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The theory of multiple public budget determination from the perspective of history of economic analysis

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Abstract

From an analitycal historical perspective, this paper deals with the important public finance issue of decentralizing the government's economic activities and functions into distinct branches, respectively devoted to collecting taxes and to allocating a given budget to different public goods. We start with the so-called Italian tradition in public finance and go on to the crucial contributions from A.C. Pigou, Paul Samuelson and Richard Musgrave, up till the modern second best and optimal taxation approaches. Starting from the Italian tradition is meaningful as this stream of literature has given important pioneristic contributions to this topic, namely by Maffeo Pantaleoni and Enrico Barone. However, it is in developing the relationships of these contributions with the modern second best optimal taxation approach that we may find rigorous solutions to the main emerging problems of the issue.

Keywords: branch model, optimal taxation, public goods

JEL Classification: B10, H21, H41

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1 Introduction

The separation between efficiency and equity aims and tools may be considered as the main topic of normative public economics. Since Pareto (1909), it has become, over time, the core of public finance theory and also of public sector organization theory [1]. Indeed, if separation holds, it is possible to decentralize the activities and the functions of a government in agencies or branches devoted respectively to collecting taxes, on the basis of the ability to pay principle (distribution function), and to allocating a given budget in providing the various public goods, on the basis of a marginal utility approach (allocation function).

In this paper we propose to deal with this issue from an analytical historical perspective, moving from the so-called Italian tradition in public finance until the modern second best approach, passing throughout the Richard Musgrave's fundamental contribution.

Starting from the Italian tradition is meaningful as it is in this stream of literature that we find the first important intuitions and some challenging contributions to this topic[2]. Also Pigou (1927, 1947), dealing with the concept of marginal cost of public funds, indirectly gave an important impulse to the issue. Musgrave published his fundamental textbook in public finance (Musgrave, 1959) a decade after *The Foundations* by Paul Samuelson (1947) and represents the application to public finance of the *new welfare economics* designed in this monumental work. Before the *Theory of Public Finance*, Musgrave (1956), as we'll see later, drew up his theory on branch division as a corollary of the theorem of public goods supply designed in the pioneristic paper by Samuelson (1954). Thus, Musgrave's multiple budget determination seems to be feasible, like the Samuelson rule, in a first best setting, where all information is publicly available, and efficiency can be achieved independently of equity concerns through lump sum transfers, without recourse to commodity taxation.

The modern second best approach, within the optimal taxation theory, relaxes these strong requirements and reaches rigorous solutions to the main emerging analytical problems. This theory, mainly started by Mirrlees (1971) and Diamond and Mirrlees (1971), on the grounds of the pioneristic Ramsey (1927) contribution, is often referred to as the *new new welfare economics,* i.e. is considered to be an extension of the Samuelsonian new welfare economics. Indeed, in a second best environment, the government faces additional constraints, in particular incentive constraints, when some individual characteristics are not observable. These new constraints restrain the scope of redistribution, the optimal rules typically differ from the first best ones and efficiency can no longer be disconnected from equity aims. There are still a number of circumstances where second best rules have a first best flavour. The crucial conditions refer to the homogeneity and separability of preference between leisure, commodities and

[1] It is sufficient to look at the main tesxtbooks from Musgrave (1959) to Atkinson and Stiglitz (1980) and up to the recent Hindriks and Myles (2006) and Bénassy-Quéré et al. (2010).

[2] For the international diffusion of the Italian tradition see Musgrave and Peacock (1958) and Buchanan (1960).

public goods. What we are going to specifically investigate is if the separability conditions, besides allowing branch separation, imply also a specific structure of taxation, namely declining a superiority of direct income taxation over indirect differential taxation. Indeed, we show that the two aspects, even in a second best environment, should be jointly considered, in order to design the optimal public finance government structure.

The plan of the paper is as follows. In section 2, we conduct a overview of the topic in the original pure efficiency context. Section 3 formally replicates the results of the overview with some propositions (the main proofs of which are in the Appendix) we may obtain within the one-consumer optimal commodity taxation framework. In section 4, we formalize Musgrave's contribution to the topic in an economy with different individual and equity concerns. In section 5, we present some almost definitive propositions on branch separation desirability along the most recent second best results. Section 5 concludes.

2 From the 'core' of traditional public finance to the optimal taxation theory

Public finance, as an economic discipline, is mainly concerned with organizing public intervention, throughout public expenditure allocation and tax burden distribution. In this respect, it is quite important to ascertain the convenience of decentralizing the various activities and functions in somewhat separated branches. In other words, given the total public goods budget, provided by a stabilization branch aiming at aggregate demand control, is it possible to determine the optimum allocation of expenditure between the various public goods independently of the magnitude of the particular taxes to finance them? In this case, the task of the public goods allocation branch is greatly simplified: it takes its budget and 'all' it has to do is to find out the optimal rule for distributing it among various public goods. Also the task of the collecting taxes ranch is simplified, as it should 'only' decide what goods and services to tax and at what rates. As a consequence, a relevant issue is to ascertain if separation and decentralization would imply a set of analytical conditions which define also the typology of optimal taxation.

The literature usually attributes to the great Italian economist Maffeo Pantaleoni one of the first attempts to assess in economic terms the issue of budget determination. From Pantaleoni (1883) we may extract the following three summarizing statements[3]

Pantaleoni statements 1. *Between total revenue and total expenditure there is a relationship of reciprocal influence as, on one hand, the total level of tax revenue is not fixed and independent from the public expenditure entity, and, on the other, the latter is not fixed and independent from the former.* 2. *Whatever the final budget allocation, it must always be considered within the*

[3]For a thorough analysis of Pantaleoni contribution to this issue, see Fossati (2003).

mind of the legislator that the marginal utilities of all the expenditure items are paired off equally. 3. *As the Parliament wants to obtain the maximum of welfare by weighing the desires of citizens with the available resources of the country, being able to appreciate the former as well as the latter, it cannot approve or not approve an expenditure if not on the basis of a judgment coming from a set of elements and propositions consisting, on one hand, of an order of decreasing marginal utilities from the various possible expenditures and, on the other, of a weighing of the marginal utilities of each expenditure item against the marginal cost to obtain it.*

The first statement establishes an unavoidable interdependency of decisions in terms of expenditure and revenues. The second one states the positive (i.e. in terms of political economy) meaning of the budget allocation, asserting the specific role of the legislator pay-off. The third one tries to establish a *marginalist rule* the Parliament should follow for optimally allocating the resources. Note that this *Pantaleoni rule*, being based on the Parliament mind, does not coincide with the pure economical and individualistic explanation of public finance decisions followed by other Italian tradition scholars such as Ricca-Salerno (1888), Mazzola (1890) and Graziani (1897). For the latter, for example, the problem of distribution of fiscal burden is to apply at individual level, the principle of *Equal absolute sacrifice* as it 'lies in determining how to organize the tax system in order to guarantee to everybody the maximum of relative utility and the equality of total utility of income paid by the tax-payers' (Graziani 1897, p. 276). Also Pigou (1927, pp. 31-32) and Dalton (1936, Ch.2) propose analogous rules of equalizing marginal benefits of public outlays for the representative household[4].

