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Memory Retrieval and Harshness of Conflict in the Hawk-Dove Game

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Abstract

We study the long-run dynamics of a repeated non-symmetric Haw-Dove type interaction between agents of two different populations. Agents choose a strategy based on their previous experience with the other population by sampling from a collective memory of past interactions. We assume that the sample size differs between populations and define a measure of harshness of conflict in the Hawk-Dove interaction. We then show how the properties of the long-run equilibrium depend on the harshness of conflict and the relative length of the sample. In symmetric interactions, if conflict is harsh, the population which samples relatively more past interactions is able to appropriate a higher payoff in the long-run, while the population with a relatively smaller sample do so if conflict is mild. These results hold subject to constraints on the sample size which we discuss in detail. We further extend our results to non-symmetric Hawk-Dove games.

Keywords: conflict, memory, hawk dove, evolution, stochastic stability

1 Introduction

Both the triggers and the dynamics of conflict have been widely investigated in theoretical biology (Hamilton, 1964, Smith, 1974), economics (Garfinkel and Skaperdas, 2012, Kimbrough et al., 2020), evolutionary anthropology (Glowacki et al., 2020), and psychology (Böhm et al., 2020). Outcomes of conflicts vary from active aggression and fighting (Archer, 1988, Huntingford and Turner, 1987) to resource sharing (Wilkinson, 1984). Among other types of interactions, the *Hawk-Dove game* is a simplified representation of conflict within the context of resources sharing (Rusch and Gavrilets, 2020, Smith and Price, 1973). If agents of a single population are randomly matched, the equilibrium state of the Hawk-Dove game is characterized by a mixed state in which a fraction of the population plays aggressively (i.e. hawk) and fight over a resources while the rest acts peacefully (i.e. dove) and avoids conflict. In a two-population setting, instead, the evolutionary stable states are defined by an equilibrium in which one population only plays hawk and the other population only plays dove (Weibull, 1997).

Since the evolutionarily stable states define a unique and different strategy for each population, two possible equilibrium states can occur: either population 1 plays dove and population 2 plays hawk, or the inverse can hold true. The existence of two possible equilibrium states imposes a selection problem. The dynamics leading to one of the two equilibria depend, at least in the medium-to-long run, on the initial state and the equilibria's respective basins of attraction. In the very long run, however, literature on stochastic stability has shown that the initial state becomes irrelevant (Foster and Young, 1990): small noise renders the dynamic system ergodic and, thus, a population keeps moving across the entire state space. Once the noise abates, the system dynamic spends most of its time at the equilibrium state to which access requires the lowest number of errors and which is hence the easiest to reach.

By using the latter approach to study the dynamics of the Hawk-Dove game, our research is situated at the intersection of the literature analyzing conflict based on the hawk-dove game (Smith, 1979, 1982), the theoretical research on the evolution of traits and behaviors in animal and human populations (Hofbauer et al., 1998, Gintis et al., 2000, Newton, 2018) and the theories on the long-run evolution of conventions and institutions (Kandori et al., 1993, Young, 1993). We further generalize results in Young (2001).

More specifically, we consider a two-population setting, where agents of one population are matched with agents of the other population to compete repeatedly over time for resources in a Hawk-Dove type of interaction. Agents sample from a collective memory of the last actions and determine an optimal strategy based on the relative frequency of each strategy - a process known as *adaptive play*. In addition, we assume that agents within the same population base their actions on samples of the same size, but the sample sizes differ between the two populations. We find that the stochastically stable equilibrium – that is the equilibrium at which the system spends most of the time in the presence of small random errors – depends on the harshness of conflict, i.e., a measure of the cost of losing a fight relative to the benefit derived from winning the resource. If conflict is harsh, the population with the larger sample size chooses strategy hawk and the population with the smaller sample size opts for dove. The reverse occurs if conflict is mild. Consequently, our results confirm recent findings in the literature that the cost of fighting can play a crucial role in Hawk-Dove type interactions (Hall et al., 2020).

