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On the Synergy of Universal Floors and Conditional Incentives: Escaping Poverty Traps

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Abstract

This paper studies why some economies remain trapped in low education and low income despite sustained social spending, and why the policy mix matters for escaping the trap. We propose a *policy synergy* framework in which human-capital accumulation depends on the interaction between (i) a broad universal income floor, τ_{univ}, and (ii) schooling-contingent incentives, summarized by an effective conditionality multiplier τ_{cond}. To this aim, we study a two-dimensional nonlinear dynamic system: an evolutionary (replicator) dynamic for the share of households investing in education, and a logistic "income-potential" dynamic featuring a transitional productivity valley (a J-curve) at low education shares. The model yields multiple steady states and identifies conditions under which a *saddle-node bifurcation* eliminates the low-education equilibrium. A key result is that universal floors and conditional incentives are *complements*: raising τ_{univ} alone may be insufficient to change long-run education when opportunity costs are high, while strong conditionality without adequate liquidity can be brittle for the poorest households.

Keywords: Bifurcation Analysis; Conditional Cash Transfers; Education and Income; Evolutionary Dynamics; Poverty Traps; Universal Transfers.
JEL classification: C72; C73; I38; O15; O40

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1. Introduction

Why do some regions remain poor for decades even when aggregate poverty declines? A substantial body of literature argues that poverty can be *self-reinforcing* when low incomes reduce investment in human capital, which, in turn, depresses productivity and wages (see, Aghion et al., 2025; Barham et al., 1995; Santos, 2011, among many others). In such environments, even sustained social spending may fail to move an economy from a low-education, low-productivity equilibrium to a high-education, high-income steady state (Ravallion, 2016; Rinehart and McGuire, 2017; Romer and Romer, 2016). But economic policies of social spending to combat poverty are often diverse in the sense that some are universal transfer (low incentive compatibility) policies and others are conditional transfer (high administrative cost) policies, i.e. Conditional Cash Transfers (CCTs) and Unconditional Cash Transfers (UCTs) are the two primary formats of cash transfer programs used to alleviate poverty (Antía et al., 2026; Bergstrom and Dodds, 2021; Fiszbein et al., 2009).

Since, a central practical debate is whether transfers should be (i) targeted and strictly conditional on human-capital investments (conditional cash transfers, CCTs) or (ii) universal and weakly conditional or unconditional (unconditional transfers, UCTs; universal floors). Based on the empirical evidence of Behrman et al. (2025) and Marquez-Padilla et al. (2025), and the theoretical framework of "Poverty Trap Regimes" by Sanchez Carrera and Risso (2024), this paper proposes a theoretical evolutionary game model to show that neither pure conditionality nor pure universality is sufficient to move the economy from a low-education equilibrium to a high-income steady state.

Our initial argument is that the decision to pursue education versus subsistence labor is not merely a static household optimization problem, but a dynamic-evolutionary process constrained by macroeconomic structures. When marginalized households pursue education, they face a severe opportunity cost: the immediate loss of informal labor income. In fragile regional economies, this creates a localized "Valley of Death" or a productivity J-Curve, where initial investments in human capital temporarily depress local household survival capabilities before long-term agglomeration benefits materialize (Brynjolfsson et al., 2021).[1] Consequently, the primary goal of this paper is to develop a macro-evolutionary model that captures the dynamics of human capital accumulation and regional income generation under structural transition costs. We aim to formalize the analytical relationship between universal income floors and conditional incentives, demonstrating why they are complements rather than substitutes.

Over the past two decades, cash transfer programs have emerged as the leading anti-poverty policy tool in the Global South. A central debate in development economics revolves around the delivery mechanism: specifically, whether transfers should be targeted and strictly conditional on human capital investments—Conditional Cash Transfers—or whether they should take the form of unconditional Universal Basic Income (Bergstrom and Dodds, 2021; Romer and Romer, 2016; Serra and Sanches, 2025; Schady et al., 2009).

For instance, the Mexican economy represents a key stylized fact—with direct theoretical relevance—due to the abrupt rollback of Progresa/Prospera after 2019 (Parker and Todd, 2017; Behrman et al., 2025; Marquez-Padilla et al., 2025; Follett and Henderson, 2023). In this vein, Marquez-Padilla et al. (2025) show that the sudden ending of the program reduced schooling and increased youth work, especially for high-school-aged boys from more disadvantaged households. In parallel, the Mexican Economic Policy (Lara et al., 2025) of 4T on educational scholarships (e.g., *Becas de Educación Básica para el Bienestar Benito Juárez*,

[1]As demonstrated by Molina et al. (2026), these opportunity costs are highly convex and heavily dependent on local labor market conditions. In fragile economies, this creates a localized "Valley of Death" or a productivity J-Curve.

PBBBJ) has been evaluated with administrative data and quasi-experimental designs. The impact assessment by the Mexican agency CONEVAL reports short-run reductions in school abandonment associated with higher scholarship exposure (e.g., about −0.48 percentage points in primary and −1.89 percentage points in secondary one cycle after treatment, depending on specification and subgroup).[2] Recent empirical evaluations of this rollback provide a stark warning: the sudden removal of conditionalities immediately reduced school enrollment and increased youth labor, particularly among high-school-aged boys from the most disadvantaged households (Marquez-Padilla et al., 2025). Despite maintaining overall social spending, the transition from a CCT to a UCT framework structurally failed to retain the poorest demographics in the educational system, effectively pushing them back into the informal labor market.

The empirical failure of replacing conditional incentives with unconditional universal floors motivates a profound theoretical re-evaluation. Traditional static microeconomic utility models struggle to fully explain why a purely unconditional transfer—which theoretically relaxes liquidity constraints and increases household budget sets—might paradoxically reduce human capital accumulation (Banerjee et al., 2019; Del Carpio et al., 2016; Gechert et al., 2020; Romer and Romer, 2016;). While foundational work by Parker and Todd (2017) and Bergstrom and Dodds (2021) established the success of CCTs in breaking intergenerational poverty, recent studies have highlighted the vulnerabilities of unconditional designs. Levasseur (2021) notes that cash incentives often fail to retain the poorest in urban settings due to high opportunity costs, while the seminal work of Marquez-Padilla et al. (2025) explicitly documents the adverse educational and labor impacts of the *Prospera* rollback in Mexico.

[2] Consejo Nacional de Evaluación de la Política de Desarrollo Social (2024).

Taking into account the empirical evidence and theoretical debate mentioned, our model innovates by applying evolutionary game theory to macroeconomic development and poverty traps, and not linear statistical inference (as are econometric exercises) or standard optimization model (such as general equilibrium models for overlapping generations).[3] Moreover, our modeling of regional income dynamics incorporates the concept of the macroeconomic J-Curve. Originally applied to the adoption of general-purpose technologies, we adapt the J-Curve (Bahmani-Oskooee and Ratha, 2004; Brynjolfsson et al., 2021; Cardi, 2007; Molina et al., 2026) to represent the "Valley of Disruption" in developing economies, where the transition from an informal agrarian/labor economy to a high-skill equilibrium incurs severe short-term transition costs.

This paper proposes a synergistic and hybrid "Push-Pull" model that combines a universal income floor to relax credit restrictions (The Push) with progressive, graduation-contingent incentives (The Pull) to overcome the opportunity cost of youth labor, and thus combat school dropout rates to improve human capital levels.[4] This paper's main contribution is the formal integration of a *Policy Synergy Term* (i.e. the complementarities between the CCTs and the UCTs) into an evolutionary game-theoretic framework of poverty traps. By embedding both a universal basic income parameter (τ_{univ}) and a conditional incentive multiplier (τ_{cond}) into a replicator dynamic equation, coupled with a quadratic J-Curve of regional income, we prove that escaping a structural poverty trap requires a saddle-node

[3]Foundational models, such as those by Brummitt et al. (2017), Carrera (2019), Samuelson (2002), and Sanchez Carrera and Risso (2024), emphasize that strategy adoption (e.g., schooling vs. informal labor) follows a replicator dynamic governed by social learning, peer effects, and bounded rationality.

[4]This paper contributes a compact, policy-ready dynamic model of *education–income poverty traps* that can map directly into the design differences between conditional and universal social programs. The model highlights an interaction often discussed informally in policy debates: *liquidity without incentives* may fail to shift long-run behavior (and in some cases may crowd it out), while *incentives without liquidity* can be "brittle" for the extreme poor, who cannot afford the transition period. When both are present, a *synergy* emerges that can eliminate a poverty trap.

bifurcation. We demonstrate analytically that universal floors and conditional transfers are strictly complementary. Crucially, we prove that under conditions of high opportunity costs, setting the conditional multiplier too low ($\tau_{cond} < 1$) transforms universal cash transfers into a regressive subsidy for the informal labor market, for example.[5] This theoretical result perfectly mirrors the foundation for why hybrid policies—combining the liquidity of a universal floor with the strict enforcement of a conditional premium—are strictly dominant in economies that achieve sustained development and escape poverty traps.

2. Model Setup

We model a representative region with a continuum of households. At each moment, households choose a strategy $S \in \{E, N\}$: E means to keep children in school (complete education, human capital investment), and N means to withdraw children early (incomplete education) into subsistence labor/informal work.

We use two reduced-form policy instruments, i.e.:

- Universal floor (UCTs), $\tau_{univ} \geq 0$: a broad transfer that relaxes liquidity constraints regardless of schooling decisions.
- Conditional intensity (CCTs) $\tau_{cond} \geq 0$: the strength of schooling-contingent incentives (conditionality, monitoring, and per-child scaling).

Specifically, we model the economy as a population of agents/households playing a simultaneous, non-cooperative coordination strategy game. In what follows, we define a two-dimensional dynamic system involving two state variables:

[5] In standard microeconomics, an unconditional cash transfer increases the reservation wage, which should theoretically reduce child labor, not increase it. We explain that because the J-curve temporarily depresses local wages, the unconditional liquidity allows households to sustain themselves while opting out of the costly, long-term investment of education, thus locking the macro-economy into the lower equilibrium.

- $x(t) \in [0,1]$ is the population share adopting the strategy of *High Education – Human Capital* (E). Consequently, $(1-x)$ adopts *Low Education – Unskilled Labor* (N).

- $y(t) \in [0,\infty)$ is the average per-capita income-productivity.

Agents/households who choose between Education (E) and No Education (N) can obtain a utility/payoff (U) that is defined as:

$$U_E(x,y) = w_H(y) - C_{edu} + \sigma x + \underbrace{\tau_{cond} \cdot \tau_{univ}}_{\text{Synergy Term}} \quad (1)$$

$$U_N(x,y) = w_L(y) + \tau_{univ} \quad (2)$$

where $w_H(y) = \phi_H y$ is the wage for educated labor (proportional to productivity), and $w_L(y) = \phi_L y$ is the wage for uneducated labor ($\phi_H > \phi_L \in (0,1)$), i.e. ($w_H > w_L$). Parameter $C_{edu} \geq 0$ measures the direct and opportunity cost of education (Aghion et al., 2025; Barham et al., 1995; Carrera, 2019; Santos, 2011). That is, highly convex education costs (C_{edu}) driven by local labor markets (Molina et al., 2026). The $\sigma x > 0$ defines the *Social Synergies/Peer Effects*. That is, the utility of education increases as more peers educate (network externality). If x is low, the social cost of studying is high. We can define σ, as the strength of social norms. Notice that, $\tau_{cond} \geq 0$ acts as a multiplier on the base floor. While $\tau_{univ} \geq 0$ is *intensity* of the universal income floor (received as a direct transfer).