According to Musgrave, Pantaleoni's statements supply the first general approach to the theory of budget allocation. 'At hearth of this new approach was the recognition that the tax and expenditure sides of the budget must be determined in a simultaneous process and that the satisfaction of social wants must be traced to the preferences of the individual member of the group' (Musgrave, 1959, p.70).

However, this approach suffers from a contradiction that arises once we realize that the first statement and the two others cannot be linked and satisfied together. Pantaleoni himself seems aware of these difficulties when he states'....such problems which theoretically would be of the utmost difficulty....are quite well solved by means of a series of administrative devices based on experience'[5]. In order to explain the nature of these difficulties we have to realize that the Pantaleoni marginalist rule can be formally derived by the

Samuelson conventional rule *For the efficient supply of a public good, the marginal rate of substitution between the public good and the numeraire for*

[4] According to Musgrave (1959, p.113), Pigou elaborated this idea extending a principle suggested by the German economist Schaffle in 1888, of course without any reference to Pantaleoni.

[5] This quotation is in Fossati (2003).

the representative consumer, or the sum of all the consumers if differentiated, must be equal to the marginal cost.

Indeed, the conventional rule leads to an allocation rule which generalizes and rationalizes that one intuitively proposed by Pantaleoni for the majority of Parliament. This result will be formally proved in section 3 below. However, the conventional rule substantially avoids the problem of interaction between revenues and expenditures underlined in the first statement, as it admits an actually unfeasible lump sum tax-price per unit of public good given by the marginal rate of substitution itself[6].

The literature on optimal taxation (OT) theory, since 1970s, has considered the problems for budget allocation coming from the funding of public expenditure with distortionary taxation, looking at a second best situation, where the taxation, although distortionary, is chosen optimally, according to Ramsey rules[7]. If we compare these modern formulations with the conventional Samuelsian rule, we derive that the latter must be revised in order to take into account the distortion caused by the way of financing public goods provision. In particular, the marginal rate of susbstitution must be equal to marginal rate of *economic* transformation (Stiglitz and Dasgupta, 1971), where the latter is given by the marginal rate of *physical* transformation, corrected with the excess burden, and the additional yield due to the change on consumption goods and productive factors demand. This result, formally derived in section 3 below, is a clear proof of another meaningful, although only intuitive, traditional public finance statement, due to Pigou (1947).

Pigou conjecture *Where there is indirect damage, it ought to be added to the direct loss of satisfaction involved in the withdrawal of the marginal unit of resources by taxation, before this is balanced against the satisfaction yielded by the marginal expenditure. It follows that, in general, expenditure ought to be carried out so far as to make the real yield of the marginal unit of resource expended by the government equal to the real yields of the last unit left in the hands of the representative citizen.*

Hence, with distortionary taxation, the marginal cost of one euro devoted to financing public expenditure, the $MCPF$, is in general greater than one.This implies also that the interdependence between the absolute and relative levels of public goods and the structure of taxation arises in the terms conjectured by the first Panteleoni statement. As a consequence, however, it is impossible to allocate the public expenditure according to the two further statements, in particular by *equalizing the weighed marginal utilities of each expenditure item against the marginal cost to obtain it*, as the legislator must foresee all the terms contained in the marginal rate of economic transformation.

[6] Pantaleoni (1883) himself was aware of this unfeasibilty when he asserted '....the impossibility to apply a system where each expenditure item is financed by a specific tax-price'.

[7] See the recent survey by Dahlby (2008).

A rigorous way of overcoming this problem, and so linking together the Pantaleoni statements, has been provided, since the first OT contributions, by Lau, Sheshinsky and Stiglitz (1978)[8]. At the conditions they found we can derive not only that all Pantaleoni statements link together but also that the intuition of another great pioneer of public finance theory, Richard Musgrave, who conjectured a subdivision of public administration in branches[9], is confirmed.

Musgrave branches model *The decisions regarding taxation and public expenditure could taken by distinct governmental agencies: The first one (Stabilization branch) fixes the amount that can be devoted to financing public expenditure; the second one (Allocation branch) allocates that budget among the expenditure items following the conventional Samuelsonian rule; while the third one (Distribution branch) decides the structure of taxation according to the welfarist ability to pay approach*[10]*.*

An interesting question refers to the specific structure of taxation implied by the Lau-Sheshinsky-Stiglitz conditions in terms of restriction of preferences. Indeed, we will show later that these conditions imply the superiority of direct income taxation over indirect commodity taxation. This result derives from elaborating and extending the famous Enrico Barone contribution which appeared in the first years of the last century (Barone 1912). It is known as:

Barone theorem *In a general equilibrium model of deadweight loss, it turns out that, at the same fiscal yield, the loss of utility of Tizio (the representative consumer) is higher with the indirect tax than with an indirect tax.*

This is considered to be the first modern approach to the excess burden issue (Steve, 1976), based on new welfare economics [11]. Indeed, Barone, by going over the traditional consumer surplus approach and applying the indifference curves and budget lines apparatus, shows that a direct tax leaves the consumer on a lower indifference curve than before the tax, but higher than an excise tax can reach at the same yield. The topic of the excess burden of a consumption tax was one of the most highly considered and studied in the Italian tradition, since Pantaleoni (1910) himself, who instead sustained the equal burden thesis. Barone (1912) actually intervened just to confute this conclusion. According to

[8] Of course these authors completely ignored the link between their analytical proofs and the intuitive statements by Pantaleoni (1883).

[9] On the actuality of this conjecture see Musgrave (2008).

[10] Notice how this way of decomposing efficiency and equity effects in the evaluation of government policy is parallel to the one known as the Hicks-Kaldor 'compensation criteria':This was motivated as a way of eschewing equity judgments that rely on much stronger value judgements than the Pareto principle on which efficiency is based. The attempt was proven to be elusive for various reasons, which Musgrave overcome by adopting the welfarist context implicit in the ability to pay approach.

[11] The Barone contribution remained unknown for a long time by anglosaxon literature as it was discovered only by Joseph (1939). The literature on excess burden is voluminous. For a classical survey see Auerbach and Hines (2002) and for a heterodox and challenging point of view see Lind and Granqvist (2010).

the complete survey by Scotto (1947), we may at least quote the contributions discussing and extending Barone theorem by Borgatta (1921), Fasiani (1930) and Ricci (1938). The main criticism by these scholars was that actually the Barone statement is true when the supply of labour is fixed and the income tax has a lump sum nature. Musgrave himself, in the textbook, affirms that with variable labour supply '...there is no general way of telling the weight of the excess burden that results from each of the two types of interference under either tax; and there is no general way of telling which of the two taxes is superior' (Musgrave, 1959, p. 151).