The intuition behind our result is that if the harshness of conflict is high, relatively few errors among dove players are needed for the other population to accommodate and switch to dove. Consequently, if the former population uses the larger sample, the latter population relies on a smaller sample and, thus, requires a smaller number of hawk plays to switch the best-response strategy to dove. In other words, fewer perturbations are required to push a population into the basin of attraction of the equilibrium in which the population with the smaller sample size plays dove and the population with the larger sample size chooses hawk. If the harshness of conflict is mild, instead, relatively few errors of initial hawk players are required to induce a transition between the equilibrium states. The easiest transition then occurs if the population with the larger sample size erroneously chooses dove, eventually leading to a stochastically stable state in which the population with the smaller sample size chooses hawk.

The remainder of this paper is organized as follows: in Section 2, we describe the main characteristics of the model and the dynamics of the unperturbed game. In Section 3 we present our main results that determine the long-run dynamics of the perturbed game. Finally, we provide a discussion in Section 4. All proofs are relegated to the Appendix.

2 The model

We define two populations of finite size: blue agents (or *blues*) denoted by B and green agents (or *greens*) denoted by G. Time is discrete $t = 1, 2, \ldots$ and in each period, one agent is drawn at random from each population to interact in the hawk-dove game depicted in Figure 1. Each agent i chooses a pure strategy s_i from a strategy set $S_i = \{H, D\}$ with $i = \{B, G\}$. Play at time t is defined as $s(t) = (s_B(t), s_G(t))$ and the payoff of each player i

is $\pi_i(s(t))$ according to the payoff matrix. We assume that $C_i > V_i$.

		GREEN POPULATION	
		H	D
BLUE POPULATION	H	$\frac{V_B - C_B}{2}$, $\frac{V_G - C_G}{2}$	V_B , 0
	D	0 , V_G	$\frac{V_B}{2}$, $\frac{V_G}{2}$

Figure 1: The hawk-dove game.

Agents recall the last m periods of play between both populations, hence m can be interpreted as the (collective) long-term *memory length*. A history of play encompassing the last m periods is described by $h(t) = (s(t), s(t-1), \dots, s(t-m+1))$, with t denoting the current period.

Furthermore, agents adjust their choices over time according to the adaptive learning assumptions in Young (1993). In general, agents select the best response to a randomly drawn sample of k opponents' plays in their memory. In case of multiple best responses, all of them have positive probability to be selected. As is standard in the literature, we refer to k as *sample size*, and we interpret it as working memory. Differently from the literature, we assume that the sample size is population dependent, with k_B denoting the sample size of blue agents, and k_G denoting the sample size of green agents. Consistently, we assume $k_B \leq m$ and $k_G \leq m$. We denote by n_B^t the number of D instances recorded in the blues' memory, and by n_G^t the number of D instances recorded in the greens' memory. At the end of each interaction period, the current play is registered in the memory and the oldest play is forgotten.

The dynamic system under consideration is a Markov chain (S, T) (see Young, 2001, for an overview of Markov chain theory), where S is the state space composed of all possible histories, i.e., sequences of m plays of the game $(s^m, \dots, s^\ell, \dots, s^1)$, with $s^\ell \in \{(H,H),(H,D),(D,H),(D,D)\}$ for all $\ell = 1, \dots, m$. Transition between states is defined by transition matrix T, with $T_{hh'}$ being the probability of moving from history h to history h' in one period of time according to the above adjustment dynamics. It must hold that

$T_{hh'} > 0$ only if h' can be obtained from h by deleting the rightmost play of the game and adding a new play of the game to the left of the sequence.

Any state h consisting of m repetitions of a strict Nash equilibrium constitutes a *convention*, that is inescapable given the defined dynamics. The hawk-dove game described in Figure 1 has two strict Nash equilibria: (H, D) and (D, H). The two corresponding conventions are defined by $h_{HD} = (s^m, \ldots, s^\ell, \ldots, s^1)$ such that $s_B^\ell = H$ and $s_G^\ell = D$ for all $\ell = 1, \ldots, m$, and $h_{DH} = (s^m, \ldots, s^\ell, \ldots, s^1)$ such that $s_B^\ell = D$ and $s_G^\ell = H$ for all $\ell = 1, \ldots, m$.

