault divorce is less costly compared to fault divorces (Ventura and Reed, 2009). Hence in our model, the introduction of a no-fault divorce reform coincides with lowering divorce costs c.

Conversely, unilateral divorce does not require mutual consent and can be granted at the request of either spouse. The right of unilateral divorce may be exercised even under fault divorce legislation, if the spouse is guilty of a grave matrimonial offense, such as physical abuse or adultery.[6] In our model, unilateral divorce occurs for asymmetric types (outcomes dm or md in Figure 7), which would be clearly banned if consensual divorce were the only option in the divorce legislation. In other words, without the unilateral divorce option, the only divorced individuals will be those who mutually choose to do it (outcomes dd).

A possible link between the introduction of no-fault and unilateral divorce legislation and the increase in the divorce rates has been an object of strong debate over the past decades: the empirical analysis has shown ambiguous evidence of these reforms on divorce rates. (Marcassa, 2013, Wolfers, 2006, González and Viitanen, 2009, Friedberg, 1998, Allen 1992 and Peters, 1986, 1992, *inter alia*). This empirical puzzle may be interpreted in the light of our theory. On the one hand the introduction of a no fault divorce legislation corresponds to the fall in c, that in turn entails a lower probability of divorcing, as the equilibrium outcome may shift from high and medium divorce to low divorce (a negative effect on divorce rate). On the other hand the introduction of unilateral divorce implies that asymmetric types may now divorce, thus increasing the chance of divorce (a positive effect on divorce rate).

[6] In this case, the applicant must exhibit proof of the committed fault in court.

7 Concluding remarks

We have analyzed divorce choice when the population distribution is non stationary and divorce entails an explicit cost. In the model, utility is non transferable and individuals are heterogeneous in their quality. We find the conditions for "divorce" and "no divorce" equilibria to emerge. Divorce takes place in the presence of asymmetry among the spouses' types or symmetric, medium-types spouses. A counterintuitive result is that lower divorce costs do not necessarily increase the probability of divorce. Our theoretical findings are consistent with some empirical facts. Finally, our results may help interpreting the empirical ambiguity in the relationship between divorce rate and the introduction of legislative measures such as unilateral and no fault divorce.

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Appendix

Appendix A: proof of Proposition 1

We prove that $\delta\theta_1 > \theta_2$ by following Smith (1992)'s approach.[7] Denote the fraction of eligible individuals with types y in the interval (a, b) in period T by $P_T(a, b)$, and the expectation operator with respect to the probability measure P_T by E_T. We can thus rewrite θ_1 as $\theta_1 = P_2(0, \theta_2)\theta_2 + P_2(\theta_2, 1)E_2(y|y \geqslant \theta_2)$, so that the difference $\delta\theta_1 - \theta_2$ can be rewritten as

$$\begin{aligned}
\delta\theta_1 - \theta_2 &= \delta\left[P_2(0,\theta_2)\theta_2 + P_2(\theta_2,1)E_2(y|y \geqslant \theta_2)\right] \\
&- \delta\left[P_3(0,\theta_2)E_3(y|y < \theta_2) + P_3(\theta_2,1)E_3(y|y \geqslant \theta_2)\right] \\
&= \delta\left[P_2(0,\theta_2)\theta_2 + (P_2(\theta_2,1) - P_3(\theta_2,1))E_2(y|y \geqslant \theta_2)\right. \\
&- \left. P_3(0,\theta_2)E_3(y|y < \theta_2)\right].
\end{aligned}$$

Note that $1-(P_2(\theta_2, 1))^2$ is the fraction of eligible individuals in period $T = 3$. Hence the probability of being married in period $T = 3$ is $(P_2(\theta_2, 1))^2$, so $1 - (P_2(\theta_2, 1))^2$ is the fraction of individuals who did not marry in $T = 2$. The expression $P_2(\theta_2, 1) - (P_2(\theta_2, 1))^2$ is the fraction of eligible individuals in period $T = 3$. Hence the difference $P_2(\theta_2, 1) - P_3(\theta_2, 1)$ is positive, since

$$P_3(\theta_2, 1) = \frac{P_2(\theta_2,1) - (P_2(\theta_2,1))^2}{1 - (P_2(\theta_2,1))^2} = \frac{P_2(\theta_2,1)}{1 + P_2(\theta_2,1)} < P_2(\theta_2, 1).$$

Therefore, we obtain

$$\begin{aligned}
\delta\theta_1 - \theta_2 &> \delta\left[(P_2(0,\theta_2) - P_3(0,\theta_2))\theta_2 + (P_2(\theta_2,1) - P_3(\theta_2,1))\theta_2\right] \\
&= 0.
\end{aligned}$$

We are now in a position to investigate the optimal threshold strategies. Begin the proof by building the expected quality of singles in $T = 2$, θ_1. In

[7] See Smith (1992), Lemma 1, p.5.