As we'll see in section 3, it was a task of OT to find this way and we'll show how this is linked with the Samuelson-Musgrave multiple budget determination theory. Further, the most recent OT literature can also extend this somewhat surprising historical link from a pure efficiency representative-agent Ramsey (1927) context (a setup where Pantaleoni, Barone and Pigou were working), to a framework where also equity concerns arise, i.e. where households differ in ability and needs (a setup where both Samuelson and Musgrave were working) (section 5). Fig. 1 summarizes the historical evolution of this topic.

$$Fig.1, \text{here}$$

3 The efficiency of branch division for collecting taxes and allocating public expenditures

Let us describe the basic model in a pure efficiency context where there is a representative household, or what Pantaleoni means as the 'Parliament mind'. Let $\mathbf{x} = (x_1, x_2, ...x_n)$ be a private consumption vector and $\mathbf{z} = (z_1, z_2, ..., z_u)$ a public one. We follow a linear general equilibrium model where production prices $\mathbf{p}$ (marginal costs) are fixed for all private goods, as well as the wage rate w. $\mathbf{c}$ is the vector of public goods marginal costs, l is labour supply and $x_0 = 1-l$ is leisure. Hence, consumer preferences are represented by the following direct utility function

$$U = U(\mathbf{x}, l, \mathbf{z}), U_{\mathbf{x}} > \mathbf{0}, U_l < 0, U_{\mathbf{z}} > \mathbf{0}. \tag{1}$$

The consumer budget constraint is

$$\mathbf{qx} = y(1-\tau) \equiv m(y), \mathbf{q} - \mathbf{p} = \mathbf{t},\ y \equiv wl \tag{2}$$

Maximizing (1) s.t. (2) we get the following indirect utility function

$$V = V(\mathbf{q}, \mathbf{z}, w(1-\tau), m(y)). \tag{3}$$

Let us start with the following

Definiton.1 *A first best one-consumer setup is one where supply of labour is fixed*: $l = l_0, y = y_0$, $m_0 \equiv (1-\tau)y_0$ *and then* $V(\mathbf{q}, \mathbf{z}, m_0)$.*Therefore, the government budget constraint is*

$$\mathbf{tx}(\mathbf{q}, \mathbf{z}, m_0) + \tau y_0 = \mathbf{cz}.$$

Then we may state the following

Proposition 1 *In the representative household setup, first best allocation conditions are as follows: (i)* $\frac{V_{z_j}}{c_j} = \frac{V_{z_k}}{c_k}$, $j, k = 1, ...u$ *(ii)* $\tau^* > 0$, $\mathbf{t}^* = \mathbf{0}$, *(iii)* $MCPF = 1$.

Proof. See Appendix

Thus, if the income tax has lump sum nature, the optimal distribution of public expenditure is given by equalizing the weighted marginal utilities of all public goods, and indirect taxation is superfluous. So both Pantaleoni's marginalist allocation rule and Barone's theorem hold. Notice that the configuration $\tau^* > 0$, $\mathbf{t}^* = \mathbf{0}$ is equivalent to a structure of uniform ad valorem tax rates: $t_i^* = \zeta q_i, i = 1..n$, with leisure, the numeraire, untaxed. Indeed, by combining consumer and government budget constraints, we have $T = \sum_i t_i^* x_i = \zeta \sum_i q_i x_i = \zeta y_0$, where $\zeta = \tau^*$. As far as the Pigou conjecture is concerned, without tax distortion, the cost of taxation, the Lagrangian multiplier attached to the government budget constraint λ, is equal to the marginal utility of income, V_m, hence the marginal cost of public funds is equal to one, $MCPF \equiv \frac{\lambda}{V_m} = 1$. This also implies the Samuelson conventional rule according to which the marginal rate of subsitution between public good j and the numeraire is equal to the marginal cost of producing it,

$$\frac{V_{z_j}}{V_m} \equiv MRS_j = c_j, j = 1, ..u. \tag{4}$$

Clearly, if Samuelson rule applies, so does Pantaleoni marginalist rule, although, as we'll see later on, the reverse is not true. Now let us consider the following

Definiton 2. *A general second best Ramsey OT setup is one where the supply of labour of the representative household is variable*, $l(\mathbf{q}, \mathbf{z}, w(1-\tau), m(y))$, *and the structure of demand of consumption goods is* $\mathbf{x}(\mathbf{q}, \mathbf{z}, w(1-\tau), m(y))$. *Therefore, the government budget constraint is*

$$\mathbf{tx}(\mathbf{q}, \mathbf{z}, w(1-\tau), m(y)) + \tau w l(\mathbf{q}, \mathbf{z}, w(1-\tau), m(y)) = \mathbf{cz}.$$

Then we have

Proposition 2 *In a general Ramsey OT setup, the Samuelson conventional rule does not hold, as the Pigou conjecture is confirmed. Further, neither the Pantaleoni marginalist allocation rule or Barone the theorem apply.*

Proof. See Appendix .

In the proof, firstly, we, derive the well known Atkinson and Stern (1974) result according to which, by financing public goods with distortionary taxes, the marginal cost of public funds is above 1, if most taxed commodities are normal goods. Secondly, we show that, if taxed commodities are generally complementary to the public goods (primary education vs. demand for textbooks), the revenue from private goods taxation, $T = \mathbf{tx}$, increases with public goods provision, $\frac{\partial T}{\partial \mathbf{z}} > 0$ [12]. Hence, by elaborating the f.o.c. (15), we have the modified one-consumer Samuelson rule:

$$\begin{aligned} MRS_j &= \Omega_j c_j, j = 1, ..u, \text{where} \\ \Omega_j &\equiv MCPF(1 - \frac{\varepsilon_{Tz_j}}{c_j z_j / T}), \end{aligned} \tag{5}$$

The term Ω_j is changing with j, and it is positive if $MCPF > 1$ and if the elasticity of revenue T w.r.t. the public good j, ε_{Tzj} is positive, but lower than the share of expenditure in j over the total revenue. The term ε_{Tz_j} describes the interdependence between optimal allocation of resources between public expenditure branch and taxation branch, as predicted in the first Pantaleoni statement. In this respect, note that if $\varepsilon_{Tz_j} = 0$, i.e if the utility function is additive separable, $U = U_1(\mathbf{x}, l) + U_2(\mathbf{z})$, condition (5) becomes,

$$MRS_j = MCPFc_j, j = 1, ...u. \tag{6}$$

Thus Samuelson conventional rule works only with proportionality. But this is how much it serves for the Pantaleoni marginalist allocation rule to hold, even if the Pigou conjecture is confirmed according to the Atkinson-Stern result. Therefore, we may summarize the result as follows

Corollary 1 *In a general Ramsey OT setup, the Pantaleoni marginalist allocation rule is not prevented by tax distortion, which modifies the Samuelson rule, but by the relationships (substitutability or complementarity) between public and private consumption goods.*

Now we introduce the following separable functional form for the utility of the representative consumer

$$U = U(F(\mathbf{x}, \mathbf{z}), l) \text{ where } F(.) \text{ is homothetic.} \tag{7}$$

[12] To qualify this rusult see Wildasin (1979, 1984), Tsunecki (2002) and Batina and Ihori (2005). The most general formulation, in mathematical terms, of Proposition 2 is provided by Guesnerie (1995). Notice that the Pigou conjecture is confirmed in the case of income normalization (leisure numeraire) and not in all other possible nomalizations (Jacobs, 2010).