Further, let α_B denote the fraction of D instances in the sample of a *green* agent. Similarly, let α_G denote the fraction of D instances in the sample of a *green* agent. For a blue agent, the expected payoff of playing H is $(1 - \alpha_B)(V_B - C_B)/2 + \alpha_B V_B$ while the expected payoff of playing D is $\alpha_B V_B/2$. Similarly, for an agent of the green population the expected payoff of playing H is $(1 - \alpha_G)(V_G - C_G)/2 + \alpha_G V_G$ while the expected payoff of playing D is $\alpha_G V_G/2$. Agents in population k are indifferent between both strategies if α_k equals

$$\alpha_k^* = 1 - \frac{V_i}{C_i}$$

In other words, if the relative frequency of D in the sample exceeds α_k^*, the optimal response is to play H in the current period. The ratio α_k^* can also be understood as a measure of conflict harshness. We call a conflict *harsh* if $\alpha^* > 0.5$, and *mild* if $\alpha^* < 0.5$.

A set of states $\mathcal{C}$ is a *recurrent class* if: (i) every pair of states in $\mathcal{C}$ communicate with each other (i.e. there is a positive probability to move from one state to the other in a finite number of steps) and (ii) no state in $\mathcal{C}$ communicates with a state not in $\mathcal{C}$ (i.e. the probability of leaving $\mathcal{C}$ is zero). By definition, a convention is a recurrent class comprised of a single state. However, there is no guarantee in general that the system will converge to a convention, since it might cycle within a set of states. The following Lemma gives a necessary and sufficient condition to ensure convergence to a convention.

LEMMA **1.** *$\{h_{HD}\}$ and $\{h_{DH}\}$ are the only two recurrent classes if and only if at least one of the following conditions holds: (i) $\min\{k_B, k_G\} < m$, (ii) there exists an integer number q such that $q = \alpha_k^* m$ for some $i \in \{B, G\}$, (iii) there exists an integer number q such that $\alpha_k^* m < q < \alpha_j^* m$, with $i, j \in \{B, G\}$, $i \neq j$.*

3 Perturbed dynamics

Now suppose that in general, a player does not choose a strategy that is a best response to the sample, but chooses one of the two strategies at random with a small probability ε close to zero. Any history of play h in t can then move to any other state h' in $t+m$ with positive probability. The Markov chain is irreducible and aperiodic, and the process thus ergodic. In the following, we determine the conditions for the convention in the long-run, first for the case in which payoffs are symmetric and thereafter for the case in which payoffs are asymmetric.

3.1 Symmetric harshness of conflict

We assume $k_G < k_B$. The *stochastic potential* of a convention is the minimum number of errors involved in the transition from the opposite convention to the former convention. The following Lemma characterizes the stochastic potentials of the two conventions if payoffs are symmetric, i.e. $V_B = V_G$ and $C_B = C_G$.

LEMMA **2.** *The stochastic potentials of conventions h_{DH} and h_{HD} are given by, respectively, $r_{DH} = \min\{\lceil(1-\alpha^*)k_B\rceil\ ,\ \lceil\alpha^* k_G\rceil\}$ and $r_{HD} = \min\{\lceil\alpha^* k_B\rceil\ ,\ \lceil(1-\alpha^*)k_G\rceil\}$.*

We have the following:

PROPOSITION **1.** *If conflict is harsh:*

(a) h_{HD} is a stochastically stable state;

(b) if $k_B - \frac{1}{1-\alpha^*} \geq k_G \geq \frac{1}{2\alpha^*-1}$ then h_{HD} is the only stochastically stable convention.

In other words, if conflict is harsh then the state in which blue agents only choose H and green agents only choose D is always stochastically stable, and there exists a region in the parameter space in which it is the unique stochastically stable convention.

In contrast:

PROPOSITION **2.** *If conflict is mild:*

(a) h_{DH} is a stochastically stable state;

(b) if $k_B - \frac{1}{\alpha^*} \geq k_G \geq \frac{1}{1-2\alpha^*}$ then h_{DH} is the only stochastically stable convention.

