Spillover Effects in a Federal Country with Vertical Tax Externalities

Lisa Grazzini and Alessandro Petretto

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Abstract

We analyse how spillover effects may affect the choice of a federal tax rate in a federal country with vertical tax externalities. Our main result shows under which conditions the federal tax rate with spillover effects is lower or higher than the federal tax rate without spillover effects. The effects of vertical tax externalities can be modified by the reaction of the federal government to the horizontal externality due to spillover effects.

Keywords: Fiscal federalism, Median voter, Positive spillovers.
JEL Classification: H71; H77; H41.

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1 Introduction

Federal countries are very common worldwide, and their main feature is that national and local governments have legislative independence, even if fiscal policies decided by each of them also affect all the others (Boadway and Tremblay (2012)). Two types of interaction are particularly important: Interdependencies between fiscal decisions taken by different levels of government (vertical fiscal externalities), and fiscal spillovers across local jurisdictions (horizontal fiscal externalities).

Vertical fiscal externalities arise when federal and local governments impose their respective taxes on the same tax base, so that a common pool problem takes place, i.e. the reduction in the common tax base, and so the loss in tax revenue, are not taken into account by neither level of government. Because of such negative vertical fiscal externalities, at the non-cooperative equilibrium, tax rates tend to be fixed inefficiently too high, i.e. governments underestimate the marginal cost of public funds, leading to excessive taxation (Dahlby (2008)).

In federal countries, such vertical interaction among different levels of government is usually combined to a horizontal interaction among local governments due to public good spillovers. When the benefits of the local public goods can spill over across communities, residents of each jurisdiction can consume the local public goods supplied by neighboring jurisdictions. As the costs of providing local public goods are borne by a jurisdiction while the benefits may, at least partly, flow out, local public goods tend to be underprovided from a social point of view.\footnote{Horizontal externalities have also been studied, for example, for capital taxation when governments at the same level tax capital investments which are highly mobile across borders. The standard ‘race to the bottom’ result shows that capital tax rates tend to be chosen inefficiently low because they are decided strategically by governments to increase their tax base, disregarding the benefit (harm) each of them does to the others by increasing (decreasing) its own tax rate.} For example, in the case of pollution, the citizens of each jurisdiction may benefit from pollution abatement programs of other jurisdictions.

Thus, an interesting question which arises is how, in a federal country, the federal fiscal policy is affected by the interplay between such vertical and horizontal fiscal externalities when the political process determining public policies is taken into account. To answer such a question, we propose a model which describes a federal country with fixed jurisdictional boundaries, and asymmetric regions where federal and regional tax rates are chosen by a national and regional assemblies by majority voting, respectively. The vertical interaction between the different levels of government is modelled as a three-stage game. At the first stage, at national level, the federal government decides a federal tax rate and the amount of a national public good. At the second stage, at the local level, each region chooses both the
level of a regional surtax, and the amount of a regional public good whose benefits spillover across regions. At the third stage, consumers take their consumption decisions. In such a framework, our main result shows under which conditions spillover effects lead to adopt a higher or a lower federal tax rate with respect to the one chosen in the case without spillover effects.

The literature on vertical fiscal externality is quite recent, and its main achievement is that “federal and state governments over-exploit the common tax base, analogous to the tragedy of commons” (Boadway and Tremblay (2012 p. 1074). Instead, the branch of the fiscal federalism literature on spillovers across jurisdictions is more traditional. The seminal contribution by Oates (1972) focuses on the trade-off between centralization and decentralization of public good provision by contrasting the merits of centralization in internalizing inter-jurisdictional spillovers and those of decentralization in making local policies corresponding to local preferences.

More recent papers on the latter issue, emphasize political economy arguments by modeling fiscal policies as being determined by a legislature of representatives of the different jurisdictions (Lockwood (2002), Besley and Coate (2003), Redoano and Scharf (2004)). In particular, Hickey (2013) analyses how inter-regional policy spillovers can be internalized through the formation of a federation starting from a set-up with decentralized provision of public policy. This paper shows that a constitutional rule based on bicameralism promotes the formation of a federation. A constitutional perspective is also followed by Schwager (1999) who analyses a framework where the federal decisions are biased in favour of one of the regions in the federation. For intermediate values of the spillovers, his major finding is that both centralization and decentralization lead to a lower welfare than under a constitutional rule, defined administrative federalism, where the federal government sets quality standards for public projects, and the regions decide which project they want to accomplish.