In household equilibrium, we have

$$\begin{aligned} l(w(1-\tau)) &= \left\{ Arg \max_{l} U(F(\mathbf{x}, \mathbf{z}), l) \; s.t. \; w(1-\tau)l = m \right\}, \\ \mathbf{x}(\mathbf{q}, \mathbf{z}, m) &= \left\{ Arg \max_{x} F(\mathbf{x}, \mathbf{z}) \; s.t. \mathbf{qx} = m \equiv w(1-\tau)l^* \right\}. \end{aligned}$$

Thus, the household firstly decides how much it has to work and the expenditure dedicated to buying other commodities, secondly he divides this expenditure among the various consumption goods. Consequently, any changes in income tax rate τ gives rise only to an income effect on demand of consumption goods, and hometheticity implies that all consumption goods demands have unitary income elasticity[13]. Separability also implies that the marginal rate of substitution between each public good is independent of the choice of hours of work, as $\frac{\frac{\partial V}{\partial z_j}}{\frac{\partial V}{\partial z_k}} = \frac{\frac{\partial F}{\partial z_j}}{\frac{\partial F}{\partial z_k}}$. Hence we have the following

Proposition 3 *In he general Ramsey OT setup, if the representative household utility function is (7), first best conditions (i) and (ii) of Proposition 1 still apply. So both the Pantaleoni marginalist allocation rule and the Barone theorem apply in this second best framework.*

Proof. See Lau, Sheshinsky and Stiglitz (1978), Sandmo (1976), Deaton (1979), Besley and Jewit (1991).

Thus separability (and hometheticity) extends the first best result to a second best one, where Pantaleoni and Barone theorems still apply, even if lump sum taxes are not feasible. In this respect, note how Pantaleoni (1883) statements implicitly admit separability in parliament preference between private and public goods. Further, the proof of superiority of income tax over indirect taxes supplied by Barone (1912), uses separability between consumption goods and labour, as in his graphic analysis the choice is made in consumption bundle space taking leisure aside (Musgrave 1959, p.Steve 1976, p. 273).

However, (7) is only a sufficient condition and it is not, for sure, the only one for fulfilling the pioneers' statements. The following Proposition establishes a perfect link between the Pantaleoni public expenditure marginalist allocation rule and Barone superiority of income tax in second best context, with also a peculiar application of Pigou conjecture.

Proposition 4 *In the general Ramsey OT setup, the Pantaleoni marginalist allocation rule and the Barone theorem apply simultaneously iff* $V_{\mathbf{q}} \propto \frac{\partial l}{\partial \mathbf{q}}$ *and* $V_{\mathbf{z}} \propto \frac{\partial l}{\partial \mathbf{z}}$[14]. *In this case, the Pigou conjecture is confirmed, as* $MCPF > 1$ *if leisure is a normal good but with a limited income effect.*

[13] The hypothesis rules out cases such as taxing movie tickets or swimsuits, relative to other goods, making leisure relatively less attractive. For a discussion of practical significance of non-separability see Kaplow (2010, p.138) and Diamond and Saetz (2011).

[14] We use $\propto$ as the proportionality symbol.

Proof. See Appendix

Thus in a linear model with identical individuals, indirect taxation can be superfluous (i.e. uniform commodity taxation is optimal), and branch decentralization can be efficient, even with the Pigou conjecture confirmed.

The necessary and sufficient conditions specified in Proposition 4 imply an interesting interpretation in optimal supply of public goods and optimal taxation, which we may consider useful to the allocation branch manager for implementing the optimum budget.

Proposition 5 *In the general Ramsey OT setup, when the Pantaleoni marginalist allocation rule and the Barone theorem apply simultaneously: (i) the first Pantaleoni statement on the interdepency applies in the sense that the change in supply of labour, and consequently the change in income tax yield, due to an increase of a public good provision, can approximate the marginal willingness to pay for it; (ii) all public goods must be complementary (substitute) with labour (leisure) and the higher (lower) the marginal willingness to pay, the higher is the degree of complementarity with labour (leisure).*

Proof. See Appendix

In conclusion, we have listed the conditions according to which the Pantaleoni and Barone statements can be rigorously realized in a Ramsey OT setup, with one decision-maker and then in a pure efficiency concern.

4 Public goods allocation and redistribution of income: modelling Musgrave's intuition

We now consider the possibility of theoretically supporting a branch division model, dealing with the rationale of evaluating policies from both efficiency and redistribution viewpoints. We start with a formalization of the original model described by Musgrave (1959, pp. 32-37), but informally anticipated in Musgrave (1956). In the subsequent section, we will link the latter model to the recent OT models with different individuals.

Let us suppose there are two administrative branches, directed by two different managers and/or ministries. The first branch (*Allocation or Service branch*) has the task of producing two public goods $z_u, u = 1, 2$ and providing them to 2 persons A and B, who have this individual separable additive utility function[15]

$$U_i(m_i, \mathbf{z}) = u(m_i) + \boldsymbol{\theta}_i \mathbf{z},$$

where $m_i \equiv y_i - T_i^a - T_i^d; i = A, B$. The parameters $\boldsymbol{\theta}_A = (\theta_{A1}, \theta_{A2}), \boldsymbol{\theta}_B = (\theta_{B1}, \theta_{B2})$ are individual marginal willingness to pay for the public goods. The

[15] Note this restriction on preferences, a special case of (7), is not necessary to support branch separation desirability. However, we admit it for simplifying the analysis and to be as near as possible to the original Musgrave (1959) presentation of the multiple budget determination.

second branch has the task of redistributing gross pre-tax incomes y_A, y_B for equity concerns (*Distribution branch*). $T_i^a, T_i^d, i = A, B$ are the taxes paid by the two households for financing respectively the allocation and distribution branches. In Musgrave's view there is also a *Stabilization branch* with the task of fixing the volume of public expenditure in order to control aggregate demand.