Remark 1 (Why a "product" term $\tau_{cond} \cdot \tau_{univ}$?)**.** *The product structure in* (1) *arises when (i) the universal floor relaxes liquidity constraints and (ii) the conditional program pays a* premium proportional to that floor*, i.e., a "top-up" on top of* τ_{univ}*. In this common policy architecture, the marginal incentive to keep children in school is the interaction between the* *base and the premium factor.*[6]

2.1. Micro-foundations for the synergy of "Push-Pull" social policies

Let us now provide the micro-foundations for the "Push-Pull" social policy model. So we derive the household decision on utilities $U_N(x,y)$ and $U_E(x,y)$. Considering a representative household with aggregate education level x and productivity y. The household must choose between two discrete strategies $S \in \{E, N\}$. To model the "survival constraint" (as per Sanchez Carrera and Risso, 2024), we employ a Stone-Geary utility function with a subsistence consumption floor $\bar{c}$. The lifetime utility V for strategy $S \in \{E, N\}$ is:

$$V_S = \underbrace{\ln(c_{1,S} - \bar{c})}_{\text{Period 1: Youth}} + \beta \underbrace{\mathbb{E}[\ln(c_{2,S} - \bar{c})]}_{\text{Period 2: Adulthood}} + \underbrace{\Psi(x)}_{\text{Social Utility}} \tag{3}$$

where $\beta \in (0,1)$ is the discount factor, $c_{t,S}$ is the consumption level in period t under strategy S, and $\bar{c}$ is the subsistence level (food/shelter). If $c < \bar{c}$, utility $\to -\infty$. Parameter $\Psi(x) = \sigma x$ measures the utility derived from social norms (peer effects).

Let us analyze the consumption vectors for both strategies (U_N vs U_E) to determine the sign of the decision inequality. Then:

1. Strategy N on Non-Education (The Trap Option). In period 1 the household receives child labor wages w_L and any universal transfer τ_{univ}, i.e. $c_{1,N} = w_L + \tau_{univ}$, and in period 2 $c_{2,N} = w_L$ (no skill premium). Then, total Utility of N is:

$$U_N = \ln(w_L + \tau_{univ} - \bar{c}) + \beta \ln(w_L - \bar{c}) \tag{4}$$

[6] In the model, the *effective policy push* that favors education is not τ_{univ} or τ_{cond} separately but their interaction. For example, this is the sense in which *Prospera*-style conditionality and 4T-style universality can be complements. If $\tau_{univ} = 0$, the conditional incentive is ineffective (binding subsistence constraint).

2. Strategy E on Education (The Escape Option). The child studies in Period 1; the household pays cost C_{edu}, foregoes wage w_L, but receives conditional transfer τ_{cond} and universal transfer τ_{univ}, i.e. $c_{1,E} = \tau_{univ} + \tau_{cond} - C_{edu}$. In Period 2, the child earns w_H, and then $c_{2,E} = w_H$. So total Utility of E is:

$$U_E = \ln(\tau_{univ} + \tau_{cond} - C_{edu} - \bar{c}) + \beta \ln(w_H - \bar{c}) + \sigma x \tag{5}$$

Please note that at the moment we are not considering the option of a synergistic policy, but rather an additive policy of conditional transfers (τ_{cond}) and universal transfers (τ_{univ}). However, their crutial interaction between the two will be discussed below.

Hence, the trap zone is when $U_N(x, y) > U_E(x, y)$. The household falls into the Poverty Trap (chooses N) if the immediate survival pressure outweighs the future return. This inequality holds strictly when the "Liquidity Constraint" is binding. The next proposition is straightforward.

Proposition 1 (The Infinite Barrier)**.** *If the net income during schooling approaches the subsistence level $\bar{c}$, the utility of education diverges to $-\infty$, guaranteeing $U_N > U_E$ regardless of the future wage w_H.*

Proof: Consider the argument of the log function in equation (5). The decision condition is:

$$\tau_{univ} + \tau_{cond} - C_{edu} \leq \bar{c} + \epsilon$$

As $\epsilon \to 0$:

$$\lim_{\text{Net Income} \to \bar{c}} \ln(\text{Net Income} - \bar{c}) = -\infty$$

Conversely, if $w_L + \tau_{univ} > \bar{c}$, then U_N is finite. Thus, $U_N \gg U_E$. $\square$

This explains the failure of pure CCTs in extreme poverty (or the 2019 rollback that occurred in Mexico (Behrman et al., 2025; Lara et al., 2025). If the transfer does not cover the gap between C_{edu} and subsistence, the agent *cannot* choose education, effectively truncating the decision set. However, the escape zone from the trap is when $U_E(x, y) > U_N(x, y)$. For the household to choose education, two conditions must be met simultaneously:

1. *Feasibility Condition (The Push):* Current consumption must exceed subsistence.
2. *Incentive Condition (The Pull):* The discounted Net Present Value (NPV) must be positive.

Let us derive the *Hybrid Policy Threshold* by setting $U_E > U_N$:

$$\ln\left(\frac{\tau_{univ} + \tau_{cond} - C_{edu} - \bar{c}}{w_L + \tau_{univ} - \bar{c}}\right) + \beta \ln\left(\frac{w_H - \bar{c}}{w_L - \bar{c}}\right) + \sigma x > 0 \tag{6}$$

Exponentiating and rearranging, we isolate the required transfers. Let $\Delta W = \beta \ln(\frac{w_H - \bar{c}}{w_L - \bar{c}}) + \sigma x$ be the *Future + Social Premium*. The condition becomes:

$$\frac{\tau_{univ} + \tau_{cond} - C_{edu} - \bar{c}}{w_L + \tau_{univ} - \bar{c}} > e^{-\Delta W} \tag{7}$$

Define the *Marginal Rate of Substitution (MRS)* between conditional and universal transfers at the subsistence boundary. From Eq (5), let $M = \tau_{univ} + \tau_{cond} - C_{edu} - \bar{c}$ (Disposable Income). The marginal impact of the conditional transfer on utility is:

$$\frac{\partial U_E}{\partial \tau_{cond}} = \frac{1}{M} = \frac{1}{\tau_{univ} + \tau_{cond} - C_{edu} - \bar{c}} \tag{8}$$

Notice that $\frac{\partial U_E}{\partial \tau_{cond}}$ is strictly decreasing in τ_{univ}. However, in terms of Decision Probability (Logit dynamics), the impact is defined by the relaxation of the constraint. If we approximate the utility gain around the indifference point using a Taylor expansion, the *interaction effect* emerges:

$$\frac{\partial^2(U_E - U_N)}{\partial\tau_{cond}\,\partial\tau_{univ}} \approx \frac{1}{(w_L + \tau_{univ} - \bar{c})^2} > 0 \tag{9}$$

Therefore, the effectiveness of τ_{cond} is strictly increasing in τ_{univ}, i.e. the Synergy Term $(\tau_{cond} \times \tau_{univ})$ which means that:

i) If τ_{univ} is low (near $\bar{c} - w_L$), the household is fighting for survival. Risk aversion is infinite. They cling to U_N.

ii) If τ_{univ} is high, the denominator stabilizes. The household can afford to "hear" the signal sent by τ_{cond}.

This validates the reduced-form term $\tau_{cond} \cdot \tau_{univ}$, i.e.: *You cannot incentivize a drowning man to swim lesson (Education) without first giving him a raft (Universal Floor).* Let us now summarizes the regimes. We define the state space partition $\Omega = \Omega_{trap} \cup \Omega_{escape}$.

Condition 1 (The Poverty Trap Region Ω_{trap})**.** *The set of states (x, y) where:*

$$y < \frac{\bar{c} + C_{edu} - \tau_{univ}}{\phi_L}$$

In this region, $U_N > U_E$ is forced by the subsistence constraint. The policy remedy is strictly $\uparrow \tau_{univ}$ (The Push).

Condition 2 (The Opportunity Region Ω_{escape})**.** *The set of states where feasibility holds, but incentives matter. Here, $U_E > U_N$ requires:*

$$\tau_{cond} > \underbrace{(w_L - \tau_{univ})}_{Opportunity\ Cost} - \underbrace{Future\ Premium}_{Discounted\ w_H}$$

In this region, the policy remedy is $\uparrow \tau_{cond}$ *(The Pull).*

Now, going back to the agents' utilities/payoffs, this is subtracting (2) from (1) yields the payoff difference:

$$\Delta U(x, y) \equiv U_E - U_N = (\phi_H - \phi_L)y - C_{\text{edu}} + \sigma x + \tau_{univ}(\tau_{cond} - 1). \tag{10}$$

Define $\Delta\phi \equiv \phi_H - \phi_L > 0$ and the *synergy index* $\Delta\tau \equiv \tau_{univ}(\tau_{cond} - 1)$.[7] Then

$$\Delta U(x, y) = \Delta\phi\, y - C_{\text{edu}} + \sigma x + \Delta\tau. \tag{11}$$

such that $\Delta\tau$ defines the *policy-synergy index* as the incremental transfer (relative to the universal floor) associated with schooling compliance, measured in the same units as income.

2.2. The Policy Mechanism: The Matching Multiplier

To support the Conditions 1 and 2 concerning the Push-Pull policy effects, and thus the *policy-synergy index*, $\Delta\tau$, let us formally define the government's transfer mechanism. The government provides a Universal Basic Income (UBI or UCTs) floor, τ_{univ}, to all youth. To incentivize schooling, the government offers a conditional transfer (CCTs). Rather than a flat additive amount, the conditional policy acts as a *matching multiplier*, τ_{cond}, on the universal base.

- A household choosing N receives exactly the universal floor: $T_N = \tau_{univ}$.
- A household choosing E receives the matched amount: $T_E = \tau_{cond} \cdot \tau_{univ}$.

[7] This reduced-form approximation meant to capture the local complementarity in a tractable way for the global phase-space analysis, rather than an exact aggregation of the non-linear household utilities.

The consumption profiles for the two strategies are determined by the policy transfers, the direct/opportunity cost of education C_{edu}, and the adult wages $w_H(y)$ and $w_L(y)$, which depend on regional macroeconomic productivity y. Hence, for the strategy N (Non-Education): $c_{1,N} = \tau_{univ}$ and $c_{2,N} = w_L(y)$. While, for the strategy E (Education): $c_{1,E} = \tau_{cond} \cdot \tau_{univ} - C_{edu}$ and $c_{2,E} = w_H(y)$.

Households derive utility according to a standard time-separable, concave utility function $u(\cdot)$, subject to a subsistence minimum $\bar{c}$ (Stone-Geary preferences). They also experience a social/peer utility premium $S(x) = \sigma x$ if they choose education, where x is the aggregate share of educated agents. The lifetime value functions for each strategy are:

$$V_N(x, y) = u(\tau_{univ}) + \beta u(w_L(y)) \tag{12}$$

$$V_E(x, y) = u(\tau_{cond}\tau_{univ} - C_{edu}) + \beta u(w_H(y)) + \sigma x \tag{13}$$

Because Equations (12) and (13) are highly non-linear, embedding them directly into a macroeconomic model yields an intractable dynamical system. To recover a robust, smoothed reduced-form payoff, we utilize a First-Order Taylor Expansion.