Other recent contributions focus, instead, on households mobility. Wellisch (1994) shows that if households are perfectly mobile across jurisdictions, then spillover effects are completely internalized via decentralization, i.e. local public goods production is efficient and central government intervention is not necessary. The focus of Bloch and Zenginobuz (2006) is, instead, on how inter-jurisdictional spillovers affect the equilibrium population distribution across communities, while Bloch and Zenginobuz (2007) analyse how local public good provision depends on the interplay between the sizes of jurisdictions and the structure of spillovers. More recently, Gregor and

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2 See, for example, Boadway and Keen (1996), Keen (1998), and Dahlby and Wilson (2003).
3 See Koethenbuerger (2008) for a paper analysing how the welfare difference under centralization and decentralization can be non-monotone in the strength of consumption spillovers.
4 Bloch and Zenginobuz (2015) show that with higher mobility and higher spillovers
Stastna (2012) compares centralization versus decentralization when tastes are homogenous across jurisdictions, and local public spending for local inputs generates a positive spillover in the other jurisdiction where it is a complement to domestic expenditure.5

To the best of our knowledge, the novelty of this paper is to consider a model which takes into account the political process to analyse how the interaction between vertical and horizontal externalities affect federal fiscal policies. Previous analyses have, instead, focused their attention on the effects on local fiscal policies. Our paper also differs with respect to that literature in fiscal federalism pioneered by Keen and Kotsogiannis (2002) which analyses the simultaneous working of vertical and horizontal externalities in the case of capital taxation but without any concern for horizontal externalities originating from local public goods’ spillovers.

The plan of this paper is as follows. Section 2 presents the model. Section 3 analyses the vertical and the horizontal interaction between the different levels of government. Section 4 compares the federal tax rate with and without spillover effects. Finally, section 5 contains some concluding remarks.

2 The model

We analyse a federal country with asymmetric regions \( i, i = 1, \ldots, I \), each of them populated by a different number \( n_i \) of identical individuals who are immobile. In each region \( i, i = 1, \ldots, I \), the following additively separable utility function describes the identical preferences of all residents,

\[
U_i = u_i(x_i) + \alpha_i b_i(g_i) + \sum_{j \neq i} \alpha_{ij} b_i(g_j) + \beta G, \quad i = 1, \ldots, I, \tag{1}
\]

where \( u_i(\cdot) \) is a strictly concave sub-utility function of private consumption, \( x_i \) (taken as the numeraire); \( b_i(\cdot) \) is a strictly concave sub-utility function with \( b_i(g_i) \) measuring the benefits of a regional public good, \( g_i \); and \( b_i(g_j) \) measuring the spillover benefits of the local public goods supplied by all other regions, \( g_j, j \neq i \); and finally, \( \beta G \) measures the benefits of a federal public good \( G \) with \( \beta > 0.6 \). Notice that, since we assume a cost of production of centralization has to be preferred when public good provision can be nonuniform under centralization, and the choice of public goods is the result of a political process in the federal legislature.

5 The possibility to internalize spillovers by charging resident and non-resident users a price for the use of regional spillover goods is explored by Fuest and Kolmar (2007). In a two-region model, they show that user fees tend to be inefficiently low or inefficiently high depending on whether the spillover goods are substitutes or complements. Such an analysis is extended to a \( n \)-region economy by Fuest and Kolmar (2013).

6 The separability assumption on the utility function implies that \( g_i, i = 1, \ldots, I \), and \( G \) do not affect households’ consumption decisions.
one unit of local (national) public good to be equal to 1, \( g_i \) \((G)\) also denotes the regional (federal) public expenditure. As the benefits of the local public goods can spill over across regions, \( \alpha_{ij} \), with \( j \neq i \), represents the spillover coefficient of region \( j \)'s local public good in region \( i \), i.e. the degree to which the benefits of \( g_j \) spill over into region \( i \). Instead, \( \alpha_{ii} = 1 - \sum_{j \neq i} \alpha_{ij} \) describes the relative weight for the residents of region \( i \) of the benefits of their local public good, \( g_i \), relative to the benefits of the spillover effects.\(^7\) For example, the regional public goods could refer to pollution abatement programs: The citizens of region \( i \) can benefit from pollution abatement programs financed by other regions. In such case, the value of the spillover coefficient \( \alpha_{ij} \) could be inversely related to the distance between region \( i \) and region \( j \), i.e. higher (lower) spillovers from closer (further) regions. Of course, in this set-up, \( \alpha_{ii} \) would take the highest value while \( \alpha_{ij} \) could be close to 0 for the furthest region \( j \). In order to insure the standard concavity property for the utility function, we assume that \( 0 \leq \alpha_{ij} \leq \frac{1}{I-1} \) so that \( 0 \leq \alpha_{ii} \leq 1 \).

