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The Romer Model with Monopolistic Competition and General Technology

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Abstract

I augment the Romer model of endogenous technological progress with a general CRS production function in labor and intermediate inputs. This determines markups and profits of the innovators in function of the number of inputs. Under imperfect substitutability the economy can converge to a steady state (as under a nested CES technology), replicating the properties of neoclassical growth due to a decreasing marginal profitability of innovation, or to contant growth linear in population growth as in semi-endogenous growth models.

Key words: Endogenous growth, technological progress, monopolistic competition, variable markups, Solow model.

JEL Code: E2, L1, O3, O4.

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The neoclassical growth model of Solow (1956) based on a constant returns to scale (CRS) aggregate production function describes a process of capital accumulation driven by the marginal productivity of capital. When this decreases, investment and growth slow down delivering convergence toward a steady state in which investment replaces depreciated capital. Only when the marginal productivity of capital is constant or bounded at a positive level, the process of capital accumulation can be consistent with a constant growth rate (the AK case) or with convergence to it (Jones and Manuelli, 1990). Instead, the endogenous growth model of Romer (1990) based on monopolistic competition *à la* Dixit and Stiglitz (1977) in the production of intermediate inputs and R&D in the creation of new varieties, describes a process of technological progress which leads to constant growth.

While it is well understood that endogenous growth models are often based on knife edge assumptions, the role of technology and monopolistic competition in driving constant growth in the Romer model is less clear. I reconsider it adopting a general symmetric technology for the production of final goods. The production function satisfies CRS in the intermediate inputs and the labor input, and I analyze monopolistic competition under the derived demand system for the intermediate inputs. Contrary to the original Romer model, markups, production levels and profits depend on the number of inputs provided in the market. The dynamics of the economy is entirely summarized by an equation of motion for the number of inputs that resembles the Solow equation for the stock of capital, but the concave production function of the neoclassical model is replaced by a non-linear function that links production to the endogenous number of inputs.

Under imperfect substitutability between inputs, the marginal profitability of innovation (the additional profit from introducing new varieties) tends to decrease along the growth process, replicating the same phenomena of the neoclassical model. Output growth can decline over time delivering convergence toward a steady state in which R&D investment replaces obsolete technologies. I show the emergence of this pattern in an example based on a nested CES technology which preserves constant markups but delivers decreasing marginal profitability of innovation.

Only when the marginal profitability is constant or bounded at a positive level, the process of technological accumulation can be consistent respectively with a constant growth rate or with gradual convergence to it. The Romer model is an example of the first kind and I present an example of the second kind where growth declines gradually while markups increase toward a constant level and the profits of the monopolistic producers decrease toward a constant level which fuels growth forever.

I notice that also more complex dynamics can emerge, and I discuss equilibrium patterns in extended versions of the model with population growth, production externalities and oligopolistic competition. The role of population growth is particularly interesting in the augmented Romer model because it allows to generate a constant growth rate for per capita income as in semi-endogenous models *à la* Jones (1995) without resorting to the exogenous spillovers in the production of new ideas used there: this happens whenever the augmented Romer model with a constant population delivers a stable steady state.

The present work is based on recent advances in the theory of monopolistic competition under general microfoundations (Bertoletti and Etro, 2016, 2017, 2018). Related applications based on homothetic preferences concern business cycle theory with endogenous entry under monopolistic competition (Bilbiie, Ghironi and Melitz, 2012, 2019) and oligopolistic competition (Etro, 2009, 2018; Colciago and Etro, 2010; Savagar, 2017). Interesting works on endogenous growth departing from constant markups are the ones by Bucci and Matveenko (2017) and Boucekkine, Latzer and Parenti (2017), but they are focused respectively on differentiation in intermediate goods with a directly additive technology and on differentiation in consumption goods with indirectly additive preferences, while here I focus on differentiation in intermediate goods with a general CRS technology.[2] In earlier work, I have extended to general microfoundations of monopolistic competition the neoclassical model of consumption growth (Etro, 2016) and the neoclassical model of trade (Etro, 2017), while here I focus on the neoclassical model of growth. The common theme is that the variability of markups and profits with costs, spending or the number of goods has crucial implications for general equilibrium models of business cycle, trade and growth (that vanish in traditional models with null or constant markups). In particular, markup variability across time and space affects the propagation of shocks and international trade, while changes in the marginal profitability of innovation affect business creation and growth.

The rest of the work is organized as follows. In Section 1 I present the simplest version of the Romer model with exogenous rates of savings and exit of firms to stress similarities with the Solow model. Then, in Section 2 I extend it to a general CRS production function and apply the results to two examples. In Section 3 I extend the model to population growth, and in Section 4 I discuss some generalizations of the baseline model before concluding in Section 5.