$T = 2$, singles are

- individuals who rejected the match in $T = 1$, with probability $(1 - \delta\theta_1)\,\delta\theta_1$, yielding an expected payoff $\frac{(1+\delta\theta_1)}{2}$;
- individuals who rejected/were rejected in $T = 1$, with probability $(\delta\theta_1 - \theta_2)$, that yield an expected payoff $\frac{\delta\theta_1+\theta_2}{2}$.
- individuals who were rejected in $T = 1$, with probability $\theta_2\,(1 - \theta_2)$, that yield an expected payoff $\frac{\theta_2}{2}$.

By Bayes' rule, the expected quality in $T = 2$ is given by:

$$\theta_1 = \frac{(1 - \delta\theta_1)\,\delta\theta_1 \frac{(1+\delta\theta_1)}{2} + (\delta\theta_1 - \theta_2)\,\frac{\delta\theta_1+\theta_2}{2} + \theta_2\,(1 - \theta_2)\,\frac{\theta_2}{2}}{(1 - \delta\theta_1)\,\delta\theta_1 + (\delta\theta_1 - \theta_2) + \theta_2\,(1 - \theta_2)}. \tag{2}$$

Consider next θ_2. Singles in $T = 3$ are

- individuals who rejected the match in $T = 1$ and $T = 2$, with probability $(1 - \delta\theta_1)\,\delta\theta_1\theta_2$, yielding an expected payoff $\frac{(1+\delta\theta_1)}{2}$;
- individuals who rejected/were rejected in $T = 1$ and rejected in $T = 2$, with probability $(\delta\theta_1 - \theta_2)\,\theta_2$, that yield an expected payoff $\frac{\delta\theta_1+\theta_2}{2}$.
- individuals who were refused in $T = 1$ and $T = 2$, with probability $\theta_2\,(1 - \theta_2)$, yielding an expected payoff $\frac{\theta_2}{2}$.

By Bayes' rule, the discounted expected quality in $T = 3$ is given by:

$$\theta_2 = \delta \frac{(1 - \delta\theta_1)\,\delta\theta_1\theta_2 \frac{(1+\delta\theta_1)}{2} + (\delta\theta_1 - \theta_2)\,\theta_2 \frac{\delta\theta_1+\theta_2}{2} + \theta_2\,(1 - \theta_2)\,\frac{\theta_2}{2}}{(1 - \delta\theta_1)\,\delta\theta_1\theta_2 + (\delta\theta_1 - \theta_2)\,\theta_2 + \theta_2\,(1 - \theta_2)}. \tag{3}$$

The solution of the system of equations (2) and (3) with respect to θ_1 and θ_2 gives the optimal threshold strategies (θ_1^n, θ_2^n). Comparing θ_1 and θ_2 from equations (2) and (3) yields

$$\delta\theta_1^n - \theta_2^n = \frac{\delta^2\theta_1(1-\theta_2)^2\left[\delta\theta_1(\theta_2+1-\delta\theta_1)-2\theta_2+1\right]}{2\left[\delta\theta_1(\delta\theta_1-2)+2\theta_2-1\right]\left[\delta\theta_1(\delta\theta_1-2)+{\theta_2}^2\right]}.$$

Assuming $\delta\theta_1 > \theta_2$, one can see, by inspection, that $[\delta\theta_1(\theta_2+1-\delta\theta_1) - 2\theta_2+1] > 0, [\delta\theta_1(\delta\theta_1-2)+2\theta_2-1] < 0$ and $\left[\delta\theta_1(\delta\theta_1-2)+{\theta_2}^2\right] < 0$, so that $\delta\theta_1^n - \theta_2^n > 0$.