The budget constraint of a consumer in region \( i \) is given by

\[
x_i = (1 - \tau_i)y_i, \quad i = 1, ..., I,
\]

where \( \tau_i \equiv t_i + T_i \), \( i = 1, ..., I \) denotes the consolidated tax rate given by the sum of \( t_i \), \( i = 1, ..., I \), a surtax on the regional fiscal base decided by each regional government, and \( T_i \), a federal tax rate decided at the national level. Further, \( y_i \) denotes the fixed income of an individual who resides in region \( i \), so that all public goods are financed through lump-sum taxation.

Each regional fiscal base is determined by \( Y_i = n_i y_i \), while the federal tax base is determined by \( Y = \sum_i Y_i \). Thus, the regional and the federal public budget constraint obtain as follows, respectively,\(^8\)

\[
t_i Y_i = g_i \quad i = 1, ..., I,
\]

and

\[
TY = G.
\]

This implies that the federal level does not consider the fiscal externalities of its decisions on the regional public budget constraints.

Elections are assumed to be simultaneous at all levels, i.e. regional and federal level, and the federal delegates from region \( i \) must be drawn from the homogeneous population in that region: Each region elects citizen-candidates as its delegates at the federal government.\(^9\) We suppose that

\(^7\)See Oddou (2014) for spillover coefficients modelled in a similar vein.

\(^8\)We do not consider public policies (such as inter-jurisdictional transfers or centralization) which could counter the inefficiencies introduced by the spillover effects.

\(^9\)See Osborne and Slivinski (1996), and Besley and Coate (1997). The assumption that the population within any region is homogenous follows Lockwood (2002), and allows the role of strategic-voting for the delegates to be abstracted (Besley and Coate (2003)).
legislators are not able to commit to a specific platform so that they are only interested in maximising their utilities once in office.

Let us also assume that both national and regional assemblies adopt majority-voting. At the regional level, since all the residents in a region are identical, their preferences are always respected by regional decisions. At the national level, we suppose an odd number of delegates so that the median delegate is pivotal for the federal government.

Having already described how the Constitution has fixed the allocation of powers on public goods’ provision and taxing authority between regional and federal level, we finally assume that the Constitution has also fixed the rules governing the interaction between the regional governments and the federal government. In particular, we suppose a three-stage game. First, at national level, the federal government chooses the national tax rate and the amount of the national public good. Second, at the local level, each region, \(i = 1, ..., I\), chooses both the level of the regional surtax, and the amount of the regional public good. Third, agents make their consumption decisions.

3 The federal tax rate with spillover effects

3.1 Stage 3: Consumers’ problem

Let us solve the game by backward induction, and thus consider the consumption decisions of the agents at the third stage of the game. In each region, \(i, i = 1, ..., I\), a consumer maximises her utility function (1) subject to her budget constraint (2). By solving such problem, the following indirect utility function obtains as

\[
V_i = v_i(\tau_i) + \alpha_{ii} b_i(g_i) + \sum_{j \neq i} \alpha_{ij} b_i(g_j) + \beta G, \quad i = 1, ..., I,
\]

where \(v_i(\tau_i) \equiv u_i((1 - \tau_i)y_i)\).