1 A benchmark model of endogenous growth

The Romer (1990) model of endogenous growth and most of its extensions (see Barro and Sala-i-Martin, 2004) are based on the following CRS production

[2] A parallel application to growth of advances in monopolistic competition concerns patent races with heterogeneous firms for Schumpeterian growth models (see Etro, 2019).

function for a perfectly competitive sector producing final goods:

$$Y = (AL)^{1-\alpha} \sum_{j=1}^{n} X_j^{\alpha} \tag{1}$$

where X_j is one of the n intermediate goods produced in a given moment, L is the constant labor input and A its productivity, with $\alpha \in (0, 1)$ representing the factor share of income from intermediate goods. One way to look at this production function is as the sum of Cobb-Douglas production units using intermediate inputs and the same labor. It is important to emphasize that the inputs are independent between themselves, in the sense that the demand of each input is independent from the others. Instead, the demand of labor is increasing in the quantity of each intermediate good. Labor market clearing implies that the real wage equates the marginal productivity of labor. Each producer of intermediates is a monopolist with an eternal patent. The final good is the *numeraire* and is used for consumption and also for the production of intermediate goods with a one-to-one technology.

To simplify the analysis of the dynamics, I follow the neoclassical model of Solow (1956) and assume that the economy saves a fraction $s \in (0, 1)$ of output to invest in the creation of new intermediate goods at cost F_e. Accordingly, in each period the number of new goods n_e must satisfy the equality of investment and savings:

$$n_e F_e = sY \tag{2}$$

and the entry of producers of new inputs is free. The intermediate goods become obsolete with probability $\delta > 0$, inducing the exit of their producers. Therefore, the rate of change of the number of firms Δn in an interval of time follows:

$$\Delta n = n_e - \delta n \tag{3}$$

The solution of the Romer model is extremely simple due to the independence between intermediate goods. Given the price of the good i, p_i, its demand satisfies $p_i = \alpha X_i^{\alpha-1}(AL)^{1-\alpha}$, which induces the monopolistic producer i to maximize variable profits $\pi_i = (p_i - 1)\left(\frac{\alpha}{p_i}\right)^{\frac{1}{1-\alpha}} AL$ by setting the price:

$$p = \frac{1}{\alpha} \tag{4}$$

in each period. This delivers production $X = \alpha^{\frac{2}{1-\alpha}} AL$ and variable profits $\pi = (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}} AL$ for any monopolist in any period. Replacing in the production function, output per capita y can be derived as:

$$y = A\alpha^{\frac{2\alpha}{1-\alpha}} n \tag{5}$$

whose growth rate g corresponds to the growth rate of the number of goods $g(n) = \Delta n/n$. It is then immediate to use (2) and (3) to obtain the growth rate as:

$$g = s\lambda\alpha^{\frac{2\alpha}{1-\alpha}} - \delta \tag{6}$$

where I defined $\lambda \equiv AL/F_e$ as the relative size of the economy,[3] and I assumed positive growth.

To close the model, the interest rate must insure that the free entry condition is met. This equates the present discounted value of future profits:

$$V = \frac{\pi}{r + \delta}$$

to the entry cost F_e. Since profits are constant, this delivers a constant real interest rate $r = (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}\lambda - \delta$.[4]

[3] As well known, this model exhibits scale effects in the population level. They disappear if the entry cost increases with the size of the market, with additional spillovers in idea production as in Jones (1995) or, as I will clarify in Section 3, under alternative CRS technologies.

[4] I consciously avoided a notation that commits to discrete or continuous time. One can indeed endogenize the savings rate either in a Ramsey model in discrete or continuous time or in an OLG model. This generates an additional feeback of the interest rate on growth.

2 General CRS technologies

I now consider a generic symmetric technology:

$$Y = F(AL, \mathbf{X}) \tag{7}$$

where $\mathbf{X}$ is the vector of n intermediate goods and L fixed labor, with F increasing and concave in each input and satisfying CRS and $F(AL, \mathbf{u}0)$, where $\mathbf{u}$ is a unit vector. The production function F can depend on n through the dimensionality of the vector and/or production externalities. Notice that these do not interefere with the CRS property, for which $F(tAL, t\mathbf{X}) = tF(AL, \mathbf{X})$ for any $t, n > 0$. The technology can be rewritten in intensive form as:

$$y = Af(\mathbf{x}) \tag{8}$$

where $x_i = X_i/AL$ is the production level of good i per effective worker and $f(\mathbf{x}) \equiv F(1, \mathbf{x}) > 0$ is a symmetric function with $f(\mathbf{u}0) = 0$, $f_i(\mathbf{x}) > 0$, $f_{ii}(\mathbf{x}) < 0$. It will be convenient to assume that $f(\mathbf{u}x)$ is differentiable in the number of goods n for any constant x, and it will be natural to focus on the case in which $f_n(\mathbf{u}x) > 0$ in virtue of (production) gains from variety. For instance, in the Romer case $f(\mathbf{x}) = \sum_j x_j^\alpha$ is homothetic and separable (i.e.: a monotonic transformation of a homogenous and additive function), with $f(\mathbf{u}k) = nx^\alpha$ increasin in n. A generalization of this form of separability of the Romer technology emerges under a production function that is directly additive in the intermediate inputs, as in:

$$Y = \sum_{j=1}^{n} G(AL, X_j) \tag{9}$$

where G is CRS in labor and an intermediate good and $f(\mathbf{x}) = \sum_{j=1}^{n} G(1, x_j)$.