Appendix B: Divorce equilibria

Characterizations of divorce equilibria

Case a: $\theta_2 > \theta_1 - c$. Begin the proof by building the expected quality of singles in $T=2$, θ_1. In $T=2$, singles are

- individuals who rejected the match in $T=1$, with probability $(1-\delta c)\,\delta c$, yielding an expected payoff $\frac{(1+\delta c)}{2}$;
- individuals who were rejected in $T=1$ with probability $\delta c\,(1-\delta c)$, that yield an expected payoff $\frac{\delta c}{2}$.
- individuals who divorced in $T=2$, with probability $(1-\theta_2)\,(\theta_1-c-\delta c)$, with expected payoff $\frac{1+\theta_2}{2}$;
- individuals who were divorced in $T=2$, with probability $(\theta_1-c-\delta c)\,(1-\theta_2)$, with expected payoff $\frac{\theta_1-c+\delta c}{2}$;

By Bayes' rule, the expected quality in $T=2$ is given by:

$$\theta_1 = \frac{(1-\delta c)\,\delta c\left(\frac{(1+\delta c)}{2}+\frac{\delta c}{2}\right)+(1-\theta_2)\,(\theta_1-c-\delta c)\left(\frac{\theta_1-c+\delta c}{2}+\frac{1+\theta_2}{2}\right)}{2\,(1-\delta c)\,\delta c+2\,(1-\theta_2)\,(\theta_1-c-\delta c)} \quad (4)$$

Consider next θ_2. In $T=3$, singles are

- individuals who divorced in $T=2$ (md) and refused the match in $T=2$, with probability $(1-\theta_2)(\theta_1-c-\delta c)\theta_2$, with expected payoff $\frac{1+\theta_2}{2}$;

- individuals who were divorced in $T=2$ (dm) and refused the match in $T=2$, with probability $(\theta_1-c-\delta c)(1-\theta_2)\theta_2$, with expected payoff $\frac{\theta_1-c+\delta c}{2}$;

- individuals who refused in $T=1$ and were refused in $T=2$ with quality $(\delta c,\theta_2)$, with probability $(\theta_2-\delta c)\delta c$, with expected payoff $\frac{\theta_2+\delta c}{2}$;

- individuals who refused in $T=1$ and refused in $T=2$ with quality $(\theta_2,1)$, with probability $(1-\theta_2)\delta c\theta_2$, with expected payoff $\frac{1+\theta_2}{2}$;

- individuals who were refused in $T=1$ and $T=2$, with probability $\delta c(1-\delta c)$, with expected payoff $\frac{\delta c}{2}$;

By Bayes' rule, the expected quality in $T=1$ is given by:

$$\theta_2=\delta\frac{(1-\theta_2)(\theta_1-c-\delta c)\theta_2\left(\frac{1+\theta_2}{2}+\frac{\theta_1-c+\delta c}{2}\right)+(\theta_2-\delta c)\delta c\left(\frac{\theta_2+\delta c}{2}\right)+(1-\theta_2)\delta c\theta_2\frac{1+\theta_2}{2}+\delta c(1-\delta c)\frac{\delta c}{2}}{2(1-\theta_2)(\theta_1-c-\delta c)\theta_2+(\theta_2-\delta c)\delta c+(1-\theta_2)\delta c\theta_2+\delta c(1-\delta c)} \tag{5}$$

The solution of the system of equations (4) and (5) with respect to θ_1 and θ_2 gives the optimal threshold strategies $\left(\theta_1^l,\theta_2^l\right)$.

Case b: If $\theta_2\in(\delta c,\theta_1-c)$, then in $T=2$, singles are

- individuals who rejected the match in $T=1$, with probability $(1-\delta c)\delta c$, yielding an expected payoff $\frac{(1+\delta c)}{2}$;

- individuals who were rejected in $T=1$ with probability $\delta c(1-\delta c)$, that yield an expected payoff $\frac{\delta c}{2}$.