As far as the *Allocation branch* budget is concerned, the manager, in setting it, will take for granted a prevailing 'proper' state of distribution, an objective met by the redistribution branch manager. Thus, a Musgrave feasible allocation branch budget (G^0, T_A^{a0}, T_B^{a0}) satisfies the following constraints

$G \equiv \mathbf{cz} = T_A^a + T_B^a,$
$T_A^a = f_A(\boldsymbol{\theta}_A, \mathbf{z}) \leq y_A - T_A^d$
$T_B^a = f_B(\boldsymbol{\theta}_B, \mathbf{z}) \leq y_B - T_B^d$

$T_i^a, i = A, B$ are benefit-taxes (Hines, 2000), i.e. taxes depending on each individual preference on public goods along a pseudo-market equilibrium approach. Indeed, '...the function of taxation in this first best context (Allocation branch) is to place the cost of public want satisfaction with those whose wants are being satisfied' (Musgrave 1956, p. 335). Hence, if the allocation branch manager knows the parameters: $y_i, \theta_{iu}, f_i(.), T_i^d, i = A, B,\ u = 1, 2$, he can define the optimal budget G^*, T_A^{a*}, T_B^{a*} by applying the tax distribution

$$T_i^a = \sum_{u=1,2} \theta_{iu} z_u, i = A, B,$$

and the Lindahl-Samuelson rule

$$\theta_{Au} + \theta_{Bu} = c_u, u = 1, 2.$$

Accordingly, we have also the extended Pantaleoni-Lau, Shenshinsky and Stiglitz solution as follows:

$$\frac{\theta_{A1} + \theta_{B1}}{c_1} = \frac{\theta_{A2} + \theta_{B2}}{c_2}. \tag{8}$$

Note if $G \equiv \mathbf{cz} \leq G^0$ is the constraint on the budget to allocation policy imposed by the *Stabilization branch*, we have a corrected Lindahl-Samuelson rule in which the total marginal benefit is proportinal to the marginal cost: $\theta_{Au} + \theta_{Bu} = (1 + \lambda^S)c_u, u = 1, 2$; λ^S being the Lagrangean multiplier of the binding constraint i.e. the marginal benefit of relaxing the limit on public expenditure by the *Stabilization branch*. However, (8) is still satisfied.

Of course, Musgrave is aware of the information asymmetry and free riding problems coming from the unapplicability of the 'exclusion principle', essential to carry out the exchange[16]. So he is aware that, without a market mechanism, a political process of decision making should be substituted and enforced. However, the voting mechanism must be designed so as to approximate a true statement of preferences, and hence come as close as possible to that solution

[16] A recent extenstion of this Musgravian model to an asymmetric information, free riding, context is provided by Bierbrauer (2010).

which would be obtained if the exclusion principle and the forces of the market could be applied. 'As professor Samuelson put it, there is a solution, but the question is how to find it (Musgrave 1956, p. 335)'. As we have seen previously, the reference to the Parliament mind was the (simplified) solution suggested by Pantaleoni.

As far as the *Distribution branch* budget is concerned, it should be defined, given the benefit-taxes established by allocation branch, according to the lump sum taxes and transfers (T_A^{d0}, T_B^{d0}) satisfying the following constraints

$Y = y_A + y_B - G$

$m_A = y_A - T_A^d - T_A^{a0} = \gamma Y$

$m_B = y_B - T_B^d - T_B^{a0} = (1-\gamma)Y$

$T_A^d + T_B^d = 0$

γ is the redistribution coefficient that the distribution branch manager receives from the politicians[17]. Therefore, by knowing the original distribution of resources (income) y_A and y_B and the taxes imposed by the allocation branch, the redistributive taxes can be univocally determined according to the ability to pay approach of *Least total sacrifice* or *Equal marginal sacrifice*[18]. Analytically the optimal γ^*can be obtained by maximizing a utilitarian-egalitarian welfare function, $W = \sum_{i=A,B} u(m_i)$, whose f.o.c. imply equal marginal utilities of after-tax income for A and B. Given the same utility function, the solution is as follows

$$m_A = y_A - T_A^{a0} - T_A^d = y_B - T_B^{a0} - T_B^d = m_B, \text{and } \gamma^* = 1/2. \tag{9}$$

In conclusion, we may summarize the analytical recostrunction of Musgrave's intuition with

Proposition 6 *The Musgrave branch separation model with different individuals supports a perfect first best outcome with a Samuelson-Lindahl pseudo equilibrium and an equal distribution of after-tax income. Hence, Pantaleoni marginalist allocation rule and Barone theorem are extended to the case where equity is taken into account.*

The essence of separation of public sector activities in autonomous branches can be left to the words of Musgrave himself. '[....] If it is to be accepted as the principle by which the cost of public services should be allocated, the same reasoning points to a general rearrangement of the distribution of income, beyond the arbitrary limit set by the size of the Service budget. In other words, the ability-to-pay approach (quite apart from the intrinsic difficulties of utility measurement and comparison) deals with matters pertaining to the Distribution rather than the Service Branch. Multiple pricing in the context of the Service

[17] The idea of representing the desired redistribution pattern with a single parameter is an old one. However the first and most explicit conceptualization of this is due to Barone (1908) who called it the 'supplementary redistribution term', necessary for the Ministry of a socialist state to redistribute the surplus, without violating the efficient structure of shadow-competitive factor prices.

[18] The original formulation of the approach is in Carver (1904) and Edgeworth (1905).

Branch reflects differences in effective demands for the satisfaction of public wants, based on a given distribution of income. As such, it is not to be confused with distributional adjustments undertaken by the Distribution Branch.' (Musgrave 1956, p. 336).

5 The optimality of branch separation in the modern Second Best theory: efficiency vs. equity

Modern second best theory has provided many results around the topics linking the Pantaleoni, Barone and Musgrave contributions. The analyses generally consider a so-called *Mirrlees framerwork*, i.e. an economy with different consumers, indexed by an unobservable skill-wage parameter, n, with cumulative distribution $\Psi(n)$, who buy private goods $\mathbf{x}$, supply labour l and enjoy public goods $\mathbf{z}$. The production costs, $\mathbf{cz}$, are financed by a set of commodity taxes, at rates $\mathbf{t}$, and a non-linear income tax $T(y)$,where $y \equiv nl$. Let utility function be weak separable between leisure and consumption goods as follows:

$$U(l, F(\mathbf{x}, \mathbf{z}), n), all\ n. \tag{10}$$

First, we may mention the celebrated result by Atkinson and Stglitz (1976).

Atkinson-Stiglitz generalization of Barone theorem *Given a Mirrlees framework with individual utility function* (10)*, differential commodities taxes are not needed if an optimal non-linear income tax is used.*

This result has even been extended to the case where income taxation is not optimal by Laroque (2005) and Kaplow (2006a). In particular, both show that one can always move to a system of non-distorting taxes coupled with an appropriate modification of the earned income tax and generate more government revenue while leaving every consumer with the same utility and the same labour supply. Indeed, by replacing the differential commodity taxes with a system of uniform ones and, at the same time, adjusting the income tax system so that all individual utilities levels are the same as before the tax change, individual labour supplies, given separability, will be unaffected. Moreover, the amount of revenue raised by the government will rise, with the rise reflecting the utiltiy gain to consumers from eliminating the commodity tax distortion.