Theorem 1 (Linearization of the Utility/Payoff)**.** *Assuming the economy operates in a neighborhood slightly above the subsistence threshold, the non-linear utility differential ΔV can be rigorously mapped to a linear payoff structure containing a multiplicative synergy term.*

Proof. We evaluate the utility difference $\Delta V = V_E - V_N$:

$$\Delta V = [u(c_{1,E}) - u(c_{1,N})] + \beta [u(c_{2,E}) - u(c_{2,N})] + \sigma x \tag{14}$$

We take a first-order Taylor series approximation of the Education utility $u(c_E)$ around the baseline consumption of the Non-Education strategy c_N:

$$u(c_{1,E}) \approx u(c_{1,N}) + u'(c_{1,N}) \cdot (c_{1,E} - c_{1,N}) \tag{15}$$

$$u(c_{2,E}) \approx u(c_{2,N}) + u'(c_{2,N}) \cdot (c_{2,E} - c_{2,N}) \tag{16}$$

Substituting the consumption constraints into the youth term:

$$(c_{1,E} - c_{1,N}) = (\tau_{cond}\tau_{univ} - C_{edu}) - \tau_{univ} = \tau_{univ}(\tau_{cond} - 1) - C_{edu} \tag{17}$$

Substituting into the adult term:

$$(c_{2,E} - c_{2,N}) = w_H(y) - w_L(y) \tag{18}$$

Thus, the approximated utility differential becomes:

$$\Delta V \approx u'(\tau_{univ}) \left[\tau_{univ}(\tau_{cond} - 1) - C_{edu}\right] + \beta u'(w_L) \left[w_H(y) - w_L(y)\right] + \sigma x \tag{19}$$

To simplify the below dynamic system while preserving the economic mechanisms, we assume marginal utility is roughly constant in the local neighborhood of the poverty trap and normalize it (or absorb it into the structural parameters). Let the wage gap be linearly proportional to regional income: $\beta u'(w_L)[w_H(y) - w_L(y)] \equiv \Delta\phi \cdot y$. Let the normalized cost be C_{edu}. We thus perfectly recover the reduced-form linear payoff differential:

$$\Delta V \approx \Delta\phi\, y - C_{edu} + \sigma x + \tau_{univ}(\tau_{cond} - 1) \tag{20}$$

□

Recall that a common flaw in macroeconomic models of poverty traps is the mathematical disconnect between household-level optimization (often featuring concave, non-linear utility) and the aggregate evolutionary dynamics (often modeled with linear reduced-form payoffs).[8] By redefining conditional cash transfers as a policy multiplier on a universal income base, and employing a first-order Taylor approximation around the uneducated subsistence steady-state, we formally derive the "Policy Synergy Term." This bridges the gap between microeconomic budget constraints (an overlapping-generations household model with subsistence constraints and) and aggregate phase-space bifurcations driven by a macro-evolutionary replicator dynamic. Thus, we are now ready to move from the microeconomic analysis of household utility to the macrodynamic evolution of the economy. We postulate that the demographic distribution of strategies (i.e. the state of the economy) is determined by a process of social learning and imitation (Apesteguia et al., 2007; Samuelson, 2002).

3. The Macro-Evolutionary Model

Households are not isolated optimizers; they observe the relative success of the two strategies, E (Education) and N (Non-Education/Labor). According to evolutionary game theory principles, the strategy that yields a payoff higher than the population average proliferates, as agents update their choices based on the observed utility/payoff differential. Consequently, the aggregate sate of the economy evolves via a replicator dynamic, i.e. the rate of change in the educated population share $\dot{x}$ (which serves as the transmission mechanism translating individual subsistence constraints into collective developmental trajectories) is

[8] See, for instance: Achdou et al. (2021), Barrett et al. (2016), Blume et al. (2020), Ikegami et al. (2017).

proportional to the difference in lifetime value functions, $\Delta V = V_E - V_N$, i.e.

$$\dot{x} = x(1-x)[V_E(x,y) - V_N(x,y)] \tag{21}$$

Having established a rigorous analytical bridge from the concave household microfoundations to the linear macro-payoffs (Theorem 1), we define the complete dynamical system. The evolution of the educated population $x(t)$ is:

$$\dot{x} = x(1-x)\left[\Delta\phi\, y - C_{edu} + \sigma x + \Delta\tau\right] \tag{22}$$

This establishes the *Policy Synergy Term*: $\Delta\tau = \tau_{univ}(\tau_{cond} - 1)$.[9] The bracketed term is the net incentive for education. When it is negative at low (x, y), education adoption declines and the economy drifts toward a low-education equilibrium. Then, we can state the next proposition.

Proposition 2 (The Synergy Condition)**.** *In a population governed by replicator dynamics* (22)*, the marginal impact of increasing the unconditional Universal Income Floor (τ_{univ}) on the rate of human capital accumulation ($\dot{x}$) is strictly dependent on the intensity of the Conditional Incentive (τ_{cond}). Specifically, unconditional transfers are regressive if $\tau_{cond} < 1$ and progressive if $\tau_{cond} > 1$. Two cases (Figure 1) are,*

- *Case 1: The Regressive Trap ($\tau_{cond} < 1$). If the conditional multiplier is weak, then $\Delta\tau < 0$. Increasing τ_{univ} makes $\dot{x}$ more negative by subsidizing the outside option (N). This explains the dropout spikes observed when strict conditionalities are removed*

[9]Because this term was derived systematically from the household budget constraints via Taylor expansion. It clearly shows that if the conditional multiplier is weak ($\tau_{cond} < 1$), increasing the universal floor τ_{univ} negatively impacts human capital accumulation by subsidizing the baseline consumption of the uneducated state.

(Aparicio-Chueca et al., 2021).

- *Case 2: The Synergistic Escape ($\tau_{cond} > 1$). If the multiplier is strong, $\Delta\tau > 0$. The universal floor provides vital liquidity, while the conditional premium ensures the marginal dollar maximizes utility in school.*

Proof. To evaluate the policy impact of increasing the universal transfer, we take the partial derivative of $\dot{x}$ with respect to τ_{univ}, holding all other state variables (x, y) and parameters constant:

$$\frac{\partial \dot{x}}{\partial \tau_{univ}} = \frac{\partial}{\partial \tau_{univ}} \left(x(1-x)[\Delta\phi, y - C_{edu} + \sigma x + \tau_{univ}(\tau_{cond} - 1)] \right) \tag{23}$$

Applying the linearity of differentiation:

$$\frac{\partial \dot{x}}{\partial \tau_{univ}} = x(1-x)(\tau_{cond} - 1) \tag{24}$$

Since we are analyzing an interior population where both strategies exist, $x \in (0, 1)$, which strictly guarantees that the variance term $x(1-x) > 0$. Therefore, the sign of the derivative is entirely determined by the term $(\tau_{cond} - 1)$. We establish three distinct parameter regimes:

1. The Regressive Trap Regime ($\tau_{cond} < 1$): If $\tau_{cond} < 1$, then $(\tau_{cond} - 1) < 0$. Consequently, $\frac{\partial \dot{x}}{\partial \tau_{univ}} < 0$. Increasing the universal transfer strictly decreases the rate of educational adoption. The universal floor acts as a subsidy for the informal labor sector.

2. The Synergistic Escape Regime ($\tau_{cond} > 1$): If $\tau_{cond} > 1$, then $(\tau_{cond} - 1) > 0$.Consequently, $\frac{\partial \dot{x}}{\partial \tau_{univ}} > 0$. Increasing the universal transfer strictly increases the rate

of educational adoption. The universal floor leverages the conditional multiplier to pull agents into the education system.

3. The Neutrality Regime ($\tau_{cond} = 1$): If $\tau_{cond} = 1$, then $\frac{\partial \dot{x}}{\partial \tau_{univ}} = 0$. The universal transfer has no net impact on strategy choice because it shifts U_E and U_N upwards by the exact same scalar amount.

□

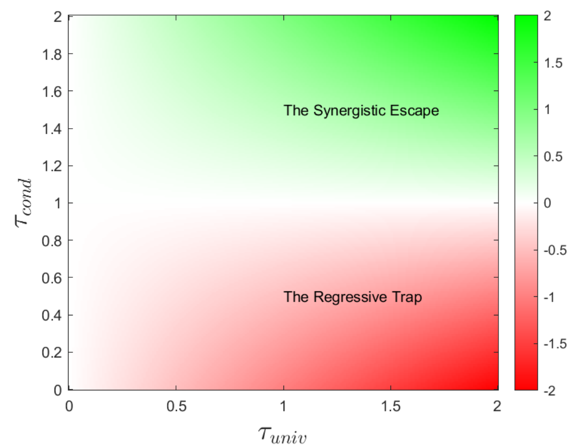

Figure 1: Illustration of the policy-synergy index $\Delta\tau = \tau_{univ}(\tau_{cond} - 1)$ in the $(\tau_{univ},\ \tau_{cond})$ plane. The locus $\Delta\tau = 0$ occurs at $\tau_{cond} = 1$ (no schooling premium).

Remark 2 (Why $\tau_{cond} > 1$ matters)**.** *In this reduced-form structure, $\tau_{cond} = 1$ means the universal transfer does not differentially reward schooling. When $\tau_{cond} > 1$, the universal floor is coupled with a positive schooling wedge ($\Delta\tau > 0$). When $\tau_{cond} < 1$, the universal floor can* reduce *the relative attractiveness of schooling ($\Delta\tau < 0$), for example when transfers relax liquidity constraints but are not paired with a sufficiently large, enforceable schooling-contingent component relative to opportunity costs.*

For escaping the trap we must emplor the synergy policy, $\Delta\tau$. Setting $\dot{x} = 0$, we find the critical income curve required to sustain education:

$$y_{crit}(x) = \frac{-\sigma x + C_{edu} - \tau_{univ}(\tau_{cond} - 1)}{\Delta\phi} \tag{25}$$

To systematically eliminate the poverty trap (ensuring $y_{crit}(0) < a_0$ –the subsistence income is sufficient to start educating–),[10] policymakers must satisfy:

$$\frac{C_{edu} - \tau_{univ}(\tau_{cond} - 1)}{\Delta\phi} < a_0. \tag{26}$$

Rearranging gives the *Minimum Universal Floor* required, given a strong conditional incentive:

$$\tau_{univ} > \frac{C_{edu} - a_0\Delta\phi}{\tau_{cond} - 1}. \tag{27}$$

This equation defines the *Iso-Success Curve*.[11] It shows that one cannot simply "spend more" (increase τ_{univ}) without "targeting better" (τ_{cond}). There is a hyperbolic trade-off between coverage and conditionality. Analyzing how expanding universal policy affects this barrier (25) by taking the partial derivative with respect to τ_{univ}:

$$\frac{\partial y_{crit}}{\partial \tau_{univ}} = \frac{-(\tau_{cond} - 1)}{\Delta\phi}. \tag{28}$$

Since the wage premium $\Delta\phi > 0$, the sign is determined by $-(\tau_{cond} - 1)$. If $\tau_{cond} < 1$, then $\frac{\partial y_{crit}}{\partial \tau_{univ}} > 0$. If $\tau_{cond} > 1$, then $\frac{\partial y_{crit}}{\partial \tau_{univ}} < 0$.