- individuals who divorced in $T=2$, with probability $(\theta_1-c-\delta c)(1-(\theta_1-c))$ with expected payoff $\frac{1+\theta_1-c}{2}$;

- individuals who divorced/were divorced in $T=2$, with probability $(\theta_1-c-\theta_2)(1-\delta c)$, with expected payoff $\frac{\theta_1-c+\theta_2}{2}$;

- individuals who were divorced in $T=2$, with probability $(\theta_2 - \delta c)(1-\theta_2)$, with expected payoff $\frac{\theta_2+\delta c}{2}$;

By Bayes' rule, the expected quality in $T=2$ is given by:

$$\theta_1 = \frac{(1-\delta c)\,\delta c\left(\frac{(1+\delta c)}{2}+\frac{\delta c}{2}\right)+(\theta_1-c-\delta c)\,(1-(\theta_1-c))\,\frac{1+\theta_1-c}{2}+(\theta_1-c-\theta_2)\,(1-\delta c)\,\frac{\theta_1-c+\theta_2}{2}+(\theta_2-\delta c)\,(1-\theta_2)\,\frac{\theta_2+\delta c}{2}}{2\,(1-\delta c)\,\delta c+(\theta_1-c-\delta c)\,(1-(\theta_1-c))+(\theta_1-c-\theta_2)\,(1-\delta c)+(\theta_2-\delta c)\,(1-\theta_2)}. \tag{6}$$

In $T=3$, singles are

- individuals who divorced in $T=2$ (md) and refused the match in $T=2$, with probability $(1-(\theta_1-c))\,(\theta_1-c-\delta c)\,\theta_2$, with expected payoff $\frac{1+\theta_1-c}{2}$;
- individuals who divorced/ were divorced in $T=2$ $(md/dd/dm)$ and refused the match in $T=2$, with probability $(\theta_1-c-\theta_2)\,(1-\delta c)\,\theta_2$ with expected payoff $\frac{\theta_1-c+\theta_2}{2}$;
- individuals who were divorced in $T=2$ (dm) and were refused the match in $T=2$, with probability $(\theta_2-\delta c)\,(1-\theta_2)$, with expected payoff $\frac{\theta_2+\delta c}{2}$;
- individuals who refused in $T=1$ and refused in $T=2$, with probability $(1-\theta_2)\,\delta c\theta_2$, with expected payoff $\frac{1+\theta_2}{2}$;
- individuals who refused in $T=1$ but were refused in $T=2$, with probability $(\theta_2-\delta c)\,\delta c$, with expected payoff $\frac{\theta_2+\delta c}{2}$;
- individuals who were refused in $T=1$ and $T=2$, with probability $\delta c\,(1-\delta c)$, with expected payoff $\frac{\delta c}{2}$;

By Bayes' rule, the expected quality in $T=1$ is given by:

$$\begin{aligned}\theta_2 \;=\;& \delta\frac{(1-(\theta_1-c))\,(\theta_1-c-\delta c)\,\theta_2\,\frac{1+\theta_1-c}{2}+(\theta_1-c-\theta_2)\,\theta_2\,\frac{\theta_1-c+\theta_2}{2}+}{(1-(\theta_1-c))\,(\theta_1-c-\delta c)\,\theta_2+(\theta_1-c-\theta_2)\,\theta_2+(\theta_2-\delta c)\,(1-\theta_2)+(1-\delta c)\,\delta c\theta_2+\delta c\,(1-\delta c)} \qquad (7)\\ &+\delta\frac{(\theta_2-\delta c)\,(1-\theta_2)\,\frac{\theta_2+\delta c}{2}+(1-\delta c)\,\delta c\theta_2\,\frac{1+\delta c}{2}+\delta c\,(1-\delta c)\,\frac{\delta c}{2}}{(1-(\theta_1-c))\,(\theta_1-c-\delta c)\,\theta_2+(\theta_1-c-\theta_2)\,\theta_2+(\theta_2-\delta c)\,(1-\theta_2)+(1-\delta c)\,\delta c\theta_2+\delta c\,(1-\delta c)}\end{aligned}$$

The solution of the system of equations (6) and (**??**) with respect to θ_1 and θ_2 gives the optimal threshold strategies (θ_1^m, θ_2^m).