As far as the conventional rule for efficient supply of public goods is concerned, we may consider the contribution from Boadway and Keen (1993), which we may represent as [19]

Boadway and Keen generalization of Samuelson rule *The conventional rule still applies in second best, with optimal non-linear income tax, if*

[19] See also Christiansen (1981).

public goods are complementary with leisure as they are private goods. A sufficient, although not necessary, condition of equi-complementarity is weak separability as in (10).

A unified and extended version of these models has been proposed by Kaplow (2010), analyzing the general issue of financing a public project or a policy reform. We may summarize his main argument as follows

Kaplow policy evaluation procedure *In a Mirrlees framework it is possible to carry out a two-step decomposition of a policy reform: (i) in the first one, identifying efficiency gains with hypothetical lump sum redistribution used for sterilizing any redistribution effects of the reform ('the distribution neutral income tax adjustment'); (ii) in the second one, evaluating, using a social welfare function approach, the redistributive effects of the reform ('the actual income tax ajustment associated with the policy').*

In other words, in the first step the reform is implemented, accompanied by a lump-sum adjustment in income tax liabilities that makes the policy both revenue-neutral and distribution neutral. This step involves a Pareto improvement and we may say that it can be carried out by the Musgravian allocation branch. In the second step, the income tax is changed so that the overall policy reform is replicated, including the tax change needed to finance the reform. This redistribution must be judged on equity grounds, say by the distribution branch, even if income tax is not optimal. However, only if income tax is actually optimized in the second step, the Samuelson condition leads to a social optimum (Boadway 2010). For our aims this can be summarized by the following:

Proposition 7 *Let us evaluate, in a Mirrlees framework, a project to produce a public good. According to the Kaplow two-step decomposition, with a separable utility function (10), the public good is produced if the Samuelson rule is satisfied, and only income tax is the needed instrument for financing the project.*

Proof. See Kaplow (2010, ch.6 and ch.8) .

The main consequence of Proposition 7 is that, '....although the distributive incidence of government expenditures is presumptively important, it may not have the effects on the analysis of optimal government provision of goods and services.[...].that one might have initially supposed' (Kaplow 2010, p.185): optimal provision is determined by the simple cost-benefit test, deriving from the Samuleson rule, and the optimal income redistribution problem is essentially unaffected. For the aims of this work, this implies the following:

Corollary 2 *From Proposition 7, it turns out that the Musgrave branch model and Pantaleoni marginal allocation rule apply, and Barone theorem is extended to a Mirrlees framework along the Kaplow two-steps procedure.*

Boadway (2010, p. 975) suggests this pertinent criticism on Kaplow's two-step decomposition: '...If one is ultimately prepared to adopt a social welfare function in the second step (of *Kaplow procedure*) to evaluate the redistribution effects of policy changes, it seems unnecessary to go through the two step procedure in the first place'. One way to overcome this criticism is by referring to the more general approach suggested by Gauthier and Laroque (2009). From their propositions, we are able to extend further the previous analyses concerning Barone's theorem on superfluous indirect taxation and the Musgrave separation branch model, as the two authors can use a more general approach than the Kaplow one, based on a marginal change and first order conditions.

In Gauthier-Laroque economy private goods and public goods are produced by labour and private inputs bought by the system of firms. If we apply their general approach to a simpler linear technology we have as *aggregate resource constraint* the following

$$\int_n \mathbf{p}\mathbf{x}_n d\Psi(n) + \mathbf{c}\mathbf{z} \leq \int_{\mathbf{n}} y_n d\Psi(n), \tag{11}$$

i.e. the total cost of producing private and public goods consumed by households must not be greater than the total income generated in the economy by supplying labour. In a first best setup, extra resources in private goods lead to a Pareto improvement, then an allocation that satisfies this property can be said to be *non-satiated*. In a second best situation, the government has more constraints, e.g. incentive compatibility (*second best constraints*), and an allocation satisfying them is a *constrained allocation*. We then have the subsequent:

Definition 3 *A feasible constrained allocation is non-satiated when an increase in the aggregate resources constraint (in our special case an increase in available income in the r.h.s of constraint (11)) allows a Pareto improvement while satisfying the second best incentives constraints.*

Therefore, we may summarize Gauthier-Laroque's contribution with the following:

Proposition 8 *Consider a non-satiated second best allocation, with a separable utility function (10): (i) indirect taxes are superfluous because this allocation is dominated by another non-satiated constrained allocation in which the consumption of each individual is a first best allocation; (ii) the provision of each public good satisfies the Samuelson Rule and the allocation rule is unaffected by the redistribution concerns of the government.*

Proof. See Lemma 1 and Lemma 3 by Gauthier and Laroque (2009)

The proof gives emphasis to non satiation and refers to an abstract general equilibrium type of argument based on the second welfare economics framework. To prove the first part (*Barone argument*), Gauthier -Laroque consider a reference incentive-compatible second best allocation, without public goods, for agent n, $(y_n^*, \mathbf{x}_n^*)$ with distorting indirect taxes. $V_n^* = V(\mathbf{x}_n^*)$ is the sub-utility derived from consumption and separability allows this utility to keep constant while suppressing indirect taxes, by adapting the non-linear after tax income schedule $R(y)$. When the indirect taxes are distortive this transformation yields a surplus. Therefore, any non-satiated second best allocation has non-distortive commodity taxes and applying first best rules would yield a Pareto improvement on constrained inefficient allocations. Also for the second part (*Pantaleoni and Musgrave argument*) they show that a first best allocation, with only a consumption good x, the numeraire, is attainable with separability and homogeneity of preference; i.e. with a sub-utility profile $V_n = V(R^*(y^*), \mathbf{z})$ associated with the second best allocation. As a consequence, when $V(.)$ is differentiable, the provision of each public good satisfies the conventional Samuelson rule in this continous form:

$$\int_n \frac{\partial V/\partial z_j}{\partial V/\partial x} d\Psi(n) = c_j, \; j = 1, ...u. \tag{12}$$

Gauthier-Laroque can also show the validity of their proof of superfluous indirect taxes also if preferences are heterogeneous[20]. Two conditions are required for this extension to be valid, i.e. that the differences in the agent's preferences can be observable by the government, as for instance in family size, and that the income tax is allowed to depend on these differences.

Consequently, nonsatiation and separability properties, and the possibility to apply a quite general non-linear income tax allow Pantaleoni, Barone and Musgrave propositions on the organization of public sector to be proven even in an asymmetric information second best context.

As far as the Pigou conjecture is concerned, a specific stream of literature following Sandmo (1998) has ascertained that the level of the $MCPF$ is not necessarily greater than one, when efficiency as well as equity concerns are taken into account (Dahlby 2008). Indeed, we have this quite general statement, recently proposed by Jacobs (2010).