[10] A poverty trap exists if the local economic potential $K(x)$ lies strictly below $y_{crit}(x)$. A poverty trap exists if the income potential $K(x) = a_2x^2 + a_1x + a_0$ lies **below** this critical threshold $y_{crit}(x)$ for low x. To escape, we must lower the y_{crit} curve: i) in the standard linear model, we just add cash, ii) in this **Synergy Model**, increasing τ_{univ} rotates the curve based on τ_{cond}.

[11] This hyperbolic relationship proves that a government cannot merely increase universal coverage (τ_{univ}) without enforcing strict conditionality (τ_{cond}). They act multiplicatively.

Corollary 1. *Both the dynamic flow ($\dot{x}$) and the structural threshold (y_{crit}) require $\tau_{cond} > 1$ for universal transfers (τ_{univ}) to have a positive impact on human capital. Therefore, the policies are shown to be synergistic and non-substitutable.*

3.1. Regarding the income dynamics

Now, let us consider that income-productivity evolves based on the accumulation of human capital (Becker, 1975; Lucas Jr, 1988). Let $y(t) \geq 0$ denote income (normalized). We assume income adjusts toward an education-dependent "income potential" $K(x)$, i.e.

$$\dot{y} = \alpha y\big(K(x) - y\big), \qquad \alpha > 0. \tag{29}$$

We allow $K(x)$ to be *non-monotone* at low x (a transitional productivity valley):

$$K(x) = a_2 x^2 + a_1 x + a_0, \qquad a_2 > 0,\ a_0 > 0. \tag{30}$$

If $a_1 < 0$ and a_0 is not too small, $K(x)$ can initially fall with x (a "valley of death", a J-curve transition cost) before rising at higher education shares: early increases in x may coincide with temporarily lower income potential.

Crucially, we allow $a_1 < 0$ to capture a transition "valley" (the J-curve). When x is low, education initially reduces local output due to labor withdrawal, skill mismatches, or brain drain before the network externalities ($a_2 x^2$) dominate. This creates a minimum at $x_{min} = \frac{|a_1|}{2a_2}$, representing the "Valley of Death" that a successful policy must bridge. Specifically, each term has a distinct structural meaning:

- a_0 (The Subsistence Floor): It represents the economic output of a completely uneducated society ($x = 0$). For example, an economy based purely on subsistence agriculture or informal manual labor.

- $a_1 x$ (The Direct Linear Return): This represents the private return to human capital. If you educate one person, they become more productive. In a linear world, 100 educated people are simply 100 times as productive as one.

- $a_2 x^2$ (The Network/Synergy Effect) represents non-linear social returns or network externalities. The value of an educated person depends on how many other educated people are around them. Example: An engineer in a village with no other engineers ($x \approx 0$) has low productivity (linear). An engineer in a technology cluster like Guadalajara or Monterrey ($x \to 1$) is exponentially more productive because they can collaborate, innovate, and specialize.

The value of an educated person depends on how many other educated people are around them. We generally assume $a_2 > 0$ (Convexity) to model "Silicon Valley" effects or industrial clusters. However, important consideration for the income dynamics equation (29) are the following:

- The "Brain Drain" Effect: When x is small (few educated people), there is no local industry ($a_2 \approx 0$) to employ them.[12] These educated agents often migrate to bigger cities or advanced economies. The region paid for their education (taxes/resources) but lost their labor. The local economy is worse off than if they had just stayed and worked in agriculture (N). Hence, $K(x)$ drops initially as resources leak out.

- Skill Mismatch and Frictional Unemployment: In a purely agrarian economy (a_0), an

[12] Zone of Regress ($0 < x < x_{min}$): In this interval, the marginal social return to education is *negative*. Social spending that increases education marginally actually lowers regional income per capita, as educated agents withdraw from traditional production without finding modern employment.

educated person who refuses manual labor but cannot find office work is unproductive. They might remain unemployed (waiting for a job) rather than contributing to the subsistence economy. This lowers the per-capita income of the municipality during the transition phase.

- Disruption of Social Structures: Traditional economies rely on tight-knit, low-skill cooperation. Introducing a few "modern" agents can disrupt these traditional networks without yet replacing them with modern markets. Consequence: The "Deep Trap." If $a_1 < 0$, the Poverty Trap becomes significantly harder to escape. The carrying capacity $K(x)$ is a parabola opening upward, but its vertex (minimum) is now at $x_{min} = \frac{-a_1}{2a_2}$. The Dip: If $x_{min} \in (0, 1)$, the economy must endure a period of declining income to reach the "Promised Land" of high development.[13]

- The Barrier to Escape: This "dip" lowers the $K(x)$ curve relative to the subsistence constraint, effectively widening the basin of attraction of the Poverty Trap.[14] Under negative linear returns, the "Push-Pull" policy must be massive enough to jump the economy directly to $x > x_{min}$. The interaction term $\tau_{cond} \cdot \tau_{univ}$ must be strong enough to sustain households *through* the valley of declining productivity.

Therefore, we consider the case where $a_1 < 0$, representing the *structural costs of transition*

[13]Stability Analysis. The Jacobian element J_{21} (Impact of Education on Growth) becomes:

$$\frac{\partial \dot{y}}{\partial x} = \alpha y(2a_2x - |a_1|)$$

At $x = 0$: The derivative is $-|a_1|\alpha y < 0$. Implication: Small increases in education from zero suppress economic growth. The economy has a natural tendency to "punish" early adopters.

[14]If $a_1 < 0$, a policy of "Gradualism" (slowly increasing scholarships) is doomed to fail. That is: i) A small increase in x pushes the economy into the recessionary zone. ii) This lowers wages (w_H, w_L), making the household budget constraint *tighter*. iii) Households respond by pulling children out of school, reverting to $x = 0$.

(e.g., brain drain, skill mismatch). The economic potential function becomes $K(x) = a_2x^2 - |a_1|x + a_0$. That is, *The Valley of Death*. This specification introduces a local minimum in regional productivity at $x_{min} = \frac{|a_1|}{2a_2}$.[15]

Remark 3 (Poverty mapping)**.** *We treat poverty as an outcome mapping from the state variables. That is, to connect the state* (x, y) *to poverty, define a reduced-form poverty index* $p(t) \in (0, 1)$ *as a decreasing function of income and education. A simple functional form is*

$$p(t) = \Pi\big(x(t), y(t)\big) \equiv \frac{1}{1 + \exp(\kappa_0 + \kappa_1 y(t) + \kappa_2 x(t))}, \qquad \kappa_1 > 0,\ \kappa_2 > 0, \tag{31}$$

so that higher income and education reduce the poverty rate. This mapping is used in the numerical section to translate predicted trajectories into poverty dynamics.

3.2. Equilibria and stability analysis

The y-nullclines are $y = 0$ and $y = K(x)$. We focus on $y = K(x) > 0$. The x-nullclines are $x = 0$, $x = 1$, and (for interior points) the *education threshold line*

$$\Delta\phi y - C_{\text{edu}} + \sigma x + \Delta\tau = 0 \quad \Longleftrightarrow \quad y = Y_E(x) \equiv \frac{C_{\text{edu}} - \sigma x - \Delta\tau}{\Delta\phi} \tag{32}$$

Interior fixed points are intersections of $y = K(x)$ and $y = Y_E(x)$. Education increases ($\dot{x} > 0$) when $y > Y_E(x)$. Define,

$$f(x, y) \equiv \Delta\phi\,(y - Y_E(x)) = 0 \tag{33}$$

$$g(x, y) \equiv K(x) - y = 0 \tag{34}$$

[15] We consider the "J-Curve" of Development (Bahmani-Oskooee and Ratha, 2004; Brynjolfsson et al., 2021; Cardi, 2007): Negative Linear Returns ($a_1 < 0$). We relax the assumption that human capital accumulation is monotonically beneficial.

and the system (22) and (29) is defined by

$$\begin{aligned} \dot{x} &= x(1-x)f(x,y) \\ \dot{y} &= \alpha\, y\, g(x,y) \end{aligned} \tag{35}$$

Proposition 3 (Boundary equilibrium)**.** *The dynamic system* (35) *admits the following boundary equilibria:*

1. *The Poverty Trap equilibrium point* $E_0 = (0,0)$*. The equilibrium* E_0 *is always unstable; a repelling node if* $C_{edu} < \Delta\tau$ *or a saddle point if* $C_{edu} > \Delta\tau$*.*

2. *The High-Skill Mismatch (high education with zero income) equilibrium point* $E_1 = (1,0)$*. The equilibrium* E_1 *is unstable whenever (*$K(1) > 0$*)* $a_1 > 0$*; a repelling node if* $C_{edu} > \Delta\tau + \sigma$ *or a saddle point if* $C_{edu} < \Delta\tau + \sigma$*.*

3. *The Skill Mismatch (education with zero income) equilibrium point* $E_x = \left(\frac{C_{edu}-\Delta\tau}{\sigma}, 0\right)$ *that exists if* $0 < C_{edu} - \Delta\tau < \sigma$*. The equilibrium* E_x *is a repelling node if* $C_{edu} > \Delta\tau$*.*

4. *The Subsistence Floor/Zone Trap equilibrium point* $E_y = (0, a_0)$ *that exists if* $0 < a_0 < 1$*. The equilibrium* E_y *is an attracting node (effectively acting as the stable low-education trap) if* $a_0\Delta\phi < (C_{edu} - \Delta\tau)$ *or a saddle node if* $a_0\Delta\phi > (C_{edu} - \Delta\tau)$

5. *The High-Level equilibrium (high-education edge equilibrium)* $E_e = (1, K(1)) = (1, a_0 + a_1 + a_2)$ *that exists if* $0 < a_0 + a_1 + a_2 \leq 1$*. The equilibrium* E_e *is an attracting node if* $(a_0+a_1+a_2)\Delta\phi < (C_{edu} - \sigma - \Delta\tau) \Longleftrightarrow K(1) > Y_E(1) \Longleftrightarrow \Delta\phi K(1) > C_{edu} - \Delta\tau - \sigma$*, or a saddle node if* $(a_0 + a_1 + a_2)\Delta\phi > (C_{edu} - \sigma - \Delta\tau) \Longleftrightarrow K(1) < Y_E(1)$*.*

Proof. The x-trivial nullclines are defined as $x = 0$ and $x = 1$, while the x-nontrivial nullcline is given by the equation $f(x, y) = 0$. For the y-trivial nullcline, we have $y = 0$, and the

y-nontrivial nullcline is represented by $g(x, y) = 0$. From the trivial nullclines, we identify two equilibrium points: $E_0 = (0, 0)$ and $E_1 = (1, 0)$. Substituting the x-trivial nullclines into the y-nontrivial nullcline, we get: $g(0, y) = 0$ and $g(1, y) = 0$. This gives us the equilibrium points $E_y = (0, a_0)$ and $E_e = (1, K(1)) = (1, a_0 + a_1 + a_2)$. Next, substituting the y-trivial nullcline into the x-nontrivial nullcline, we have $f(x, 0) = 0$, leading to the equilibrium point $E_x = \left(\frac{C_{edu} - \Delta\tau}{\sigma}, 0\right)$. The Jacobian matrix of system (35) at (x, y) is

$$J(x, y) = \begin{pmatrix} (1 - 2x)\Delta\phi\,(y - Y_E(x)) + x(1 - x)\sigma & x(1 - x)\Delta\phi \\ \alpha y K'(x) & \alpha(K(x) - 2y) \end{pmatrix} \tag{36}$$

where $K'(x) = 2a_2 x + a_1$. Each equilibrium is substituted into the Jacobian matrix to assess its stability, as detailed in the proposition. Notice that, for the subsistence-floor equilibrium $E_y = (0, a_0)$, the eigenvalues are

$$\lambda_1 = \Delta\phi a_0 - (C_{edu} - \Delta\tau), \qquad \lambda_2 = -\alpha a_0. \tag{37}$$