Given the price of each intermediate good p_i, its inverse demand satisfies:

$$p_i = f_i(\mathbf{x})$$

which is decreasing in x_i, and changes with the production levels of the other intermediate goods in various ways depending on the sign of $f_{ij}(\mathbf{x})$. In particular, the intermediate goods are substitutes if $f_{ij}(\mathbf{x}) < 0$, independent if $f_{ij}(\mathbf{x}) = 0$ and complements if $f_{ij}(\mathbf{x}) > 0$. The associated variable profits:

$$\pi_i = [f_i(\mathbf{x}_i) x_i - x_i] AL \tag{10}$$

are maximized by each firm i chosing the production level per effective worker x_i under monopolistic competition.[5] According to the traditional definition of Dixit and Stiglitz (1977), this means that each firm considers as negligible the impact of its strategy on the aggregators in computing the demand elasticity. When such aggregators do not exist, an alternative definition requires one to approximate the demand elasticity considering market shares as negligible. The two definitions are equivalent with a large number of firms (see the discussion in Bertoletti and Etro, 2016, 2018). In either case, under symmetry the relevant elasticity has been shown to be the symmetric version of the Morishima elasticity of complementarity:

$$\epsilon_{ij} \equiv -\frac{\partial(p_i/p_j)}{\partial x_i}\frac{x_i}{(p_i/p_j)} = \frac{-f_{ii}(\mathbf{x})x_i}{f_i(\mathbf{x})} + \frac{f_{ji}(\mathbf{x})x_i}{f_j(\mathbf{x})}$$

The symmetric version of this function, $\epsilon(n, x)$, depends on the number of goods n and the common value of x, and is assumed smaller than unity, implying a positive markup. This delivers the optimal price:

$$p = \frac{1}{1-\epsilon(n,x)} \quad \text{with} \quad \epsilon(n,x) = \frac{f_{ji}(\mathbf{u}x)x - f_{ii}(\mathbf{u}x)x}{f_i(\mathbf{u}x)} \tag{11}$$

The equilibrium wage satisfies:

$$w = A\left[f(\mathbf{u}x) - nxf_i(\mathbf{u}x)\right]$$

[5] We assume that the labor input is taken as given and that the second order condition for an interior solution is satisfied.

The elasticity $\epsilon(n, x)$ and therefore the price of each monopolist depends only on n if $f(\mathbf{x})$ is homothetic and only on x if $f(\mathbf{x})$ is separable, therefore it is a constant when $f(\mathbf{x})$ is both homothetic and separable as in the Romer case.[6]

In equilibrium of monopolistic competition for a given number of firms, the symmetric demand system provides the condition:

$$[1 - \epsilon(n, x)] f_i(\mathbf{u}x) = 1$$

which implicitly defines the equilibrium production of each input per effective worker $x \equiv \phi(n)$ in function of the number of firms, which I assume to be unique and twice differentiable.[7] It is easy to verify that the number of firms is actually neutral on the equilibrium production (which is therefore a constant) if $f(\mathbf{x})$ is separable, as in the Romer model or, more in general, with the technology (9): this is what insures the existence of a constant growth rate. Otherwise, production depends on the number of firms.

As a consequence, the equilibrium elasticity $\epsilon(n, \phi(n))$ is entirely determined by the number of firms,[8] and also the variable profits can be expressed as a function of n:

$$\pi(n) = \frac{\epsilon(n, \phi(n))\phi(n)}{1 - \epsilon(n, \phi(n))} AL$$

At this level of generality, prices and profits can either decrease or increase in

[6] Alternatively, one can adopt a cost function $c(\mathbf{p}, w) = c(\frac{\mathbf{p}}{w}, 1)w$, and derive the direct demand of inputs from the Shephard's lemma. This allows one to express markups in terms of the symmetric Morishima elasticity of substitutability. Indirect additivity delivers markups in function of the wage. The formal analysis is analogous to the one of Bertoletti and Etro (2017).