This means that if conflict is mild then the state in which blue agents choose D and green agents H is a stochastically stable state and the unique convention if the sample size fulfills condition *(b)*. We note that, if $k_B = k_G$, then h_{HD} and h_{DH} are both long-run conventions.

3.2 Asymmetric harshness of conflict

We now consider the asymmetric case in which $\alpha^*_B \neq \alpha^*_G$, implying that $V_B \neq V_G$ or $C_B \neq C_G$ in Figure 1. We assume that $k_B + k_G \leq m$ and $k_G < k_B$. The stochastic potential of each convention is given in the next Lemma.

LEMMA **3.** *The stochastic potentials of conventions h_{DH} and h_{HD} are given by, respectively, $r_{DH} = \min\{\lceil(1-\alpha^*_B)k_B\rceil\ ,\ \lceil\alpha^*_G k_G\rceil\}$ and $r_{HD} = \min\{\lceil\alpha^*_B k_B\rceil\ ,\ \lceil(1-\alpha^*_G)k_G\rceil\}$.*

We maintain the assumption that $k_G < k_B$. The following Proposition provides the sufficient conditions for the conventions to be stochastically stable.

PROPOSITION **3.** *Assuming that $k_G < k_B$. We have:*

(a) if $\alpha^*_G > T(\alpha^*_B; k_B, k_G)$, state h_{HD} is stochastically stable;

(b) if $\alpha^*_G < T(\alpha^*_B; k_B, k_G)$, state h_{DH} is stochastically stable;

where:

$$T(\alpha^*_B; k_B, k_G) = \begin{cases} \alpha^*_B \frac{k_B}{k_G}, & \text{if } \alpha^*_B \leq \frac{k_G}{2k_B} \\ 0.5 & \text{if } \frac{k_G}{2k_B} < \alpha^*_B \leq 1 - \frac{k_G}{2k_B} \\ \alpha^*_B \frac{k_B}{k_G} + \frac{k_G - k_B}{k_G} & \text{if } 1 - \frac{k_G}{2k_B} < \alpha^*_B \end{cases}$$

The results are summarized in Figure 2. Notice that the slope of the oblique intervals of $T(\alpha^*_B; k_B, k_G)$ is given by the ratio k_B/k_G. If k_B (along with m) tends to infinity (i.e. the population can keep all past play in the memory and no play is forgotten), the slope is vertical and only the horizontal interval of $T(\alpha^*_B; k_B, k_G)$ matters. In this case the sufficient conditions for both the conventions to be stochastically stable depend only on the harshness of conflict of the population with the shortest length of memory. The second limit case is $k_B = k_G$. In this case the two populations draw samples of the same length and the slope of the two oblique intervals is equal to one. The convention in which the population with the lower level of harshness plays hawk is always stochastically stable. The following Proposition defines the conditions for the uniqueness of the stochastically stable convention.

PROPOSITION **4.** *Assuming that $k_G < k_B$. We have:*

(a) if $\alpha^*_G > T_{HD}(\alpha^*_B; k_B, k_G)$ then the convention h_{HD} is the only stochastically stable convention;

(b) if $\alpha^*_G < T_{DH}(\alpha^*_B; k_B, k_G)$ then the convention h_{DH} is the only stochastically stable convention;

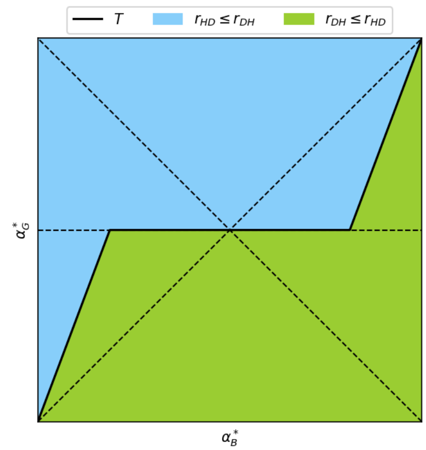

Figure 2: In the blue area the convention h_{HD} is stochastically stable, while in the green area the convention h_{DH} is stochastically stable.