Hence E_y is locally attracting if $\Delta\phi a_0 < C_{edu} - \Delta\tau$. For the high-education edge equilibrium $E_e = (1, K(1))$, the relevant eigenvalues are

$$\lambda_1 = -\Delta\phi K(1) + C_{edu} - \Delta\tau - \sigma, \qquad \lambda_2 = -\alpha K(1). \tag{38}$$

Thus E_e is locally attracting (in the x direction) if

$$C_{edu} - \Delta\tau - \sigma < \Delta\phi K(1) \quad \Longleftrightarrow \quad K(1) > Y_E(1). \tag{39}$$

That is, stability requires *higher* income potential at $x = 1$, not lower. $\square$

The interior equilibria satisfy $y = K(x)$ and $f(x,y) = 0$. Substituting $y = K(x)$ into $f(x,y) = 0$ yields

$$a_2\Delta\phi\, x^2 + (a_1\Delta\phi + \sigma)x + (a_0\Delta\phi - C_{edu} + \Delta\tau) = 0. \tag{40}$$

Let $h(x)$ denote the left side. The following proposition outlines the conditions for existence.

Proposition 4 (Interior Equilibrium – Existence)**.** *The dynamic system* (35) *admits at least one interior equilibrium defined by $E_i = (x_i, Y_E(x_i))$, where x_i is the positive solution of*

$$h(x_i) := a_2\Delta\phi {x_i}^2 + [a_1\Delta\phi + \sigma]x_i + a_0\Delta\phi - (C_{edu} - \Delta\tau) = 0. \tag{41}$$

The equilibrium E_i exists if the following three conditions are satisfied:

(C1) $\big(a_1\Delta\phi + \sigma\big)^2 \geq 4a_2\Delta\phi\big(a_0\Delta\phi - C_{edu} + \Delta\tau\big)$

(C2) $(a_0 + a_1 + a_2)\Delta\phi > C_{edu} - \Delta\tau - \sigma \quad \Leftrightarrow \quad K(1) > Y_E(1)$

(C3) $a_0\Delta\phi - C_{edu} + \Delta\tau < 0$

Proof. Combining the results from both nontrivial nullclines, we find that the equilibrium points E_i can be determined as solutions to the system of equations $f(x,y) = g(x,y) = 0$. The quadratic equation has two real roots if the discriminant is non-negative, which is condition (C1). The interior point is the intersection of a dashed curve $f(x,y) = 0$ and a solid line $g(x,y) = 0$, so it exist if $y_f > y_g$ where $f(1, y_f) = 0$ and $g(1, y_g) = 0$, see Figure 2 or equivalent with (C2). According to the Intermediate Value Theorem, if $h(0)h(1) < 0$ or $h(0)$ and $h(1)$ have different signs, then there exist at least one $x_i \in (0,1)$ such that $h(x_i) = 0$. The condition $h(0)h(1) < 0$ is equivalent to condition $\big[a_0\Delta\phi - C_{edu} + \Delta\tau\big]\big[(a_0 + a_1 + a_2)\Delta\phi + \sigma - C_{edu} + \Delta\tau\big] <$

0. Since condition (C2) implies that $\left[(a_0+a_1+a_2)\Delta\phi+\sigma-C_{edu}+\Delta\tau\right]>0$ we can conclude that $\left[a_0\Delta\phi-C_{edu}+\Delta\tau\right]<0$ (C3). □

Remark 4 (The existence of interior equilibrium implies the instability of high-level equilibrium.)**.** *Since (C2) in Proposition 4 is equivalent to $K(1)>Y_E(1)$, we can conclude that if the interior equilibrium (E_i) exists, then the High-Level equilibrium (E_e) is a saddle point.*

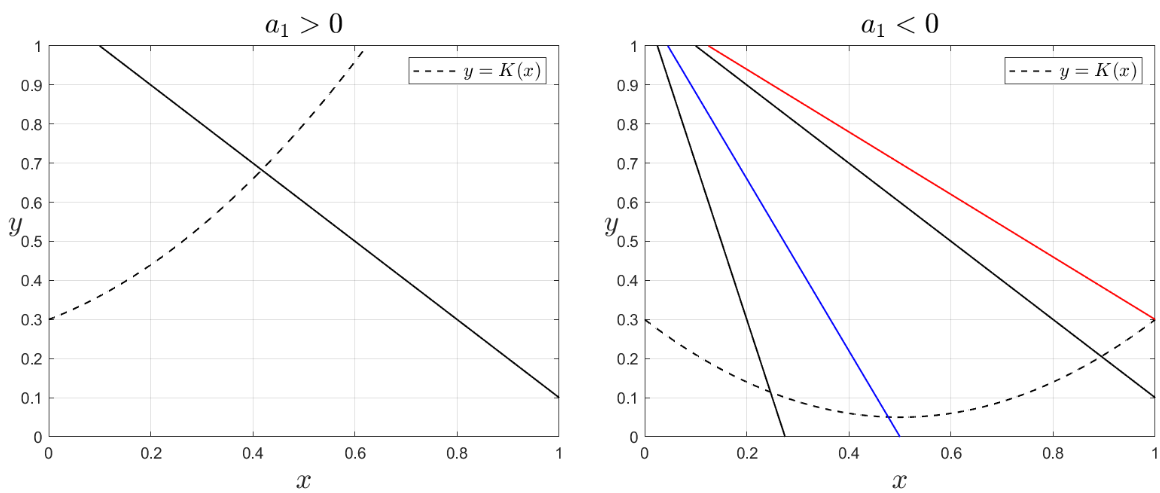

Figure 2: Illustrating when an interior equilibrium can exist depending on the slope parameter a_1 in the income-potential function $K(x)$. For $a_1>0$, i.e. monotone $K(x)$ tends to yield at most one interior intersection, and for $a_1<0$, i.e. a low-x valley in $K(x)$ can create multiple intersections with $Y_E(x)$.

Remark 5 (Why a J-curve creates a stable poverty trap)**.** *When $a_1<0$, the slope $K'(x)=2a_2x+a_1$ can be negative for low x. If $K'(x)$ is sufficiently negative, then $\sigma+\Delta\phi K'(x)<0$, making the low-$x$ interior equilibrium a stable node (a "poverty trap") rather than a boundary equilibrium at $x=0$.*

By analyzing the Jacobian matrix J, the system undergoes a *Saddle-Node Bifurcation* when the discriminant of Equation (41) equals zero. The threshold parameter dictates where the interior saddle point and poverty trap collide and annihilate, freeing the vector field to flow directly to the high equilibrium. At an interior equilibrium $E_i = (x^*, y^*)$ we have $\Delta U(x^*, y^*) = 0$ (by (32)), so the Jacobian simplifies to

$$J(x^*, y^*) = \begin{pmatrix} \sigma x^*(1-x^*) & \Delta\phi x^*(1-x^*) \\ \alpha y^* K'(x^*) & -\alpha y^* \end{pmatrix}. \tag{42}$$

Its trace and determinant are

$$\operatorname{tr} J = \sigma x^*(1-x^*) - \alpha y^*, \tag{43}$$

$$\det J = -\alpha y^* x^*(1-x^*)\Big(\sigma + \Delta\phi\, K'(x^*)\Big). \tag{44}$$

Because $\alpha y^* x^*(1-x^*) > 0$, the sign of $\det J$ is the opposite of $\sigma + \Delta\phi K'(x^*)$. In particular, $\det J < 0$ (a saddle) iff $\sigma + \Delta\phi K'(x^*) > 0$.

Remark 6 (Interpretation of Interior Equilibria)**.** *When $a_1 < 0$ and $\Delta\tau$ is below the saddle-node threshold derived below, there can be two interior equilibria: typically one saddle and one attracting node or focus. This corresponds to* bistability*: a low-education attractor coexists with a high-education attractor, separated by the stable manifold of the interior saddle.*

3.3. Bifurcation Analysis

Transcritical–collision with boundary equilibria. Because $x = 0$ and $x = 1$ are always nullclines, interior equilibria can collide with boundary equilibria as parameters vary. For example, the interior equilibrium collides with $E_y = (0, a_0)$ when $Y_E(0) = a_0$, i.e.

$$\Delta\phi a_0 = C_{edu} - \Delta\tau. \tag{45}$$

Similarly, collision with E_e occurs when $Y_E(1) = K(1)$, i.e.

$$\Delta\phi K(1) = C_{edu} - \Delta\tau - \sigma. \tag{46}$$

In the following propositions, we analytically show the conditions that cause bifurcations to occur.

Proposition 5 (Transcritical bifurcations)**.** *We identify the critical threshold at which two equilibria coalesce and exchange their stability. This phenomenon is known as a transcritical bifurcation, which causes the system's dynamics to change qualitatively.*

a) For $C_{edu} = \Delta\tau$, E_x coalesces with E_0.

b) For $C_{edu} = \Delta\tau + \sigma$, E_x coalesces with E_1.

c) For $a_0 = 0$, E_y coalesces with E_0.

d) For $a_0 + a_1 + a_2 = 0$, E_e coalesces with E_1.

e) For $a_0 + a_1 + a_2 = \frac{C_{edu} - \Delta\tau - \sigma}{\Delta\phi}$, E_i coalesces with E_e.

Proof. A transcritical bifurcation occurs when an equilibrium is non-hyperbolic at a critical parameter value, indicating that the Jacobian has a zero eigenvalue, and two equilibrium branches intersect and exchange stability at that point.

a) For $C_{edu} = \Delta\tau$, the abscissa of E_x becomes zero. One of the eigenvalues of $J(E_0)$ and $J(E_x)$ is zero.

b) For $C_{edu} = \Delta\tau + \sigma$, the abscissa of E_x becomes 1. One of the eigenvalues of $J(E_1)$ and $J(E_x)$ is zero.

c) For $a_0 = 0$, the ordinate of E_y becomes zero. One of the eigenvalues of $J(E_0)$ and $J(E_y)$ is zero.

d) For $a_0 + a_1 + a_2 = 0$, the ordinate of E_e becomes 1. One of the eigenvalues of $J(E_1)$ and $J(E_e)$ is zero.

e) For $a_0 + a_1 + a_2 = \frac{C_{edu} - \Delta\tau - \sigma}{\Delta\phi}$, one of the solution of (41) is $x_i = 1$ and the ordinate of E_i equals to $a_0 + a_1 + a_2$. One of the eigenvalues of $J(E_e)$ is zero.