[7] Assuming substitutability ($f_{ij}(\mathbf{x}) < 0$), $f_i(\mathbf{u}x)$ must decrease with x. Then, the conditions $f_i(\mathbf{u}0) \to \infty$ and $f_i(\mathbf{u}\infty) \to 0$ are sufficient for uniqueness under homotheticity or when $\epsilon(n, x)$ is non-decreasing in x.

[8] This happens also for a demand system derived from homothetic preferences (as noticed in Benassy, 1996, and Bilbiie, Ghironi and Melitz, 2012). It holds here for any aggregator due to the CRS of the original production function.

the number of goods (as well as remain independent from them, as in the Romer case). Replacing in the production function, I obtain:

$$y = Ah(n) \quad \text{with} \quad h(n) \equiv f(\mathbf{u}\phi(n)) \tag{12}$$

The function $h(n)$ is not a production function, but an equilibrium relation between the output per effective worker and the number of monopolistically produced inputs. The essentiality of inputs implies $h(0) = 0$, and I assume this function to be twice differentiable and increasing. Given the existence of production gains from variety, the assumption that:

$$h'(n) = f_n(\mathbf{u}\phi(n)) + nf_i(\mathbf{u}\phi(n))\phi'(n) > 0$$

requires only that $\phi(n)$ does not decrease too quickly. None of our assumptions insures concavity, though the second derivative:

$$\begin{aligned} h''(n) &= f_{nn}(\mathbf{u}\phi(n)) + [2nf_{ni}(\mathbf{u}\phi(n)) + f_i(\mathbf{u}\phi(n))]\,\phi'(n) + \\ &\quad +n^2 f_{ii}(\mathbf{u}\phi(n))\phi'(n)^2 + nf_i(\mathbf{u}\phi(n))\phi''(n) \end{aligned}$$

is negative when the marginal gains from variety are decreasing in the number of inputs ($f_{nn} < 0$) but increasing in each quantity ($f_{ni} > 0$) and the equilibrium production of each input is decreasing ($\phi'(n) < 0$) and not too much. These conditions hold in our examples below, but they do not necessarily hold in general or globally.

The growth rate of output $y = Ah(n)$ is directly related to the growth rate of the number of firms. The equality of savings and investments (2) provides $n_e F_e = sALh(n)$, that using (3) delivers the dynamics of the number of firms:

$$\Delta n = s\lambda h(n) - \delta n \tag{13}$$

Due to the non-linearity of the $h(n)$ function, this equation of motion can give raise to a variety of dynamic paths, including stable or cycling convergence to a steady state, complex dynamix, multiple steady states and a long run growth.[9] The model is closed by the free entry condition equating the entry cost in each period to the present discounted value of future profits, which always pins down the interest rate in function of the state variable n as:

$$r(n) = \frac{\pi(n)}{F_e} - \delta$$

Therefore the interest rate represents the rate of return of innovation.

If there is a steady state for the number of inputs n^*, it must satisfy:

$$s\lambda h(n^*) = \delta n^* \tag{14}$$

with output $y^* = Ah(n^*)$. Moreover, I have stable convergence to the steady state if $\vartheta(n) \in (0,1)$ for any $n \geq n^*$, where $\vartheta(n) \equiv h'(n)n/h(n)$ is the elasticity of the equilibrium production with respect to the number of inputs.

Between steady states associated with different saving rates, one can also determine the steady state associated with the number of inputs (and therefore the investment in R&D) that maximizes net consumption per capita. Since output of final goods net of expenditure for inputs and R&D is $C^* = (1 - s)y^* - \phi(n^*)A$, such a golden rule satisfies:

$$\lambda \left[h'(n^{GR}) - \phi'(n^{GR}) \right] = \delta$$

equating the net marginal productivity of innovation to the rate of exit of firms. The intuition is that the additional contribution of the new goods to total production net of the cost in intermediate goods should be compared with the cost of replacement of obsolete technologies.

In the long run a positive growth rate can be sustainable only if the following

[9] For an important analysis of cycles within the Romer model based on different sources, see Matsuyama (1999).

growth rate remains positive for $n \to \infty$:

$$g(n) = s\lambda \frac{h(n)}{n} - \delta \tag{15}$$

Therefore, the possibility of long run growth depends on the shape of the function $h(n)/n$, which represents output per effective worker and per intermediate good. In the Romer model this is constant and growth is constant as well, and the same happens if $f(\mathbf{x})$ is separable, as with the technology (9), which provides the growth rate:

$$g = s\lambda G(1,\phi) - \delta$$

where ϕ is a constant that satisfies $\phi = [G_x(1,\phi) + G_{xx}(\phi)]^{-1}$. However, when the function $h(n)/n$ is decreasing and asymptotically constant, the balanced growth path is reached through a gradual process of declining growth. In the following subsections I will exemplify the two main patterns of convergence to zero and positive growth.