3.2 Stage 2: Regional governments’ problem

Let us now solve the second stage of the game where each regional government votes on the level of the surtax, \(t_i\), and the amount of the local public good, \(g_i\), according to majority-voting. Since each region acts as a Stackelberg follower with respect to the federal level, it takes as given the federal tax rate, \(T\), and accordingly it does not take into account how the federal government budget constraint may be affected by such decision. As all agents in a region are identical, the regional government maximisation problem is as follows

\[
\max_{t_i} \quad v_i(\tau_i) + \alpha_{ii} b_i(g_i) + \sum_{j \neq i} \alpha_{ij} b_i(g_j) + \beta G \\
\text{s.t.} \quad t_i Y_i = g_i.
\]

(6)
Since $\frac{\partial v_i}{\partial t_i} = -u_i'(x_i)g_i$, the first order condition of this problem obtains as\(^{10}\)

$$\frac{u_i'(x_i)}{\alpha_{ii} b_i'(g_i)} = n_i, \quad i = 1, ..., I, \quad (7)$$

where the L.H.S. describes the marginal rate of substitution between private consumption, $x_i$, and the local public good, $g_i$.\(^{11}\) This condition shows that the more highly (lower)-populated regions will prefer a higher (lower) regional tax rate because they will be willing to give up a greater (smaller) amount of private consumption for an additional unit of regional public good.

From the first order conditions in (7), it follows that $t_i = t_i(T; \alpha_i)$, where $\alpha_i = (\alpha_{i1}, ..., \alpha_{ij}, ..., \alpha_{iI})$, $i, j = 1, ..., I$, so that the local tax rate decided by region $i$ depends on the federal tax rate, $T$, and the vector of spillovers coefficients, $\alpha_{ij}$.

We can now state the following

**Lemma 1.**

(i) $\frac{\partial t_i(T; \alpha_i)}{\partial T} < 0, \quad i = 1, ..., I,$

(ii) $\frac{\partial t_i(T; \alpha_i)}{\partial \alpha_{ij}} < 0, \quad j = 1, ..., I, \quad i \neq j$,

(iii) $\frac{\partial t_i(T; \alpha_i)}{\partial \alpha_{ii}} > 0$.

**Proof.** See the Appendix.

Lemma 1 (i) describes a *vertical fiscal substitutability*, i.e. a standard vertical tax externality between federal and regional units. An increase (decrease) in the federal tax rate leads to a decrease (increase) in the regional tax rate so that regional and federal tax rates are strategic substitutes.\(^{12}\) This also implies that regional and federal public goods are substitutes.

Lemma 1 (ii), instead, describes a *horizontal fiscal substitutability*, and it is more novel because it enlightens how the regional tax rate is affected by changes in the vector of spillover coefficients. It shows that an increase in the relative weight of a region $j$ spillover effect leads to a reduction in the region $i$ tax rate because region $i$ residents now attach a higher weight to the spillover effect with respect to their own local public good given that $\alpha_{ii} = 1 - \sum_{j \neq i} \alpha_{ij}$, and thus they are willing to reduce their own local tax rate. Thus, each regional government tends to reduce its own local tax rate, thinking that its residents will be able to consume the local public goods produced by all other regions so that, at the Nash subgame equilibrium, all

\(^{10}\)Derivatives are denoted by a prime for functions of one argument.

\(^{11}\)It can be easily checked that the second order condition is negative. Thus, agents’ preferences are single-peeked on the whole range of the regional surtax.

\(^{12}\)See, for example, also Grazzini and Petretto (2012).
regions would reduce their local tax rates, and accordingly their local public good supply.

Finally, Lemma 1 (iii) shows that, if, instead, the relative weight, $\alpha_{ii}$, attached by a region $i$ to its own local public good increases, this will lead to an increase in region $i$ local tax rate.

To sum up, in our set-up, the standard negative vertical externality from federal tax rates to regional ones is reinforced by the existence of positive spillovers across regions. Regional tax rates can be fixed at inefficient low levels which may be even lower with respect to what predicted taking just negative vertical externalities into account.

3.3 Stage 1: Federal government’s problem

We now turn to a description of the first stage of the game, when the federal government has to choose the federal tax rate $T$ by majority voting.\footnote{The federal government’s choice on $T$ automatically determines the level of the federal public good $G$ required to satisfy the federal public budget constraint so that the policy problem is one-dimensional. Further, to apply the median voter theorem, $\frac{\partial^2 t}{\partial T^2} < 0$ is a sufficient condition to have that the second order condition is negative, and thus single-peaked preferences.} Let us define $F$ as the median representative who, accordingly, lives in region $F$, and who will be pivotal with respect to the choice on $T$.\footnote{We assume that, at stage two of the game, the residents of region $F$ do not know that the national median delegate comes from their region, otherwise they could anticipate that they will be decisive at the federal government, and thus could strategically adjust the choice on the regional tax rate. As pointed out by Grazzini and Petretto (2012, 2015), such an assumption seems not too strong because in the real world the median region of a federal country may change in the course of time.}