□

On the *saddle-node bifurcations.* According to the procedure outlined in Owen and Tuwankotta (2022), we employ the Lagrange multiplier method to identify the saddle-node bifurcation point, using a_1 as the bifurcation parameter. This bifurcation point corresponds to the solution of the constrained optimization problem given by:

$$\begin{aligned} \text{Min/Max} &: \; G(x, y, a_1) \\ \text{Subject to} &: \; f(x, y) = 0 \end{aligned} \tag{47}$$

Here, $G(x, y, a_1) = \frac{1}{x}\left(y - a_2 x^2 - a_0\right)$, which is derived by solving $g(x, y) = 0$ in the a_1 term. Using Lagrange multiplier method, the solution of the problem (47) is

$$\begin{aligned} x_{\text{sn}} &= \sqrt{\frac{a_0\Delta\phi - (C_{edu} - \Delta\tau)}{a_2\Delta\phi}}, \quad y_{\text{sn}} = Y_E(x_{\text{sn}}) \\ a_{1\text{sn}} &= -\frac{\sigma}{\Delta\phi} - \frac{2}{\Delta\phi}\sqrt{\Delta\phi a_2(\Delta\tau - C_{edu} + \Delta\phi a_0)} \end{aligned} \tag{48}$$

This $a_{1\text{sn}}$ in equation (48) is the saddle node bifurcation point which defines the critical policy level.

Another approach to identify the saddle-node bifurcation point is by solving the discriminant in equation (41) and setting it equal to zero,[16] and we get

$$a_1 = -\frac{\sigma}{\Delta\phi} \pm \frac{2}{\Delta\phi}\sqrt{\Delta\phi a_2(\Delta\tau - C_{edu} + \Delta\phi a_0)}$$

A saddle-node bifurcation occurs when $\Delta = 0$. Solving for $\Delta\tau$ gives the critical synergy threshold. That is, solving for the policy-synergy index gives the critical synergy needed for a saddle-node given (a_0, a_1, a_2):

$$\Delta\tau_{\text{sn}} = C_{edu} - a_0\Delta\phi + \frac{(a_1\Delta\phi + \sigma)^2}{4a_2\Delta\phi}. \tag{49}$$

For $\Delta\tau > \Delta\tau_{\text{sn}}$, the discriminant is negative and there are no interior fixed points: the system becomes globally attracted to one of the boundary equilibria, and (under mild additional conditions ensuring $K(x) > Y_E(x)$ for all x) the high-education equilibrium becomes the unique attractor. Hence, expression (49) is policy-relevant: it is the minimum effective schooling premium (in income units) required to remove the low-education trap, given structural parameters and the shape of $K(x)$, see Figure 3.

[16] Details can be found in Appendix A. However, this method doesn't provide information about positive equilibria. If we select the negative sign for the $\pm$ notation, we arrive at the same results as $a_{1\text{sn}}$ in eq. (48).

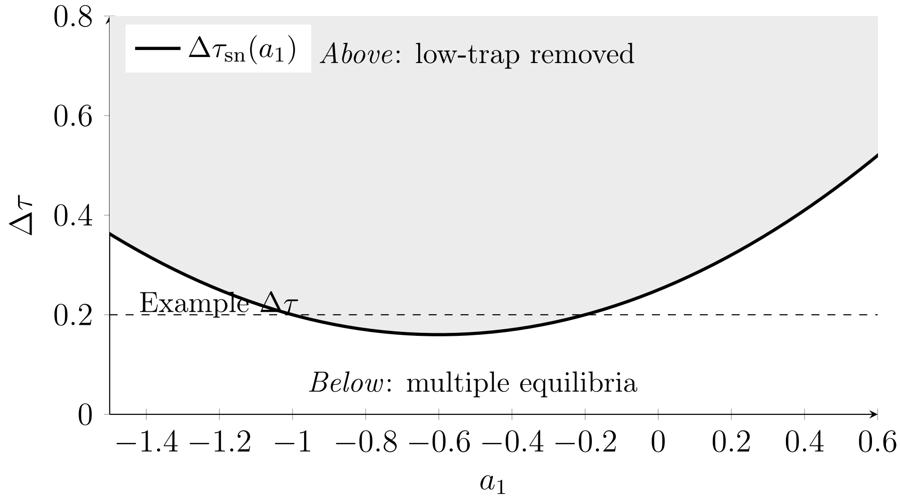

Figure 3: Vectorized saddle-node locus from (49): combinations of a_1 (valley depth) and the effective schooling premium $\Delta\tau$ at which the two interior equilibria merge and disappear. Above the curve ($\Delta\tau > \Delta\tau_{\text{sn}}$) the interior roots vanish (discriminant negative).

4. Simulation Analysis

This section presents numerical examples to illustrate (i) coexistence of equilibria, (ii) how the basin of attraction changes with policy, and (iii) the role of hybrid "push–pull" designs. The numerical parameters are defined: $\Delta\phi = 0.7, C_{edu} = 0.6, \sigma = 0.4, a_0 = 0.97, a_2 = 1.0$. Subtitute into equation (48) and the saddle-node bifurcation curve parameterized by

$$a_{1\text{sn}} = -0.5714285716 - 1.428571429\sqrt{2.8\Delta\tau + 0.2212}.$$

The radicand is non-negative for $\Delta\tau \geq -0.079$.

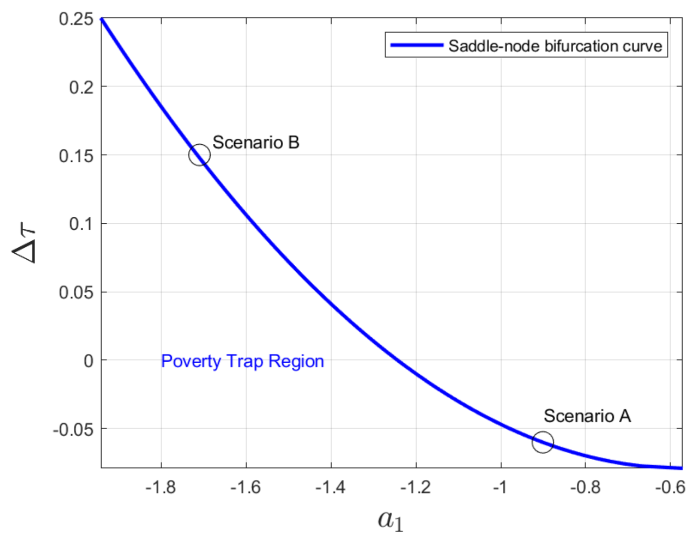

Figure 4: Saddle-node bifurcation curve in $(a_1, \Delta\tau)$-parameter space.

For parameter coordinates below the blue curve in Figure 4, no interior equilibrium exists, indicating that the system converges to the poverty trap equilibrium. Then, we offer two scenarios.

Scenario A: Current Policy ($\tau_{univ} = 0.3, \tau_{cond} = 0.8$) Here $\tau_{cond} < 1$. The policy term is $0.3(0.8 - 1) = -0.06$. Result: The effective cost of education *increases*. The system converges to $x = 0$. Using Lagrange multiplier method, the solution of the problem (47) is

$$x_{\text{sn}} = 0.02857142857\sqrt{1225a_0 - 1155} \tag{50}$$

$$y_{\text{sn}} = 0.9428571429 - 0.01632653061\sqrt{1225a_0 - 1155} \tag{51}$$

In the context of dynamical systems, the analysis of the constraints $0 \le x_{\text{sn}}, y_{\text{sn}} \le 1$ reveals that a saddle-node bifurcation occurs within the parameter range $0.9428571429 \le a_0 \le 1.942857143$. This analysis underscores the significance of parameter selection in the identification of bifurcation phenomena in the system under consideration.

For $a_0 = 0.97$, a saddle-node bifurcation is observed at $a_{1\text{sn}} = -0.9009303593$. The corresponding equilibrium point is characterized by the coordinates $(x_{\text{sn}}, y_{\text{sn}}) = (0.1647508942, 0.8487137748)$. In Table 1, we display the illustration of saddle node bifurcation where the two interior fixed points collide and either appear or disappear as the a_1 parameter changes. In addition to these interior fixed points, we have two saddle points $E_0(0,0)$ and $E_y(0,0.97)$ and one repelling node $E_1(1,0)$, see Figure 5.

Table 1: The stability of the interior fixed point of Scenario A with respect to variations in a_1 near $a_{1\text{sn}}$.

Parameter	Interior fixed point	Real part of Eigenvalue 1	Real part of Eigenvalue 2	Stability
$a_1 = -0.91$	(0.1303653351, 0.8683626657)	$-$	$-$	Stable node
	(0.2082060935, 0.8238822323)	$+$	$-$	Saddle point
$a_1 = a_{1\text{sn}}$	(0.1647508942, 0.8487137748)	0	$-$	
$a_1 = -0.9$	(not exist)			

Then, for $a_1 > a_{1\text{sn}}$, the interior fixed point doesn't exist.

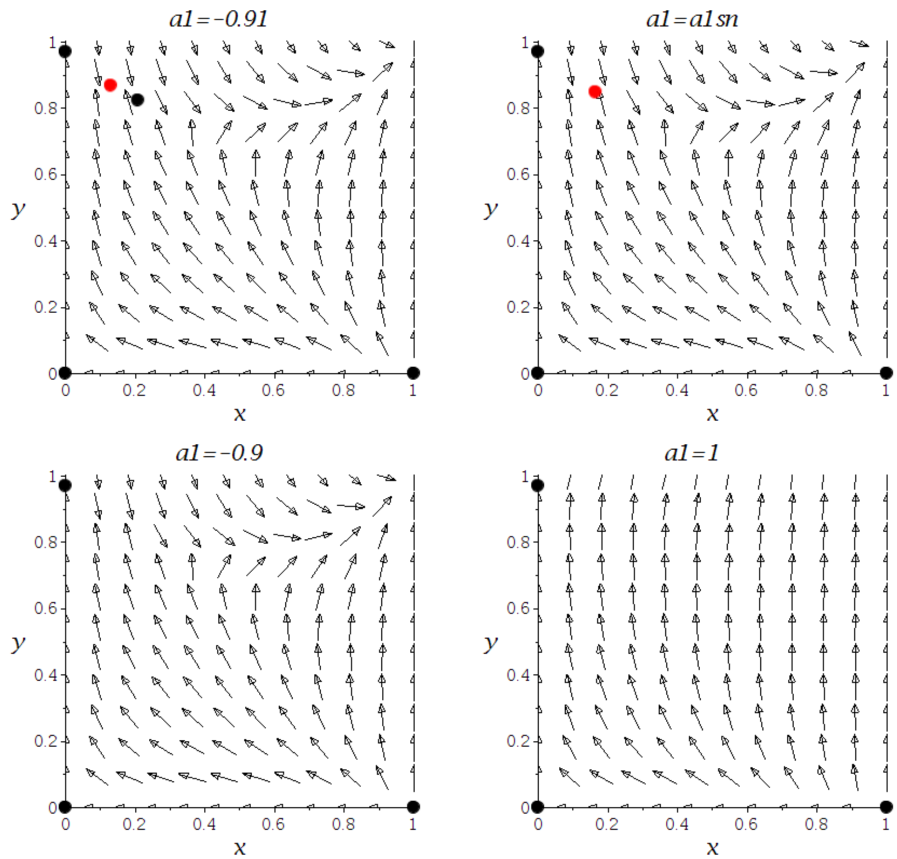

Figure 5: Illustrating the possibility of the interior point in Simulation A.