2.1 An example of stable steady state

As an example of a process of technological progress that does not lead to permanent growth, let me consider the following generalization of the production function of Romer (1990):

$$Y = (AL)^{1-\alpha} \left(\sum_{j=1}^{n} X_j^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta\alpha}{\theta-1}} \tag{16}$$

which derives from a Cobb-Douglas in a CES index of intermediate goods and labor, where $\theta > 1$ is the elasticity of substitution between the intermediate goods. The Romer case is nested when $\theta = 1/(1-\alpha)$. Beyond that case, here the total production is not the sum of the output of each production unit, because the intermediate inputs must be combined to produce the final good, with a substitutability parametrized by θ. The different inputs are imperfect substitutes for $\theta \in \left(\frac{1}{1-\alpha}, \infty\right)$, implying $f_{ij}(\mathbf{x}) < 0$, while they are independent for $\theta = \frac{1}{1-\alpha}$, and complements otherwise. My focus will be on the case of substitutability assuming $\theta > 1/(1-\alpha)$. Notice that $f(\mathbf{x})$ is homothetic and separable with $f(\mathbf{u}x) = n^{\frac{\theta\alpha}{\theta-1}} x^{\alpha}$ increasing and concave in n.

The inverse demand of intermediate goods satisfies:

$$p_i = \frac{\alpha x_i^{\frac{-1}{\theta}}}{\left[\sum_{j=1}^{n} x_j^{\frac{\theta-1}{\theta}}\right]^{1-\frac{\theta\alpha}{\theta-1}}}$$

and the aggregator at the denonimator is taken as given by firms acting under monopolistic competition, therefore the perceived elasticity of demand is simply θ, and profits are maximized by the price:

$$p = \frac{\theta}{\theta - 1} \tag{17}$$

which is again constant. This delivers:

$$\phi(n) = \left[\frac{(\theta-1)\,\alpha}{\theta}\right]^{\frac{1}{1-\alpha}} n^{\frac{1-\theta(1-\alpha)}{(\theta-1)(1-\alpha)}} \quad \text{and} \quad \pi(n) = \frac{\phi(n)AL}{\theta - 1}$$

Under our assumptions $\phi(n)$ and both the demand of each intermediate good and the profits of its producer are decreasing in the number of intermediate goods. This insures that the marginal profitability of product creation is decreasing while new products are created along the growth process, exactly as in the neoclassical model of growth, where the marginal productivity of capital is decreasing while new capital is accumulated through investment.

Replacing the quantity of inputs in the production function we have output $y = Ah(n)$ with:

$$h(n) = n^{\frac{\alpha}{(\theta-1)(1-\alpha)}} \left(\frac{\alpha\,(\theta-1)}{\theta}\right)^{\frac{\alpha}{1-\alpha}} \tag{18}$$

satisfying $h'(n) > 0$ and $h''(n) < 0$. The growth rate of output depends on the growth rate $g(n)$ of the number of goods as follows:

$$g = \frac{\alpha g(n)}{(\theta-1)(1-\alpha)} \tag{19}$$

which is lower than $g(n)$ under the same assumptions. The equality of savings and investments provides the equation of motion for the number of firms:

$$\Delta n = s\lambda \left(\frac{\alpha\,(\theta-1)}{\theta}\right)^{\frac{\alpha}{1-\alpha}} n^{\frac{\alpha}{(\theta-1)(1-\alpha)}} - \delta n \tag{20}$$

or the growth rate:

$$g(n) = \frac{sAL}{F_e}\left(\frac{\alpha\,(\theta-1)}{\theta}\right)^{\frac{\alpha}{1-\alpha}} n^{\frac{1-\theta(1-\alpha)}{(\theta-1)(1-\alpha)}} - \delta \tag{21}$$

In this case with substitutability between inputs there is a unique stable steady state with zero growth of income. Innovation keeps creating new varieties replacing obsolete ones, but the number of intermediate goods used in the production reaches the steady state value:

$$n^* = \tilde{\alpha}\left(\frac{s\lambda}{\delta}\right)^{\frac{(\theta-1)(1-\alpha)}{\theta(1-\alpha)-1}} \tag{22}$$

where $\tilde{\alpha} \equiv \left(\frac{\alpha(\theta-1)}{\theta}\right)^{\frac{(\theta-1)\alpha}{\theta(1-\alpha)-1}}$. Output per capita approaches the following long run level:

$$y^* = \tilde{\alpha}\left(\frac{s\lambda}{\delta}\right)^{\frac{\alpha}{\theta(1-\alpha)-1}} A \tag{23}$$

This case replicates the convergence property of the neoclassical model.[10] Growth decreases over time and output per capita reaches a steady state that is positively related to the savings rate and negatively to the rate of obsolescence. However, here the steady state levels for number of firms and output are also positively related to the relative size of the market, namely population and