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In (10), the L.H.S. describes the marginal rate of substitution for the median representative \( F \) between the federal public good and her own region public good. This condition shows that, *ceteris paribus*, the higher-populated and/or richer (lower-populated and/or poorer) the \( F \) region, the lower (higher) the federal government preferred federal tax rate because the median representative is willing to give up a greater (smaller) amount of federal public good for an additional unit of her own regional public good. This result together with the one following the regional government maximisation problem at stage 2 shows that, *ceteris paribus*, the higher-populated (lower-populated) the \( F \) region, the higher (lower) its preferred regional tax rate, and the lower (higher) its preferred federal tax rate. Finally, condition (10) defines region \( F \)'s reaction function, given a vector \( \alpha_F \) of spillover effects:

\[
T^F = T(t_1(T; \alpha_1), ..., t_i(T; \alpha_i), ..., t_I(T; \alpha_I); \alpha_F).
\]

We can now analyse more deeply how the first order condition (10) varies according to a different degree of the spillover effects. In particular, we identify two extreme cases: The first one without any spillover effect, and the second one with the maximum level of spillover effects. The first case is when there is no spillover effect in favour of the agents of region \( F \), i.e. \( \alpha_{Fj} = 0, \forall j = 1, ..., I \), with \( j \neq F \), so that \( \alpha_{FF} = 1 \). In such a case, condition (10) rewrites as

\[
\beta = b'_F(g_F) \frac{Y_F}{Y} \equiv MC|_{\alpha_{FF} = 1}.
\]

In (11), the L.H.S. represents the marginal benefit of the federal public good for region \( F \), and the R.H.S. describes its marginal cost in terms of foregone regional public good. Let us define \( \overline{G} \), and \( \overline{T} \) the level of the federal public good and the federal tax rate which result by solving (11), respectively.

Instead, the second case is when there is a maximum level of spillover effects, and thus agents of region \( F \) just care of the spillover effects and attach no utility to their own regional public good, i.e. \( \alpha_{Fj} = 1, \forall j = 1, ..., I \), with \( j \neq F \), so that \( \alpha_{FF} = 0 \). In this case, condition (10) rewrites as

\[
\beta = \frac{1}{I - 1} \left| \sum_{j \neq F} b'_F(g_j) \frac{Y_j}{Y} \frac{\partial t_j}{\partial T} \right| \equiv MC|_{\alpha_{FF} = 0}.
\]

In (12), the L.H.S. still represents the marginal benefit of the federal public good, and the R.H.S. now describes its marginal cost in terms of foregone local public goods provided by all regions different from \( F \) whose benefits spillover into region \( F \).

Now consider the intermediate case when there are some spillover effects, i.e. \( 0 < \alpha_{Fj} < \frac{1}{I - 1} \). By rearranging its terms, condition (10) can be
rewritten as

$$\beta = \left(1 - \sum_{j \neq F} \alpha_{Fj}\right) b'_F(g_F) Y_F \frac{1}{Y} - \sum_{j \neq F} \alpha_{Fj} b'_F(g_j) Y_j \frac{\partial t_j}{\partial T} \equiv MC|_{0<\alpha_{FF}<1}.$$  

(13)

In (13), the R.H.S. represents the marginal cost of the federal public good which is now given by the sum of the forgone region $F$'s local public good and the spillover effects of the local public goods provided by all other regions. Let us define $\hat{G}$, and $\hat{T}$ the level of the federal public good and the federal tax rate which result by solving (13), respectively.

By comparing the marginal costs in (11), (12), and (13), we can state the following

**Lemma 2.** $MC|_{\alpha_{FF}=1} < (>) MC|_{0<\alpha_{FF}<1} < (>) MC|_{\alpha_{FF}=0}$ if and only if $b'_F(g_F) Y_F < (>) \frac{1}{1-\sum_{j \neq F} \alpha_{Fj} b'_F(g_j) Y_j \frac{\partial t_j}{\partial T}}$.