Scenario B: Synergy Policy ($\tau_{univ} = 0.3, \tau_{cond} = 1.5$) Here $\tau_{cond} > 1$. The policy term is $0.3(1.5 - 1) = +0.15$. Result: The effective cost decreases significantly. The system crosses the saddle point and converges to $x = 1$ (High Equilibrium). Using Lagrange multiplier method, the solution of the problem (47) is

$$x_{\text{sn}} = 0.07142857143\sqrt{196a_0 - 126} \tag{52}$$

$$y_{\text{sn}} = 0.6428571429 - 0.04081632653\sqrt{196a_0 - 126} \tag{53}$$

In the context of dynamical systems, the analysis of the constraints $0 \leq x_{\text{sn}}, y_{\text{sn}} \leq 1$

reveals that a saddle-node bifurcation occurs within the parameter range $0.6428571429 \leq a_0 \leq 1.642857143$. This analysis underscores the significance of parameter selection in the identification of bifurcation phenomena in the system under consideration.

For $a_0 = 0.97$, a saddle-node bifurcation is observed at $a_{1\text{sn}} = -1.715356641$. The corresponding equilibrium point is characterized by the coordinates $(x_{\text{sn}}, y_{\text{sn}}) = (0.5719640349, 0.3160205516)$. In Table 2, we display the illustration of saddle node bifurcation where the two interior fixed points collide and either appear or disappear as the a_1 parameter changes. In addition to these interior fixed points, we have two saddle points $E_0(0,0)$ and $E_y(0,0.97)$, one repelling node $E_1(1,0)$ and one attracting node E_e, see Figure 6.

Table 2: The stability of the interior fixed point of Scenario B with respect to variations in a_1 near $a_{1\text{sn}}$.

Parameter	Interior fixed point	Real part of Eigenvalue 1	Real part of Eigenvalue 2	Stability
$a_1 = -1.8$	(0.3902230408, 0.4198725481)	$-$	$-$	Attracting spiral
	(0.8383483877, 0.1638009213)	$+$	$-$	Saddle point
$a_1 = a_{1\text{sn}}$	(0.5719640349, 0.3160205516)	0	$-$	
$a_1 = -1.5$	-			

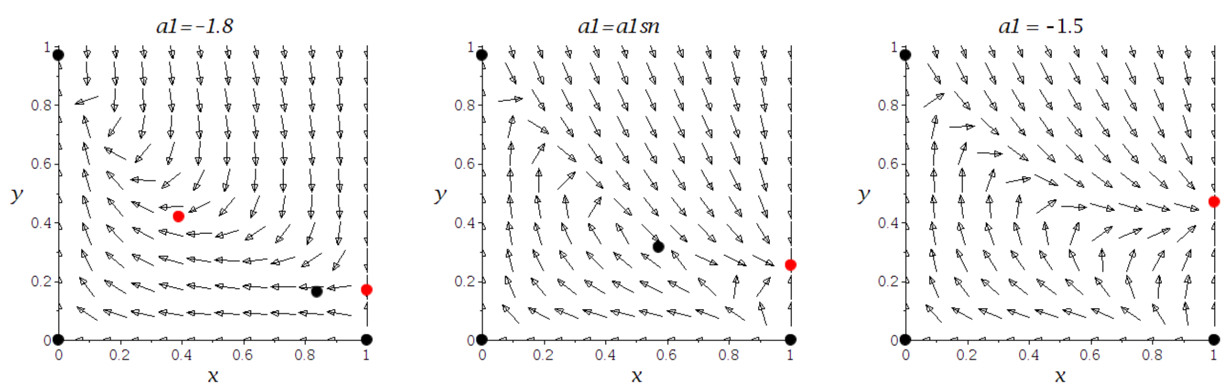

Figure 6: Illustrating the possibility of the interior point in Simulation B.

Then, for $a_1 > a_{1\text{sn}}$, the interior fixed point doesn't exist and for $a_0 + a_1 + a_2 > 1$ or $a_0 + a_1 + a_2 < 0$, the edge fixed point E_e doesn't exist, see Figure 7.

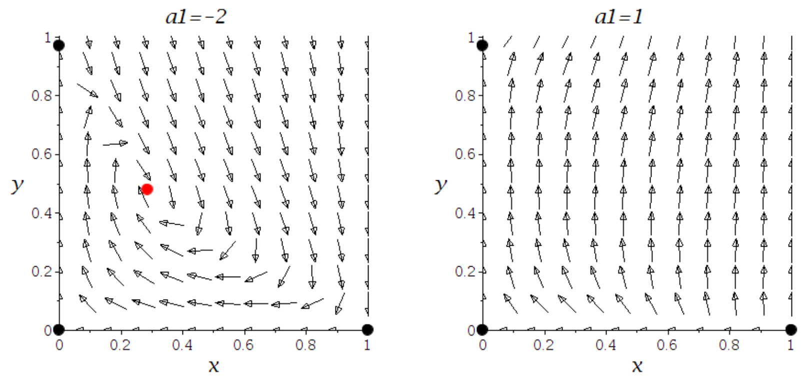

Figure 7: Illustrating the possibility of the interior point in Simulation B.

4.1. Hybrid escape vs trapped dynamics

Let us offer an illustrative calibration to highlight the mechanisms discussed analytically in the preceding sections. We now focus on the policy interpretation. We use the quadratic income potential $K(x) = a_2x^2 + a_1x + a_0$ with $a_1 < 0$ to create a "valley" for low education shares. A baseline choice is: $a_0 = 0.2,\ a_1 = -0.3,\ a_2 = 1.2$. We set education costs and social multiplier to: $C_{\text{edu}} = 0.6,\ \sigma = 0.4,\ \Delta\phi = 0.7,\ \alpha > 0$. Policy is represented by $(\tau_{univ}, \tau_{cond})$ through $\Delta\tau = \tau_{univ}(\tau_{cond} - 1)$. We simulate the system for an economy in the "Trap" $(x_0 = 0.1, y_0 = 0.2)$. The results are given below.

Figure 8: threshold crossing and S-shaped takeoff. This figure visualizes the core escape mechanism of the model. In the "universal-only" configuration (red curve; $\tau_{cond} = 1$ so $\Delta\tau = 0$), the education incentive

$$f(x, y) = \Delta\phi\, y - C_{edu} + \sigma x + \Delta\tau$$

remains negative at low (x, y), so replicator dynamics drive the education share downward (equation (22)). Lower x then depresses income potential through $K(x)$ (equation (29)), reinforcing high poverty through the mapping (31). In the hybrid configuration (green curve), a higher schooling premium raises $\Delta\tau$ and shifts the threshold line $y = Y_E(x)$ downward (equation (32)). Once the trajectory crosses the separatrix associated with the interior saddle (right panel), the feedback becomes self-reinforcing: higher x raises y, which increases $f(x, y)$ and accelerates adoption of education. This generates the characteristic S-shaped "takeoff" in $x(t)$.

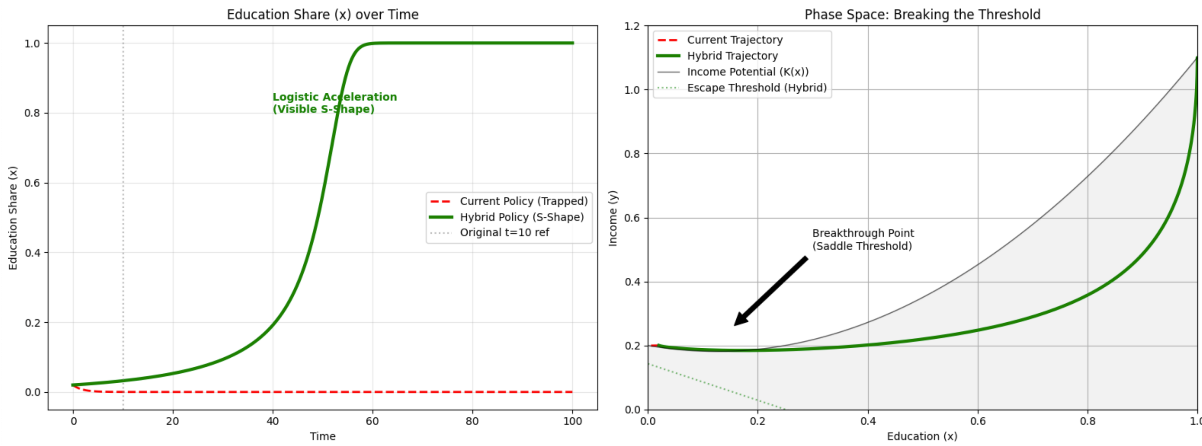

Figure 8: Hybrid escape versus trapped dynamics. Left: $x(t)$ under a universal-only regime ($\tau_{cond} = 1$, $\Delta\tau = 0$) versus a hybrid regime with $\tau_{cond} > 1$ (positive $\Delta\tau$). Right: phase diagram with income potential $K(x)$ and the education-incentive threshold $Y_E(x)$.

In Figure 8 the left panel shows that a universal-only regime ($\tau_{cond} = 1$, $\Delta\tau = 0$) may stabilize at a low education share when opportunity costs remain high. Under a hybrid regime with $\Delta\tau > 0$, education can display a visible S-shape: slow initial growth, acceleration near the threshold, and saturation as $x \to 1$. The right panel plots the same logic in the (x, y) plane: the trajectory must cross the unstable manifold associated with the interior saddle (the "breakthrough" point). Policy works by shifting $Y_E(x)$ downward (increasing $\Delta\tau$), making threshold crossing feasible.

Figure 9: why the same policy can succeed in one region and fail in another. This figure illustrates three stylized structural regimes obtained by changing

$(a_0, a_1, a_2, \sigma, C_{\text{edu}})$:[17]

1. Regime I ("Asian Tiger"): high social multiplier and convex returns produce escape even under weak policy.
2. Regime II ("LATAM Trap"): bistability; universal-only stagnates, synergy escapes.
3. Regime III ("Fragile State"): deep J-curve valley; a "big push" is required to subsidize households through the income dip.

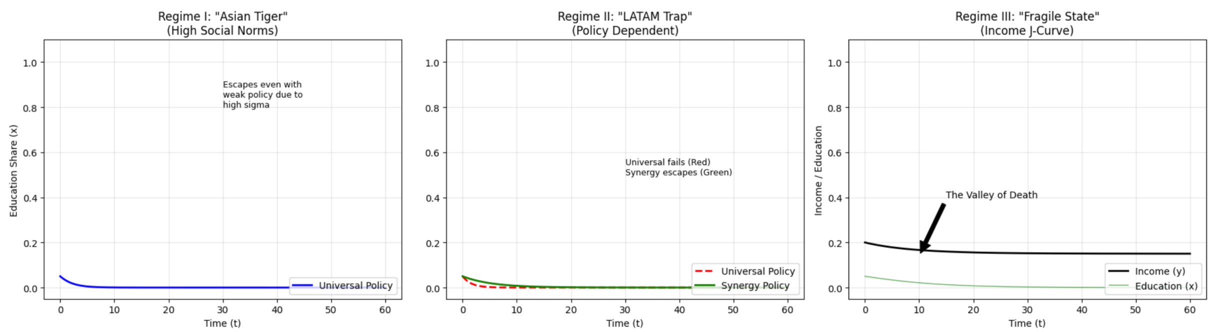

Figure 9: Three stylized development regimes generated by different structural parameters (e.g., the depth of the low-x "valley" in $K(x)$ and the strength of social interactions σ). The figure is used to interpret heterogeneity across economic regions: some are structurally resilient, some are policy-dependent (bistable), and some exhibit a "valley of death" where early education gains may not immediately translate into higher income potential.