[10] Of course, in the knife-edge case where $\theta = 1/(1-\alpha)$ growth is constant. If $\theta > 1/(1-\alpha)$, the larger is θ the lower are profits and the faster is the convergence to zero growth. In principle, if $\theta < 1/(1-\alpha)$ complementarity between production units implies that demand and profits of each intermediate good increase with the number of goods and growth becomes explosive.

productivity relative to the fixed cost of R&D. Computing the steady state consumption $C^* = \tilde{\alpha} A \left[1 - s - \frac{\alpha(\theta-1)}{\theta}\right] \left(\frac{s\lambda}{\delta}\right)^{\frac{\alpha}{\theta(1-\alpha)-1}}$, one can also derive the golden rule saving rate as:

$$s^{GR} = \frac{\alpha\left[1 - \frac{\alpha(\theta-1)}{\theta}\right]}{(\theta-1)(1-\alpha)}$$

which is higher when the factor share of intermediate inputs is higher (α is high) and these inputs are less substitutable between themselves (θ is low). Finally, the declining path of profits generates a declining path for the equilibrium interest rate, once again in line with the neoclassical growth model.

The CES example is simple due to the fact that during the growth process the markups remain constant, but this is not enough to avoid that demand and profits for each firm keep decreasing due to the substitutability between inputs. This is destined to stop growth exactly as it happens in the neoclassical model where a decreasing marginal productivity is destined to terminate the process of capital accumulation. The same would happen with other CRS technologies which imply also declining markups and marginal profitability, as in case of translog specifications.

2.2 An example of convergence to constant growth

Technologies generating equilibrium demand and profits that decrease along the growth path can still generate long run growth. This requires only that the profits have a lower positive bound. Once again, this is quite similar to what happens in the neoclassical growth model as long as the marginal productivity of capital has a lower bound that is high enough to sustain capital accumulation in the long run (Jones and Manuelli, 1990). Consider the production function:

$$Y = (AL)^{1-\alpha}\left[\sum_{j=1}^{n} X_j^{\alpha} + \left(\sum_{j=1}^{n} X_j\right)^{\alpha}\right] \tag{24}$$

This satisfies CRS and essentially combines our earlier examples of Cobb-Douglas production units and production units using perfectly substitutable inputs. Now $f(\mathbf{x})$ is homothetic but non-separable with $f(\mathbf{u}x) = (n + n^{\alpha})x^{\alpha}$ convex in the number of inputs. Such an economy behaves asymptotically as in the Romer model with positive long run growth, but markups are variable with the number of intermediate goods and the growth rate decreases while reaching its long run value. To verify this, notice that the inverse demand of intermediate goods satisfies:

$$p_i = \alpha x_i^{\frac{-1}{\theta}} + \frac{\alpha}{\left(\sum_{j=1}^{n} x_j\right)^{1-\alpha}}$$

implying substitutability due to $f_{ij}(\mathbf{x}) < 0$. Monopolistic competition delivers the symmetric equilibrium price:

$$p = \frac{1 + n^{\alpha-1}}{\alpha + n^{\alpha-1}} \tag{25}$$

which is actually increasing in the number of intermediate goods (in this case substitutability between inputs decreases when there are more inputs) and approaches the constant (4) when that number grows unbounded. Demand per effective worker and profits for each monopolist are:

$$\phi(n) = \alpha^{\frac{1}{1-\alpha}} \left(\alpha + n^{\alpha-1}\right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad \pi(n) = (1-\alpha)\,\alpha^{\frac{1}{1-\alpha}} \left(\alpha + n^{\alpha-1}\right)^{\frac{1}{1-\alpha}} AL$$

both of which decrease in the number of intermediate goods and converge to the corresponding constants of the Romer model. The growth rate of the number of goods is therefore:

$$g(n) = s\lambda\alpha^{\frac{\alpha}{1-\alpha}} \left(1 + \frac{1}{n^{1-\alpha}}\right) \left(\alpha + \frac{1}{n^{1-\alpha}}\right)^{\frac{\alpha}{1-\alpha}} - \delta \tag{26}$$

which decreases over time toward the growth rate of the Romer model $g(\infty) = s\lambda\alpha^{\frac{2\alpha}{1-\alpha}} - \delta$. As the neoclassical model can generate long run growth when the marginal productivity of capital is bounded below, endogenous growth models deliver the same result when the marginal profitability of innovation is bounded below.

3 Population growth

In this section I introduce growth of the labor input, assumed equal to population growth. As well known, the Romer model with scale effects delivers explosive growth in case of a positive population growth, therefore I will focus on versions of the augmented Romer model that exhibit a stable steady state without population growth. The important result is that these versions of the Romer model generate a balanced growth path when population increases at a constant rate. In other words, the model can deliver endogenous technological progress without scale effects as in models of semi-endogenous growth (Jones, 1995), but without resorting to any spillover effects typical of those models.