Where:

$$T_{HD}(\alpha^*_B; k_B, k_G) = \begin{cases} \alpha^*_B \frac{k_B}{k_G} + \frac{1}{k_G}, & \textit{if } \alpha^*_B \leq \frac{k_G}{2k_B} \\ \frac{k_G+1}{2k_G} & \textit{if } \frac{k_G}{2k_B} < \alpha^*_B \leq 1 - \frac{k_G}{2k_B} \\ \alpha^*_B \frac{k_B}{k_G} + \frac{1+k_G-k_B}{k_G} & \textit{if } 1 - \frac{k_G}{2k_B} < \alpha^*_B \end{cases}$$

and

$$T_{DH}(\alpha^*_B; k_B, k_G) = \begin{cases} \alpha^*_B \frac{k_B}{k_G} - \frac{1}{k_G}, & \textit{if } \alpha^*_B \leq \frac{k_G+1}{2k_B} \\ \frac{k_G-1}{2k_G} & \textit{if } \frac{k_G+1}{2k_B} < \alpha^*_B \leq 1 - \frac{k_G-1}{2k_B} \\ \alpha^*_B \frac{k_B}{k_G} + \frac{k_G-k_B-1}{k_G} & \textit{if } 1 - \frac{k_G-1}{2k_B} < \alpha^*_B \end{cases}$$

Results are summarized in Figure 3.

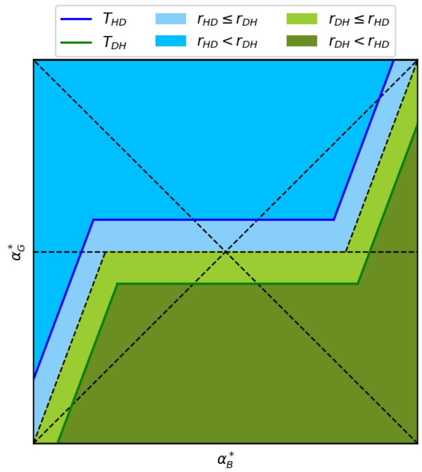

Figure 3: In the dark-blue area, h_{HD} is the unique stochastically stable convention, while in the dark-green area, h_{DH} is the unique stochastically stable convention.

4 Discussion

In this paper, we studied the long-run dynamics of the two-population Hawk-Dove game under perturbed adaptive learning. We demonstrated that information heterogeneity between both populations arising from a difference in the number of past interactions sampled by the members of each population affects the long-run dynamics and hence, the attribution of a resource. In particular, we showed that the harshness of conflict plays a critical role: if the cost of losing a fight is small/large relative to the benefit of the resource, the population with the smaller/larger sample plays hawk and the other population plays dove. Consequently, the impact of an information advantage matters in a non-trivial way, and our results indicate that is is an essential component that needs to be carefully considered when modelling the dynamics of conflict (Rusch and Gavrilets, 2020). Since we obtain conditions under random matching of the members of two populations, future research should investigate the robustness of our results if mixing is assortative, on social networks or if agents are spatially segregated (Aydogmus, 2018).

Common wisdom suggests that having more abundant cognitive or physical resources is, ceteris paribus, beneficial for the evolutionary success of any living species. The reason why we observe species with rather limited cognitive capacities is generally attributed to the increasing cost of such an apparatus. Yet, our results suggest cognitive limitations can result in a relative fitness advantage even in the absence of costs of sustenance. In conflict situations, similar to the Hawk-Dove game, the population with a smaller working memory tends to be more aggressive and earn higher payoffs than the population with larger working memory which is more peaceful, if conflict is mild. This finding is in line with recent results in the literature (Doi and Nakamaru, 2018).

For simplicity, we assumed that population sizes are identical and fixed. In a more realistic context, increased payoffs translate into higher fitness. At the same time, different population sizes alter the frequency of pairwise interactions and thus affect the updating process. A larger group size, on the other hand, reduces the cost of conflict and thus the harshness measure. Future research should find that the interplay between population dynamics and harshness of conflict may be conducive to interesting insights.

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Author contributions statement

E.B., L.B., S.I., and E.V. contributed equally to setting the model and developing theoretical results, as well as to writing the paper.