**Proof.** See the Appendix.

Lemma 2 shows that two alternative set-ups can arise. When from the region $F$’s viewpoint, ceteris paribus, the marginal benefit coming from its local public good is sufficiently low (high) with respect to the marginal benefit coming from all spillover effects on average, the marginal cost for providing an additional unit of the federal public good without any spillover effect, $MC|_{\alpha_{FF}=1}$, is lower (higher) than that with the maximum level of spillover effect, $MC|_{\alpha_{FF}=0}$. Of course, in the intermediate case when there are some spillover effects, the marginal cost for having one unit more of the federal public good, $MC|_{0<\alpha_{FF}<1}$, will be between such extreme threshold values.

We are now in a position to describe how the federal tax policy may be affected by the existence of spillover effects at the local level. In particular, the following proposition shows how the federal tax rate, $T$, is affected by changes in the evaluation of the spillover effects made by the pivotal region $F$, i.e. a decrease (increase) in the sum of spillover coefficients, $\sum_{j \neq F} \alpha_{Fj}$, or equivalently an increase (decrease) in the relative weight, $\alpha_{FF}$, of the region $F$ local public good, given that $\alpha_{FF} = 1 - \sum_{j \neq F} \alpha_{Fj}$.

In this respect, let us define $\xi_{t_F, \alpha_{FF}} \equiv \frac{\partial T}{\partial \alpha_{FF}}$, the elasticity of the region $F$ local tax rate with respect to the relative weight of its local public good, with $\xi_{t_F, \alpha_{FF}} > 0$ by Lemma 1 (iii). Let us also define $\varphi_F \equiv \left| \frac{b'_F(g_F) g_f}{b'_F(g_F)} \right|$ the degree of the concavity of the benefit function of the local public good relative to its level. Then, we can state the following

**Proposition 1** $\frac{\partial T}{\partial \alpha_{FF}} \geq 0$ if and only if $\varphi_F \xi_{t_F, \alpha_{FF}} \geq 1$. 

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Proof. See the Appendix.

While at the second stage of the game the effect of the spillovers on the local tax rate is unambiguous (see Lemma 1 (ii) and (iii)), at the first stage of the game, its effect on the federal tax rate depends on the relative intensity of the desirability of the local public good, $\varphi_F$, and the elasticity of the region $F$ local tax rate with respect to the relative weight of its local public good, $\xi_{t_F, \alpha_{FF}}$. Specifically, when the relative intensity of the preference for its own local public good is sufficiently high (low), and/or the elasticity of the local tax rate with respect to its own public good is sufficiently high (low), a decrease in the relative weight of the region $F$ local public good, i.e. an increase in the relative weight of the spillovers, leads to a decrease (increase) in the federal tax rate.

To grasp the intuition behind this proposition, let us suppose for simplicity that $\varphi_F$ is constant, and that $\xi_{t_F, \alpha_{FF}}$ is sufficiently high. Consider, for example, a marginal increase in the relative weight of all spillovers, $\sum_{j \neq F} \alpha_{Fj}$, i.e. a marginal decrease in the relative weight of the region $F$ local public good, $\alpha_{FF}$. Proposition 1 shows that this negatively affects the federal tax rate, i.e. $\frac{\partial T}{\partial \alpha_{FF}} > 0$. By manipulating the federal tax rate, region $F$ can thus indirectly affect the local tax rates of all other regions to face the marginal increase in the relative weight of the spillovers associated to the local public goods supplied by all other regions. Indeed, region $F$ can induce an increase in the local tax rates of all other regions, and thus also in the level of their local public goods, because $\frac{\partial t_i(T_{\alpha_i})}{\partial T} < 0$ by Lemma 1 (i). Of course, a decrease in $T$ also induces an increase in the region $F$ local tax rate, but this indirect positive effect must be contrasted to the large direct negative effect on $t_F$ due to $\frac{\partial t_F(T_{\alpha_F})}{\partial \alpha_{FF}} > 0$ by Lemma 1 (iii), and $\xi_{t_F, \alpha_{FF}}$ sufficiently high. When $\xi_{t_F, \alpha_{FF}}$ is sufficiently low, an analogous reasoning holds.

Finally, notice that this result is independent on the level $\beta$ of the marginal benefit of the national public good $G$.

4 The comparison between federal tax rates with and without spillovers

We now check whether the equilibrium federal tax rate with spillover effects resulting from (13) is higher or lower than the equilibrium federal tax rate without spillover effects resulting from (11). From the viewpoint of the national legislator $F$, when the local public goods produced in all other regions different from $F$ generate positive spillovers in region $F$, we can state the following

Proposition 2 The federal tax rate with spillover effects, $\hat{T}$, is lower (higher) than the federal tax rate without spillover effects, $\bar{T}$, if and only if $MC|_{\alpha_{FF}=0} > (\leq) MC|_{\alpha_{FF}=1}$.
Proof. See the Appendix.