Figure 9 shows that in the "Asian Tiger" regime, social interactions are strong (high σ) and/or $K(x)$ is sufficiently monotone so that the high-education state is a robust attractor even with relatively weak policy incentives. In the "LATAM trap" regime, the system is

[17] This figure provides a qualitative typology of regional heterogeneity generated by different structural parameters $(\sigma, a_0, a_1, a_2, C_{edu}, \Delta\phi)$. In "resilient" regions, social interactions are strong (high σ) and/or $K(x)$ is sufficiently monotone so that the high-education state is a robust attractor. In "policy-dependent" regions, the system can be bistable and the basin of attraction can be shifted by hybrid designs. In "fragile" regions, a deep low-x valley in $K(x)$ creates a transitional "valley of death" where early increases in education do not immediately raise income potential, so $\Delta\tau$ must be sufficiently large (and/or complementary investments must raise K) to ensure escape.

policy-dependent and can be bistable: a low-education equilibrium coexists with a high-education equilibrium (cf. Figures 6-7), so hybrid "push–pull" designs can shift the basin of attraction and facilitate escape. In the "Fragile State" regime, a deep low-x valley in $K(x)$ (large negative a_1 in (30)) creates a transitional "valley of death" in which modest increases in education can temporarily lower income potential, keeping $f(x, y)$ negative unless policy incentives are sufficiently strong and/or complementary investments raise returns to education.

Figure 10: conditionality vs. universal floors vs. hybrid designs.[18] This figure compares three stylized regimes for the education-share dynamic $x(t)$: a universal-only regime ($\tau_{cond} = 1$), a Prospera-like strict CCT (higher τ_{cond}), and a hybrid design (higher $\Delta\tau$). When initial conditions are inside the low-education basin, all trajectories can converge downward, but stronger conditionality and hybrid designs slow the decline and preserve higher $x(t)$ longer. This is policy-relevant because preserving $x(t)$ increases the likelihood that complementary shocks or reforms (raising $K(x)$ or lowering C_{edu}) can push the system across the threshold.

[18]The figure is naturally interpreted as a *resilience* comparison: conditionality and synergy slow the fall and can "buy time" for complementary interventions (supply-side school quality, transport, enforcement, local labor demand) to move the state across the threshold.

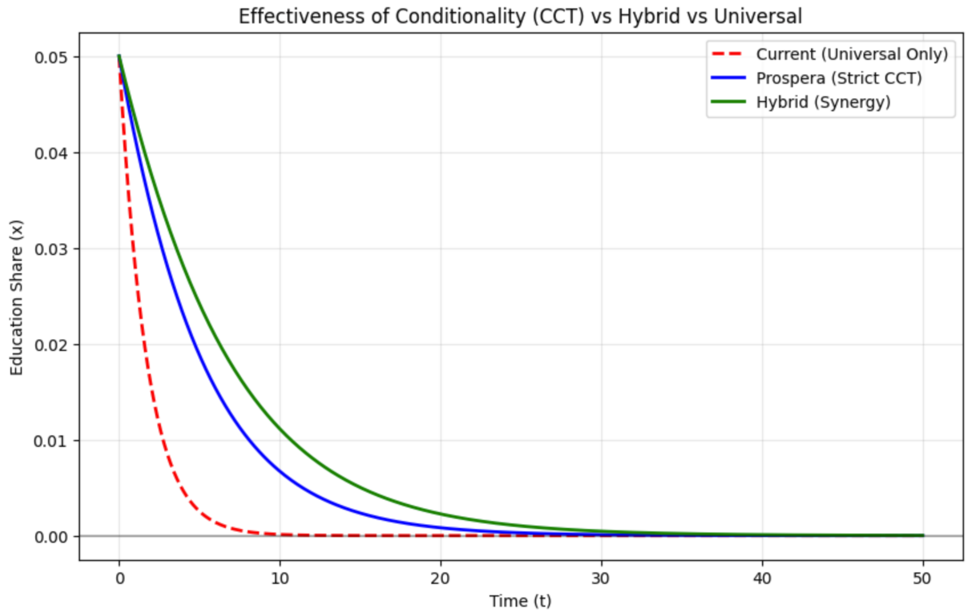

Figure 10: Illustrative comparison of policy regimes for the education-share dynamics $x(t)$: universal-only baseline ($\tau_{cond} = 1$), a Prospera-like strict CCT ($\tau_{cond} > 1$), and a hybrid design (larger effective schooling premium $\Delta\tau$). In this calibration, initial conditions lie in the low-education basin, so all trajectories converge toward low education, but stronger conditionality and hybrid designs slow the decline and preserve higher $x(t)$ for longer—raising the likelihood of eventual threshold crossing when combined with shocks or complementary structural changes (e.g., higher $K(x)$ or lower C_{edu}).

The hybrid design dominates by preserving a higher education share for longer, which increases the probability of threshold crossing in the presence of shocks and makes complementary policies (raising $K(x)$ or lowering C_{edu}) more effective. Analytically, robust escape requires meeting the saddle-node threshold $\Delta\tau \geq \Delta\tau_{\text{sn}}$ in equation (49).

What does "escaping the poverty trap" mean here?

In this framework, a region "escapes" when trajectories move from the basin of attraction of the low-education equilibrium (e.g., E_y) to the basin of the high-education equilibrium (near E_e), implying permanently higher long-run income y and thus lower poverty $p(x, y)$. Policy can change escape prospects by shifting the threshold line $Y_E(x)$ downward (via higher $\Delta\tau$) and by raising the income-potential curve $K(x)$ (via supply-side development that increases returns to education). The policy-relevant diagnostic is the critical synergy threshold (49). If $\Delta\tau < \Delta\tau_{\text{sn}}$, the trap persists (multiple equilibria). If $\Delta\tau \geq \Delta\tau_{\text{sn}}$, the low-education equilibrium can disappear via a saddle-node bifurcation, making escape robust.

5. Concluding Remarks and Future Research

The dilemmas posed by the application of Conditional Cash Transfers (CCTs) and Unconditional Cash Transfers (UCTs) policies represent one of the most significant shifts in contemporary development economics. This paper has provided a rigorous macro-evolutionary framework to evaluate such an issue, fundamentally challenging the assumption that unconditional and conditional transfers are simple additive substitutes.

By integrating a *Policy Synergy Term* ($\Delta\tau$) into a replicator dynamic equation, coupled with a macroeconomic J-Curve of regional productivity, we have analytically demonstrated that universal floors and conditional incentives are strict complements. Hence, we can claim that in economies characterized by fragile local labor markets and high opportunity costs—where initial investments in human capital plunge households into a transitional "Valley of Death"—unconditional transfers alone are insufficient. In fact, our model proves that if the conditional multiplier is weak ($\tau_{cond} < 1$), increasing universal cash transfers can perversely act as a regressive subsidy that accelerates exit into the informal labor market.

This theoretical result perfectly mirrors the recent stylized facts observed in Mexico following the rollback of *Prospera* and the introduction of the semi-universal *Benito Juárez Scholarships*. As documented by recent empirical literature (Anagol et al., 2025; Marquez-Padilla et al., 2025; Parker and Todd, 2017), the sudden removal of conditionality led to immediate spikes in school dropouts and a regressive reallocation of resources. Our model formally explains this phenomenon: without the conditional price-signal to force the system through a saddle-node bifurcation, the evolutionary dynamic naturally gravitates back toward the stable, low-education subsistence trap. The optimal policy for developing nations is therefore a multiplicative hybrid—a universal base to alleviate extreme liquidity constraints, amplified by strict conditionalities to override the immediate opportunity costs of education. Hence, for economies facing structural transition costs (the J-Curve), the optimal policy is a multiplicative hybrid: a broad universal base to eliminate extreme liquidity constraints, amplified by a strong conditional factor ($\tau_{cond} > 1$) to override immediate opportunity costs. This alignment forces the necessary saddle-node bifurcation to permanently escape the poverty trap, providing a robust theoretical backing to recent empirical findings regarding the rollback of conditional cash transfer programs.

5.1. Future Research Directions

While this model provides a robust theoretical foundation for the synergy of social programs, it opens several promising avenues for future research:

1. Endogenizing Local Labor Market Dynamics: Following the empirical insights of Molina et al. (2026), future theoretical extensions should endogenize the opportunity cost of education (C_{edu}). By explicitly modeling the labor demand side—such as the arrival of export-manufacturing shocks—researchers could analyze how exogenous shifts in low-skilled wages dynamically alter the required threshold for the conditional policy multiplier (τ_{cond}).

2. Spatial and Network Econometrics: The evolutionary model assumes a uniform social multiplier (σ) within the population. Future empirical work should leverage spatial econometrics and network analysis to estimate how peer effects operate across different regional topologies (e.g., urban vs. rural settings). Calibrating the σ parameter with

real-world network data would allow policymakers to identify precise, region-specific thresholds for the saddle-node bifurcation.

3. Expanding the Synergy Framework Beyond Education: The mathematical architecture of the *Policy Synergy Term* is not limited to human capital accumulation in youth. As demonstrated by Galvez-Soriano and Peralta (2026), the universalization of social pensions paradoxically induced labor force withdrawal and increased extreme poverty among the elderly. Future macro-evolutionary models could adapt this framework to analyze the tradeoffs between universal pensions and conditional active-labor-market policies for older demographics, testing the robustness of the "leisure trap" hypothesis across the entire life cycle.

4. Optimal Dynamic Taxation: Finally, this model currently treats the fiscal budget as exogenous. A vital next step is to close the macroeconomic loop by introducing dynamic taxation. Investigating the optimal tax trajectory required to fund the "big push" hybrid policy—without depressing the convex returns of the high-skill sector (a_2x^2)—will be critical for determining the long-term fiscal sustainability of synergistic social programs.

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Appendix A. Identifying the saddle-node bifurcation

The discriminant of quadratic equation (41) as

$$D = (\Delta\phi a_1 + \sigma)^2 - 4\Delta\phi a_2(\Delta\phi a_0 + \Delta\tau - C_{edu})$$

A qualitative change in the number of equilibria occurs when $D = 0$, which defines the critical policy level $\Delta\tau_{\text{sn}}$. We get

$$\Delta\tau_{\text{sn}} = C_{edu} - \Delta\phi a_0 + \frac{(\Delta\phi\, a_1 + \sigma)^2}{4\Delta\phi a_2}$$

When $\Delta\tau < \Delta\tau_{\text{sn}}$, the discriminant negative and no interior point exists, implying that the system converges to the poverty trap equilibrium. When $\Delta\tau > \Delta\tau_{\text{sn}}$, the discriminant positive and two interior equlibria exists. To express the bifurcation condition in the terms of parameter bifurcation a_1, solving the latest equation for a_1 and we get

$$a_{1\text{sn}} = \frac{\sigma}{\Delta\phi} \pm \frac{2}{\Delta\phi}\sqrt{\Delta\phi a_2(\Delta\tau_{\text{sn}} - C_{edu} + \Delta\phi a_0)}$$

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