Proposition 9 *In a Mirrlees framework, the marginal cost of public funds under optimal non-linear income taxation is always equal to one*

Proof. See Jacobs (2010)

The intuition of this apparently surprising result relies on the fact that in the presence of a non-linear income tax '...reliance on Ramsey principles is misplaced......' and '...results derived in the original Ramsey framework fail to provide a proper guidance' (Kaplow, 2010, pp. 147-148). More precisely, a non-linear income tax does not deliver a direct measure for the $MCPF$. To determine this, the welfare effect can be obtained with reference to a unit increase in the intercept of the tax function, i.e. a poll tax or a no tax area equal for all tax-payer types, $-T(0)$. At the optimum, the specific $MCPF$ for all taxes

[20]For an empirical viewpoint rejecting the Atkinson-Stiglitz theorem, given heterogeneity, see Diamond and Saez (2011).

must be equalized. Since the manoeuvre is equivalent to a non-distortionary non-individualized lump sum tax, at the optimum, the $MCPF$ for all other tax rates must be equal to the marginal cost of $-T(0)$. Thus, at each point in the income distribution, tax distortions should be equal to distributional gains for all marginal tax rates. Hence the marginal cost of public funds must be equal to one.

Note that, while the Barone theorem can be proved, following Kaplow (2006a) and Laroque (2005), even when income tax is not optimal, to have $MCPF = 1$, and then rejecting Pigou conjecture, we need, following Jacobs (2010), an optimal income tax schedule.

6 Concluding remarks

In this paper we have discussed the following issue, from a historical perspective: is it possible and useful to decentralize the activities and the functions of a government in agencies or branches devoted to respectively collecting taxes, on the basis of some ability to pay principle (*distribution branch*), and to allocating a given budget in providing the various public goods, on the basis of their relative utilities (*allocation branch*)? And moreover, is the income tax the only fiscal instrument compatible with this multiple budget determination rule?

This involves the old and glorious topic in public finance theory running from the Italian tradition at the end of 19th century, namely through the contribution from Maffeo Pantaleoni and Enrico Barone, to the fundamental Musgrave's intuition in the mid of the 20th century which explicitly raised the conjecture of branch division. Musgrave gave positive answers to the two questions as a corollary of another keystone of Public finance theory: the Samuelson conventional rule of efficient supply of public goods, which, however, remains valid in a first best environment.

In a second best environment, modern optimal taxation theory, since the 1970s until today, has shown as follows. If government is constrained to apply only linear commodity taxes and it is only concerned with efficiency aims, the answer is still affermative if the preferences of the representative household are homothetic and separable between commodities and leisure. Also the Pigou conjecture according to which the marginal cost of one euro of public expenditure obtained via taxation is in general greater than one, is related to this issue. When households are instead differentiated according to a skill parameter, as in the *Mirrlees framework*, the answer to the question of the optimality of the separation is again affirmative if preferences are separable and homogenous among people and the government can apply a general unconstrained nonlinear income tax. In particular, Kaplow (2010) two-step procedure, with the distribution-neutral (offsetting) income tax adjustment acting as a sort of benefit taxation, supplies the nearest istitutional solution in second best to the Musgrave (1959) multiple budget determination theory. However, in this case, the Pigou conjecture loses its original meaning as the marginal cost of public funds must simultaneously take into account efficiency and equity concerns.

The circumstances under which, in a more realistic second best, first best conditions still apply are rather restrictive. So we may conclude that the division between allocation and distribution branches can be only an *approximate* normative guide for organizing the public intervention. And the degree of this approximation is largely dependent on the information set actually available to the government, exactly what distinguishes a first best from a second best economic environment. Indeed, the solution provided by Kaplow, Larouque, Jacobs and others relies on imagining the availability of a 'slightly' distorsive income tax, very similar to a non-distorsive lump sum tax.

We may reasonably guess that the dimension of the information set available to the government will go on widening in the future with the development of techniques and tools of people selection. Thus, in the same perspective, we may expect that the Musgravian multiple budget determination, together with a fine tuning application of income taxation, will see its coherence and applicability increasing also in the real world. However, if we move from a normative to a positive approach to public finance and we look at the evolution of fiscal systems we find that another story has been prevailing, in some sense contrary to the normative prescription of the Barone theorem. After having reached a relevant weight and a high level of sophistication in industrialized countries, income tax is now losing its predominant role against the increasing weight of differential indirect taxation. Indeed the actual progressive income taxation is, around the world, very far from having the flexibility and the quasi-lump sum tax nature desired by the theory presented in this work, thus we mayl say that its superiority over indirect taxes is not widely accepted and even rejected (Diamond and Saez 2011).

7 Appendix

Proof of Proposition 1. The solution of the FB problem is given by finding $\mathbf{t}$, $\mathbf{z}$ and τ in order to

$$Max\ L = V(\mathbf{q}, \mathbf{z}, m_0) + \lambda[\mathbf{tx}(\mathbf{q}, \mathbf{z}, m_0) + \tau y_0 - \mathbf{cz}].$$

The f.o.c.s are: $(\mathbf{z})$ $V_{\mathbf{z}} - \lambda \mathbf{c} + \lambda \frac{\partial(\mathbf{tx})}{\partial \mathbf{z}} = \mathbf{0}$; (τ) $-V_m y_0 + \lambda y_0 = 0$; $(\mathbf{t})$ $V_{\mathbf{q}} + \boldsymbol{\lambda} \frac{\partial(\mathbf{tx})}{\partial \mathbf{q}} = \mathbf{0}$.

From the second condition it obtains $V_m = \lambda$, and from the third one, given Roy identity, $V_{\mathbf{q}}/V_m = -\mathbf{x}$, it obtains the OT formula asserting that the marginal cost of public funds raised by each tax is equal for all commodities and equal to one :

$$MCPF_i \equiv \frac{x_i}{\frac{\partial(\mathbf{tx})}{\partial q_i}} = \frac{\lambda}{V_m} \equiv MCPF = 1, i = 1, ...n. \tag{13}$$

Then (i) $\mathbf{x} = \mathbf{x} + \mathbf{t}\frac{\partial \mathbf{x}}{\partial \mathbf{q}} => \mathbf{t}^* = \mathbf{0}$, as $\frac{\partial \mathbf{x}}{\partial \mathbf{q}} \neq \mathbf{0}$, and $\mathbf{cz} = \tau y_0$ and (ii) from the f.o.c for $\mathbf{z}$, $\frac{V_{\mathbf{z}}}{V_m} = \mathbf{c}$.□

Proof of Proposition 3. In the SB framework, where $\mathbf{t}^* \geq \mathbf{0}$, $\tau = 0$, given the normalization $p_0 = q_0, t_0 = 0$, the OT formula becomes as follows (Hindriks and Myles 2006, p. 458):

$$\frac{\Delta x_k^c}{x_k^c} \simeq \frac{\sum_i t_i S_{ki}}{x_k} = -\Gamma \equiv -[1 - \frac{1}{MCPF} - \sum_i t_i \frac{\partial x_i}{\partial m}] < 0, k = 1, ...n. \quad (14)$$