Case c: Suppose $\theta_2 < \delta c$. The expected quality of singles in $T = 2$, θ_1 is given by

- individuals who rejected the match in $T = 1$, with probability $(1 - \delta c)\,\delta c$, yielding an expected payoff $\frac{(1+\delta c)}{2}$;
- individuals who reject/were rejected in $T = 1$, with probability $(\delta c - \theta_2)$, that yield an expected payoff $\frac{\delta c + \theta_2}{2}$.
- individuals who reject/were rejected in $T = 1$, with probability $\theta_2\,(1 - \theta_2)$, that yield an expected payoff $\frac{\theta_2}{2}$.
- individuals who divorced in $T = 2$ with probability $(1 - (\theta_1 - c))\,(\theta_1 - c - \delta c)$, with expected payoff $\frac{1+\theta_1-c}{2}$;
- individuals who divorced/were divorced in $T = 2$, with probability $(\theta_1 - c - \delta c)\,(1 - \delta c)$, with expected payoff $\frac{\theta_1-c+\delta c}{2}$;

By Bayes' rule, the expected quality in $T = 2$ is given by:

$$\theta_1 = \frac{(1-\delta c)\,\delta c\frac{(1+\delta c)}{2} + (\delta c - \theta_2)\,\frac{\delta c+\theta_2}{2} + \theta_2\,(1-\theta_2)\,\frac{\theta_2}{2} + (1-(\theta_1-c))\,(\theta_1-c-\delta c)\,\frac{1+\theta_1-c}{2} + (\theta_1-c-\delta c)\,(1-\delta c)\,\frac{\theta_1-c+\delta c}{2}}{(1-\delta c)\,\delta c + (\delta c-\theta_2) + \theta_2\,(1-\theta_2) + (1-(\theta_1-c))\,(\theta_1-c-\delta c) + (\theta_1-c-\delta c)\,(1-\delta c)}. \tag{8}$$

Consider next θ_2. In $T = 3$, singles are

- individuals who divorced in $T = 2$ (md) and refused the match in $T = 2$, with probability $(1 - (\theta_1 - c))\,(\theta_1 - c - \delta c)\,\theta_2$, with expected payoff $\frac{1+\theta_1-c}{2}$;
- individuals who divorced in $T = 2$ (dd/dm) and refused the match in $T = 2$, with probability $(\theta_1 - c - \delta c)\,(1 - \delta c)\,\theta_2$, with expected payoff $\frac{\theta_1-c+\delta c}{2}$;

- individuals who refused in $T=1$ and refused in $T=2$, with probability $(1-\delta c)\,\delta c\theta_2$, with expected payoff $\frac{1+\delta c}{2}$;

- individuals who refused/were refused in $T=1$ and refused in $T=2$, with probability $(\delta c-\theta_2)\,\theta_2$, with expected payoff $\frac{\delta c+\theta_2}{2}$;

- individuals who were refused in $T=1$ and $T=2$, with probability $\theta_2\,(1-\theta_2)$, with expected payoff $\frac{\theta_2}{2}$;

By Bayes' rule, the expected quality in $T=1$ is given by:

$$\begin{aligned}\theta_2 \quad = \quad & \delta\frac{(1-(\theta_1-c))\,(\theta_1-c-\delta c)\,\theta_2\frac{1+\theta_1-c}{2}+(\theta_1-c-\delta c)\,(1-\delta c)\,\theta_2\frac{\theta_1-c+\delta c}{2}+}{(1-(\theta_1-c))\,(\theta_1-c-\delta c)\,\theta_2+(\theta_1-c-\delta c)\,(1-\delta c)\,\theta_2+(1-\delta c)\,\delta c\theta_2+(\delta c-\theta_2)\,\theta_2+\theta_2\,(1-\theta_2)} \qquad (9)\\ & \delta\frac{(1-\delta c)\,\delta c\theta_2\frac{1+\delta c}{2}+(\delta c-\theta_2)\,\theta_2\frac{\delta c+\theta_2}{2}+\theta_2\,(1-\theta_2)\,\frac{\theta_2}{2}}{(1-(\theta_1-c))\,(\theta_1-c-\delta c)\,\theta_2+(\theta_1-c-\delta c)\,(1-\delta c)\,\theta_2+(1-\delta c)\,\delta c\theta_2+(\delta c-\theta_2)\,\theta_2+\theta_2\,(1-\theta_2)}.\end{aligned}$$

The solution of the system of equations (8) and (9) with respect to θ_1 and θ_2 gives the optimal threshold strategies $\left(\theta_1^h,\theta_2^h\right)$.