Proposition 2 shows that the federal tax rate with spillover effects, $\hat{T}$, is lower (higher) than the federal tax rate without spillover effects, $\bar{T}$, if and only if the marginal cost of providing the federal public good in case of maximum spillovers, $MC|_{\alpha_{FF}=0}$, and thus also in the intermediate case of some spillovers, is higher (lower) than its marginal cost without any spillover effect, $MC|_{\alpha_{FF}=1}$.

The intuition behind such result is as follows. Consider, for example, the case when $MC|_{\alpha_{FF}=1} < MC|_{0<\alpha_{FF}<1} < MC|_{\alpha_{FF}=0}$ which arises if and only if $\frac{1}{I-1} \sum_{j \neq F} b_F'(g_j) Y_j \frac{\partial t_j}{\partial T} > b_F'(g_F) Y_F$ by Lemma 2. In this case, an additional unit of the federal public good implies an higher marginal cost in the case with some spillover effects rather than without spillover effects, and thus region $F$ will prefer a lower supply of the federal public good or, equivalently, a lower federal tax rate in the case with some spillover effects rather than without them, i.e. $\hat{T} < \bar{T}$. In other words, for a given level $\beta$ of the marginal benefit of the federal public good, an increase in the effects of the spillovers will induce region $F$ in preferring a lower federal tax rate which, in its turn, will lead to higher local taxes necessary to finance higher amounts of the other regions’ local public goods which, in this case, at the margin and ceteris paribus, are valued more with respect to the region $F$’s local public good.

Further, notice that a tax base effect may also play a role in determining the federal tax rate. For example, when the region $F$ tax base, $Y_F$, is sufficiently low with respect to the average tax base of the other regions, $\frac{1}{I-1} \sum_{j \neq F} Y_j$, ceteris paribus, the national legislator $F$ will prefer a lower federal tax rate in case with some spillovers rather than without them, in order to induce higher regional tax rates so to take advantage of the benefits from the other regions public goods.

We may thus conclude that region $F$ can use the power to decide the federal tax rate as a tool to internalize the horizontal externality due to the spillover effects. This, however, implies that the vertical externality may be mitigated (reinforced) by the reaction of the federal government to the horizontal externality due to spillovers when, the marginal valuation of all spillover effects is higher (lower) than the marginal valuation of the local public good supplied in the region of residence of the federal median voter, and/or region $F$ is sufficiently poor (rich) in tax base with respect to all other regions.

5 Concluding remarks

In a federal country, we have analysed how the choice of the federal tax policy by the national government may be affected by the existence of spillover
effects among regions. Our analysis has been cast in a set-up with vertical tax externalities, i.e. when different tiers of government tax the same base. When the benefits of the local public goods do not spill over across regions, we know that at the noncooperative equilibrium, tax rates decided by each level of government depend on the vertical interaction between them.

In a set-up where spillover effects are taken into account, our main result shows that whether the federal tax rate with spillover effects is higher or lower than the one without spillover effects also depends on a comparison made by the national legislator between the marginal valuation of the local public good of her region of residence and the marginal valuation of the spillover benefits from the other regions’ local public goods. In particular, Proposition 1 shows how the federal tax rate depends on the extent of the spillover effects while Proposition 2 compares the equilibrium of the game with and without spillovers to point out under which conditions the federal tax rate with spillover effects is lower or higher than the federal tax rate without spillover effects.

Finally, we are aware of the fact that our model rests on several simplifying assumptions. For example, we assume that households are homogeneous within regions, that local public goods supplied by different regions are substitutes, and we exclude the possibility that inter-jurisdictional transfers can alleviate the inefficiencies due to spillovers. Possible extensions could relax some of these assumptions.

6 Appendix

Proof of Lemma 1.