Additional information

The authors declare no competing interests.

Appendix

In the previous sections we identify sufficient conditions for h_{HD} and h_{DH} to be the unique stochastically stable conventions. We do not provide necessary conditions but through graphical analysis we can assess the goodness of these sufficient conditions (see Figure 4).

If $\min\{k_B, k_G\}$ is an even number, the sufficient conditions under which a convention is the unique stochastically stable state can be refined for both symmetric and asymmetric

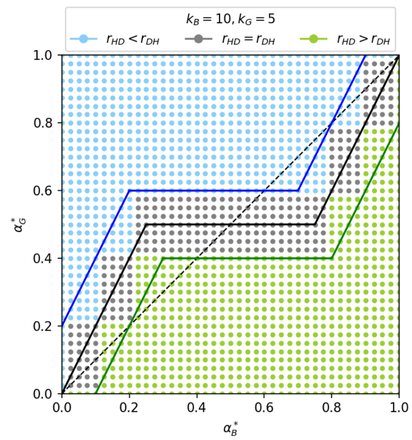

Figure 4: Each point on the grid represents a different combination of (α_B^*, α_G^*). Blue and green points represent values for which h_{HD} and h_{DH} are the unique stochastically stable convention respectively. Grey points represent values of (α_B^*, α_G^*) for which both the conventions are stochastically stable. The three polygonal chains represent, starting from the left, $T_{HD}(\alpha_B^*; k_B, k_G)$, $T(\alpha_B^*; k_B, k_G)$, and $T_{DH}(\alpha_B^*; k_B, k_G)$.

conflicts. In particular conditions (c) and (d) of Propositions 1 and 2 can be rewritten as:

Proposition 5. *If conflict is harsh and $k_G < k_B$ is an even number:*

(a) if $k_B - \frac{1}{1-\alpha^*} \geq k_G$ then h_{HD} is the only stochastically stable convention.

If conflict is mild and $k_G < k_B$ is an even number:

(b) if $k_B - \frac{1}{\alpha^*} \geq k_G$ then h_{DH} is the only stochastically stable convention.

Without loss of generality we can consider an example in which $\alpha^* > 0.5$ in part (a) in Proposition 5. In this case, from Lemma 2, we know that $r_{HD} = \lceil(1-\alpha^*)k_G\rceil$, then h_{HD} is the unique stochastically stable convention if:

$$\begin{cases} \lceil(1-\alpha^*)k_G\rceil < \lceil(1-\alpha^*)k_B\rceil \\ \lceil(1-\alpha^*)k_G\rceil < \lceil\alpha^* k_G\rceil \end{cases}$$

The second condition in the system above is always verified if k_G is an even number in fact we have:

$$\lceil(1-\alpha^*)k_G\rceil \leq \frac{k_G}{2} < \lceil\alpha^* k_G\rceil$$

We can apply the same reasoning to the case of asymmetric harshness of a conflict. In Figure 5, we plot the cases in which $\min\{k_B, k_G\} = k_G$ is an even number on the left and an odd number on the right. Around the value $\alpha_G^* = 0.5$ appears a grey area if k_G is an odd number, and in this case the sufficient conditions of Proposition 4 are strongly informative. If instead k_G is an even number, Proposition 4 can be refined.