The condition is the celebrated Ramsey rule, according to which the *discouragement index*, given by the compensated demand percentage reduction due taxation[21], is constant for each good. The latter implies that, in general, the optimal tax rates are differentiated (not uniform). Moreover, as $\Gamma > 0$, because of the need to get revenues, if $\sum_i t_i \frac{\partial x_i}{\partial m} > 0$ it is $MCPF > 1$ (Atkinson and Stern 1974). Hence, the optimal provision of public goods condition runs as follows

$$\frac{V_{\mathbf{z}}}{V_m} = -MCPF[\frac{\partial(\mathbf{tx})}{\partial \mathbf{z}} - \mathbf{c}] \neq \mathbf{c}. \quad (15)$$

Thus neither the Pantaleoni nor the Barone rules is satisfied as long as $MCPF1 > 1$, and $\frac{\partial(\mathbf{tx})}{\partial \mathbf{z}} \neq \mathbf{0}$ □

Proof of Proposition 4. Let us start with the necessity part. Barone is equivalent to sayig that $t_i = \zeta q_i$, $i = 1, ..n, t_0 = 0$, are optimal tax rates. Sadka (1977) has shown that this occurs, i.e. Ramsey rule (14) is satisfied, iff all consumption goods are equally substitute with leisure (commodity 0). Then $S_{0i} = \beta x_i$, where S_{0i} is the Slutsky term of the leisure w.r.t. commodity i, and $\beta > 0$ is independent from i. Then, from Slutsky equation, we have $\frac{\partial x_0}{\partial q_i} = (\beta - \frac{\partial x_0}{\partial m})x_i$, i.e. $\frac{\partial x_0}{\partial \mathbf{q}} = \upsilon \mathbf{x}$. By Roy's identity $\frac{V_{\mathbf{q}}}{V_m} = -\mathbf{x}$, so $\frac{\partial x_0}{\partial \mathbf{q}} = -\upsilon(V_{\mathbf{q}}/V_m)$, and then

$$V_{\mathbf{q}} = \frac{V_m}{\upsilon} \frac{\partial l}{\partial \mathbf{q}} \propto \frac{\partial l}{\partial \mathbf{q}}. \quad (16)$$

Now, if (i) of Proposition 1 (Pantaleoni) holds, it must be $V_{\mathbf{z}} = \mu \mathbf{c}$. Moreover, the optimum public goods provision condition $V_{\mathbf{z}} - \lambda(\mathbf{c} + \frac{\partial(\mathbf{tx})}{\partial \mathbf{z}}) = \mathbf{0}$, by substituting $t_i = \zeta q_i$, $i = 1, ..n$, and $V_{\mathbf{z}}/\mu = \mathbf{c}$, becomes $V_{\mathbf{z}} = \lambda(\zeta \mathbf{q}\frac{\partial \mathbf{x}}{\partial \mathbf{z}} - V_{\mathbf{z}}/\mu)$. By differentiating the consumer budget constraint with respect to $\mathbf{z}$, we have $\mathbf{q}\frac{\partial \mathbf{x}}{\partial \mathbf{z}} = -\frac{\partial x_0}{\partial \mathbf{z}}$, thus $V_{\mathbf{z}} = -\lambda(\zeta \frac{\partial x_0}{\partial \mathbf{z}} + V_{\mathbf{z}}/\mu)$. Therefore, it turns out

$$V_{\mathbf{z}} = -\frac{\zeta \lambda \mu}{(1 + \mu)} \frac{\partial x_0}{\partial \mathbf{z}} = \Phi \frac{\partial l}{\partial \mathbf{z}} \propto \frac{\partial l}{\partial \mathbf{z}}. \quad (17)$$

Let us now prove the sufficiency part of the Proposition. If $V_{\mathbf{q}} \propto \frac{\partial x_0}{\partial \mathbf{q}}$, by Roy's identity we have $\frac{\partial x_0}{\partial \mathbf{q}} \propto \mathbf{x}$, so Sadka (1977) theorem once again applies and

[21] As S_{ik} is the Slutsky term, for small tax rates, $t_i S_{ik}$ approximates the compensated demand reduction of good k due to the tax rate in good i.

uniform tax rates are optimal, i.e. Barone theorem. Now, if $V_{\mathbf{z}} \propto \frac{\partial l}{\partial \mathbf{z}}$ and $\frac{\partial l}{\partial \mathbf{z}} = \mathbf{q}\frac{\partial \mathbf{x}}{\partial \mathbf{z}}$ from the consumer budget constraint, there exists a scalar ψ s.t. on the optimum, the condition $V_{\mathbf{z}}\psi = \mathbf{q}\frac{\partial \mathbf{x}}{\partial \mathbf{z}}$ is satisfied. However, as uniform tax rates are optimal we have $\mathbf{q} = \mathbf{t}/\zeta$, therefore $V_{\mathbf{z}}\psi\zeta = \mathbf{t}\frac{\partial \mathbf{x}}{\partial \mathbf{z}}$. By substituting in the condition $V_{\mathbf{z}} = \lambda\mathbf{c} - \boldsymbol{\lambda}\frac{\partial(\mathbf{tx})}{\partial \mathbf{z}}$, we obtain $V_{\mathbf{z}} = \lambda\mathbf{c} - \lambda V_{\mathbf{z}}\psi\zeta$,or

$$V_{\mathbf{z}} = \frac{\lambda}{1+\lambda\psi\zeta}\mathbf{c}.$$

Accordingly, $\frac{V_{z_j}}{c_j} = \frac{V_{z_k}}{c_k}, j,k = 1..u$, i.e. the Pantaleoni marginalist rule. To prove the Pigou conjecture, recall that, from (13) and $\mathbf{q} = \mathbf{t}/\zeta$, it is

$$MCPF = \frac{x_i}{x_i + \sum_j t_j \frac{\partial x_j}{\partial q_i}} = \frac{x_i}{x_i + \zeta \sum_j q_j \frac{\partial x_j}{\partial q_i}}, i = 1, ...n.$$

But from individual budget constraint, $\sum_j q_j \frac{\partial x_j}{\partial q_i} + x_i = -\frac{\partial x_0}{\partial q_i}$, so $MCPF = \frac{x_i}{x_i - \zeta(x_i + \frac{\partial x_0}{\partial q_i})}$. Hence, given $S_{0i} = \beta x_i > 0$, $MCPF -> 1$ if $(1+\beta) > \frac{\partial x_0}{\partial m} > 0$.□

Proof of Proposition 5. From (17) of the previous proof, we may write the marginal rate of substitution of each public good as follows

$$MRS_j \equiv \frac{V_{z_j}}{V_m} = \frac{\Phi}{V_m}\frac{\partial l}{\partial z_j} \approx \frac{\partial y}{\partial z_j}, j = 1...u.$$

Further, as $\frac{\Phi}{V_m} > 0, MRS_j > (<)0 \Longrightarrow \frac{\partial l}{\partial z_j} > (<)0, j = 1, ..u.$□

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Fig. 1. The theory of multiple budget determination in the history of economic analysis