Assume that population grows at the costant rate $g_L = \frac{\Delta L}{L}$ and, for simplicity, that there is no obsolescence of intermediate goods, namely $\delta = 0$. Then output per capita $y = Ah(n)$ grows at the rate:

$$g = \vartheta(n)g(n)$$

where I already defined $\vartheta(n)$ the elasticity of the $h(n)$ function. To insure the existence of a stable steady state when population is constant, I also assume that this elasticity satisfies $\vartheta(n) \in (0, 1)$ for any $n \geq n^*$. I now define the limit of this elasticity as $\bar{\vartheta} = \vartheta(\infty) \in (0, 1)$.

Then, the equation of motion of the number of firms $\Delta n = s\lambda h(n)$ implies:

$$g(n) = \frac{sALh(n)}{F_e n}$$

which is constant if and only if:

$$g_L = [1 - \vartheta(n)]g(n)$$

in the long run. This implies the growth rate of the number of firms $g(\infty) = \frac{g_L}{1-\bar{\vartheta}}$. Replacing in the expression above evaluated for $n \to \infty$, I obtain the growth rate of output per capita in the long run as:

$$g = \frac{\bar{\vartheta} g_L}{1 - \bar{\vartheta}} \tag{27}$$

which is always positive and linear in the growth rate of the population. The coefficient of proportionality is entirely dependent on the intrinsic properties of the technology (i.e.: independent from the savings rate or from the amount of inputs, which is increasing over time).

To exemplify the analysis I focus on the nested CES technology with imperfect substitutability ($\theta > 1/(1-\alpha)$). Then, output per capita grows at the rate (19), but the growth rate of $g(n)$ is constant if:

$$g_L = \frac{\theta(1-\alpha) - 1}{(\theta - 1)(1-\alpha)} g(n) \tag{28}$$

which provides the per capita growth rate:

$$g = \frac{\alpha g_L}{\theta(1-\alpha) - 1} \tag{29}$$

consistent with constant profits for each firm. Growth is decreasing in the elasticity of substitution θ, which reduces market power and therefore the incentives to innovate, and increasing in the parameter α, representing the importance of the intermediate inputs in the technology. The novel lesson is that constant growth driven by R&D is compatible with population growth under standard technologies featuring imperfect substitutability between inputs, without resorting to the spillover effects of models *à la* Jones (1995). Scale effects, which are empirically implausible, emerge in the original Romer model, but not in its augmented versions with more general technologies.

One can augment this analysis with a simple microfoundation for savings and fertility in the style of Becker (1960) and Barro and Becker (1989) to analyze an endogenous market structure with endogenous market size and growth.[11] As in earlier related studies (Jones, 2001; Chu *et al.*, 2013), the equilibrium growth rate decreases in the discount factor of the consumers since more patient agents save more and have less children, which reduces the expected gains from innovation. Finally, adding endogenous investment in education by the parents to increase productivity of the children delivers a classic trade-off between quantity and quality of children, and allows one to reproduce the inverse relation between fertility and growth which characterizes the modern era, as in Galor and Weil (2000).[12]

[11] See Cigno (1991) on the theory of fertility and Galor (2011) on the application to growth theory.

[12] Consider an OLG framework where each young agent decides savings S and number of children b to maximize utility:

$$U = \log\left[w(1-eb)-S\right] + \beta\log\left[S(1+r)\right] + \gamma\log Ab$$

Here $e > 0$ parametrizes the cost of raising children in terms of lost wage, $\beta \in (0,1)$ is the discount factor and γ parametrizes the utility from children and their productivity. Then, savings are $S = \frac{\beta w}{1+\beta+\gamma}$ and population grows at the rate:

$$g_L = \frac{\gamma(\frac{1}{e}-1)-(1+\beta)}{1+\beta+\gamma}$$

If human capital $A(e,n)$ increases in education e and decreases in technological complexity n, one can even endogenize the investment in education and obtain that fertility decreases and human capital increases with technological progress (see Galor, 2011, for a discussion).

4 Externalities and strategic interactions

I will briefly consider further extensions using again the case of a nested CES technology and imperfect substitutability ($\theta > 1/(1-\alpha)$) with a constant labor force as a useful benchmark.

Until now I have neglected *technological externalities* of the number of inputs on labor productivity, but these can have an interesting role in growth dynamics. Let me consider a productivity $A(n)$ affected by the number of intermediate goods. This can be due either to positive Marshallian externalities *à la* Romer (1986) and Becattini (1987) or to negative congestion effects that slow down innovation (remember that externalities in the production function due to changes of n do not interfere with private CRS).[13] I will only mention two examples, defining $\chi = \frac{\alpha}{(\theta-1)(1-\alpha)} < 1$.