Rewrite condition (7) as follows

\[ G(\alpha_i, t_i, T) \equiv -u'_i(x_i) + \left(1 - \sum_{j \neq i} \alpha_{ij}\right) b'_i(g_i)n_i = 0. \]  

(14)

To show part (i) of Lemma 1, we differentiate equation (14) with respect to \(t_i\) and \(T\), i.e. \(\frac{\partial t_i}{\partial T} = -\frac{\partial G/\partial T}{\partial G/\partial t_i}\) (see, for example, Andersson et al. (2004)). Since \(\partial G/\partial t_i < 0\), by the second order condition of the problem in (6), \(\text{sign} \ \frac{\partial t_i}{\partial T} = \text{sign} \ \frac{\partial G}{\partial T}\). From (14), it follows that

\[ \frac{\partial G}{\partial T} = u''_i(x_i)y_i < 0, \]

because \(u''_i(x_i) < 0\) by assumption. Then, \(\frac{\partial t_i}{\partial T} < 0\).

To show part (ii) of Lemma 1, we follow the same procedure, i.e. we differentiate equation (14) with respect to \(t_i\) and \(\alpha_{ij}\), \(i, j = 1, \ldots, I\), i.e. \(\frac{\partial t_i}{\partial \alpha_{ij}} = -\frac{\partial G/\partial \alpha_{ij}}{\partial G/\partial t_i}\). Since \(\partial G/\partial t_i < 0\), \(\text{sign} \ \frac{\partial t_i}{\partial \alpha_{ij}} = \text{sign} \ \frac{\partial G}{\partial \alpha_{ij}}\). From (14), it follows that
\[ \frac{\partial G}{\partial \alpha_{ij}} = -b'_i(g_i) n_i < 0, \]
because \( b'_i(g_i) > 0 \) by assumption. Then, \( \frac{\partial t_i}{\partial \alpha_{ii}} < 0. \)
The same reasoning also shows part (iii) of Lemma 1. Equation (14) is differentiated with respect to \( t_i \) and \( \alpha_{ii}, \ i = 1, ..., I, \) i.e. \( \frac{\partial t_i}{\partial \alpha_{ii}} = -\frac{\partial G/\partial \alpha_{ii}}{\partial G/\partial t_i}, \) and thus \( \text{sign} \frac{\partial t_i}{\partial \alpha_{ii}} = \text{sign} \frac{\partial G}{\partial \alpha_{ii}}. \) From (14), it follows that \( \frac{\partial G}{\partial \alpha_{ii}} = b'_i(g_i) n_i > 0, \) so that \( \frac{\partial t_i}{\partial \alpha_{ii}} > 0. \)

**Proof of Lemma 2.**
By direct inspection of (11), (12), and (13).

**Proof of Proposition 1.**
Rewrite condition (9) as follows
\[ F(t_i(T, \alpha_i), T, \alpha_i) \equiv \beta Y - \alpha_{FF} b'_F(g_F) Y_F + \sum_{j \neq F} \alpha_{Fj} b'_F(g_j) \frac{\partial t_j}{\partial T} Y_j = 0. \quad (15) \]
Following the same procedure of the proof of Lemma 1, we differentiate equation (15) with respect to \( T \) and \( \alpha_{FF}, \) i.e. \( \frac{\partial T}{\partial \alpha_{FF}} = -\frac{\partial F/\partial \alpha_{FF}}{\partial T/\partial \alpha_{FF}}. \) Since \( \partial F/\partial T < 0 \) by the second order condition of the problem in (8), \( \text{sign} \frac{\partial T}{\partial \alpha_{FF}} = \text{sign} \frac{\partial F}{\partial \alpha_{FF}}. \) From (15), it follows that \( \frac{\partial F}{\partial \alpha_{FF}} = -b'_F(g_F) Y_F - \alpha_{FF} b''_F(g_F) \frac{\partial t_F}{\partial \alpha_{FF}} Y_F^2. \quad (16) \)
Then, it is easy to check that \( \frac{\partial T}{\partial \alpha_{FF}} \geq 0 \Leftrightarrow \varphi_F \xi_{t_F, \alpha_{FF}} \geq 1. \)

**Proof of Proposition 2.**
By Lemma 2, and (13), it immediately follows that \( \hat{T} < < ) T \Leftrightarrow MC|_{\alpha_{FF}=0} = \frac{1}{r_{T-T}} \left| \sum_{j \neq F} b'_F(g_j) \frac{\partial t_j}{\partial T} Y_j \right| > ( ) b'_F(g_F) Y_F = MC|_{\alpha_{FF}=1}. \)

**References**