A positive externality such as $A(n) = (a+n)^{1-\chi}$ implies the following equation of motion for the model based on CES technology:

$$\Delta n = \frac{sL}{F_e}\left(\frac{\alpha\left(\theta-1\right)}{\theta}\right)^{\frac{\alpha}{1-\alpha}}(an + n^{\frac{1}{\chi}})^{\chi} - \delta n$$

This dynamic path restores convergence to a constant long run growth rate $g = s\frac{L}{F_e}[\frac{\alpha(\theta-1)}{\theta}]^{\frac{\alpha}{1-\alpha}} - \delta$.

A negative externality such as $A(n) = (a-n)^{\chi}$ generates an inverse-U shape for the equation of motion of the number of firms:

$$\Delta n = \frac{sL}{F_e}\left(\frac{\alpha\left(\theta-1\right)}{\theta}\right)^{\frac{\alpha}{1-\alpha}}\left(an - n^2\right)^{\chi} - \delta n$$

that leads to a cycles or more complex dynamics, whose analysis is beyond the scope of this work.

[13] Externalities of the number of goods on homothetic preferences have been emphasized under monopolistic competition by Benassy (1996) and also applied in dynamic models by Bilbiie, Ghironi and Melitz (2012).

The baseline analysis can be extended to *oligopolistic competition*, applying the equilibrium concepts derived for a general microfoundation of the demand system in Bertoletti and Etro (2016). Symmetry insures that markups, production and profits of each intermediate good depend on the number of intermediate goods as before. In the nested CES example with $\theta > 1/(1-\alpha)$, the equilibrium price under Bertrand competition becomes:

$$p^B = \frac{\theta}{\theta - \frac{n}{n-\Gamma(\theta,\alpha)}} \quad \text{where } \Gamma(\theta,\alpha) = \frac{\theta(1-\alpha)-1}{(\theta-1)(1-\alpha)}$$

and the one under Cournot competition becomes:

$$p^C = \frac{\theta n}{(\theta-1)(n-1)+\theta\alpha}$$

In both cases, the markups decrease in the number of intermediate goods n, in the substitutability θ, and in the factor share of income from intermediate goods α. They tend to the markups with monopolistic competition (17) while the number of intermediate goods increases, therefore growth approaches zero in the long run (business cycle properties of related models are analyzed in Etro, 2009, and Colciago and Etro, 2010).

Finally, one could extend the analysis to *vertical innovations* that provide better versions of existing goods rather than new goods (as in Barro and Sala-i-Martin, 2004). Formalizing patent races for the development of higher quality goods would endogenize the rate of obsolescence and deliver endogenous growth under knife edge technological conditions (the case of the nested CES technology is examined in Etro, 2009, Ch. 5). This would open the door to a more general situation where goods are genuinely asymmetric and each new good is an imperfect substitute for the old ones. This is a scenario where the same optimality of the sequence of innovatons cannot be taken as given, because the market may postpone the introduction of goods that are highly productive for the economy but not much profitable from the point of view of the monopolistic suppliers. While this extension is beyond the scope of this work, the spirit of the above insights should be robust: the economy can converge gradually to long run growth if the marginal profitability of innovations decreases but remains large enough or if there is population growth.[14]

5 Conclusion

There are different ways to read the previous results. The first is that the basic insights of the neoclassical growth model of Solow (1956) are inherited by R&D-based models of endogenous growth once one allows for a general neoclassical production function in an expanding set of intermediate goods: in particular, the model is consistent with convergence to a steady state due to decreasing marginal profitability of innovation and constant growth emerges only under knife-edge conditions. This is indeed a main message of this work, but I should also remind the reader that more complex dynamics can emerge in R&D-based models (which cannot emerge in the neoclassical growth model) and would deserve further investigations.

A second way to read the results above is that the key idea of Romer (1990) remains unchanged after generalizing its peculiar technological assumptions: market power is necessary to incentivize R&D investment that leads to long growth. This applies with the qualification that technology and the process of monopolistic competition can affect drammatically the nature and the same existence of the link between R&D and long run growth.

A third and complementary way of reading this work is that when population grows the augmented Romer model can generate constant growth without any of the spillovers of the semi-endogenous growth models of Jones (1995). This

[14] For an important contribution in this spirit see Peretto (2018).

happens whenever imperfect substitutability between inputs delivers a stable steady state under a constant population, and it implies a positive growth rate linear in the population growth rate with a coefficient of proportionality that depends on the technological parameters. This should de-emphasize the critique to the first-generation models of endogenous growth based on the implausibility of scale effects in the population level: under more general technologies these scale effects disappear naturally preserving endogenous growth.

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