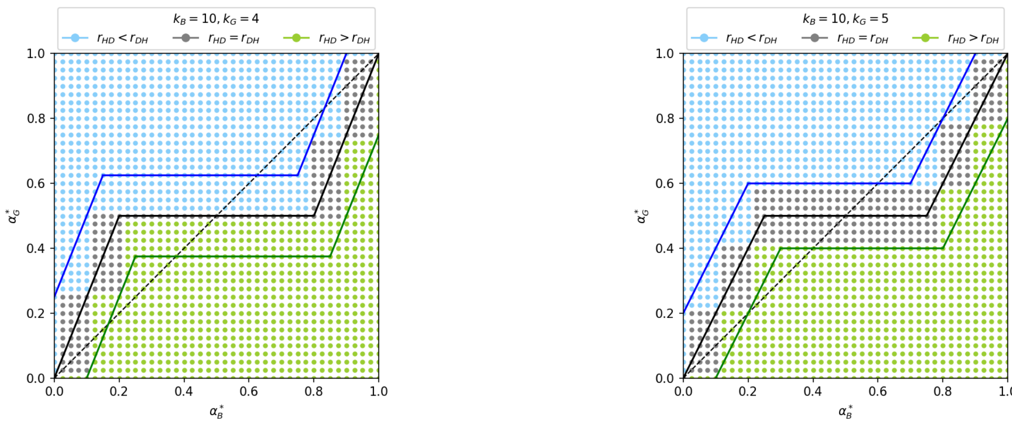

Figure 5: On the left the case in which k_G is an even number ($k_G = 4$). On the right the case in which k_G is an odd number ($k_G = 5$).

Proposition **6.** *If k_G is an even number:*

(a) if $\alpha_G^* > T_{HD}(\alpha_B^*; k_B, k_G)$ then the convention h_{HD} is the only stochastically stable convention;

(b) if $\alpha_G^* < T_{DH}(\alpha_B^*; k_B, k_G)$ then the convention h_{DH} is the only stochastically stable convention;

where:

$$T_{HD}(\alpha_B^*; k_B, k_G) = \begin{cases} \alpha_B^* \frac{k_B}{k_G} + \frac{1}{k_G}, & \text{if } \alpha_B^* \leq \frac{k_G - 2}{2k_B} \\ 0.5 & \text{if } \frac{k_G-2}{2k_B} < \alpha_B^* \leq \frac{2k_B - k_G - 2}{2k_B} \\ \alpha_B^* \frac{k_B}{k_G} + \frac{1 + k_G - k_B}{k_G} & \text{if } \frac{2k_B - k_G - 2}{2k_B} < \alpha_B^* \end{cases}$$

and

$$T_{DH}(\alpha_B^*; k_B, k_G) = \begin{cases} \alpha_B^* \frac{k_B}{k_G} - \frac{1}{k_G}, & \text{if } \alpha_B^* \leq \frac{k_G + 2}{2k_B} \\ 0.5 & \text{if } \frac{k_G+2}{2k_B} < \alpha_B^* \leq \frac{2k_B - k_G + 2}{2k_B} \\ \alpha_B^* \frac{k_B}{k_G} + \frac{k_G - k_B - 1}{k_G} & \text{if } \frac{2k_B - k_G + 2}{2k_B} < \alpha_B^* \end{cases}$$

4.1 Proofs

Proof. Lemma 1

We first show the "if" part of the statement.

Consider a generic history h, which represents the state of the system at time t, and select a pair of agents to play the hawk-dove game. If there is a positive probability that they play either (H, D) or (D, H), and they actually do so, then the following pair of agents that is drawn to play the game has a positive probability to play as the previous pair. Indeed, suppose (without loss of generality) that they play (D, H). Then, we note that $n_B^{t+1} \geq n_B^t$ and $n_G^{t+1} \leq n_G^t$. By repeating this argument for m times, we conclude that with positive probability a convention is reached.

Suppose now that, starting from h, with probability 1 the pair of selected agents plays either (H, H) or (D, D). At the following period, if the pair of selected agents can play (H, D) or (D, H) with positive probability, then we can apply the argument of the previous paragraph. Otherwise, we move to the following period. At period $t + m$, either at some period agents have played (H, D) or (D, H) (so that with positive probability a convention is reached), or all plays of the game in memory are either (H, H) or (D, D). In the latter case, we note that $n_G^{t+m} = n_B^{t+m}$. At period $t + m$, if the pair of selected agents has a positive probability to play (H, D) or (D, H), then we are done. Otherwise, with probability 1 they play either (H, H) or (D, D). Without loss of generality, we assume that they play (D, D). This means that $n_B^t/k_B < \alpha_B^*$ and $n_G^t/k_G < \alpha_G^*$. After such agents play (D, D), we have $n_B^{t+1} \geq n_B^t$ and $n_G^{t+1} \geq n_G^t$. The following pair of agents either plays (D, D) with probability 1, or not. If (D, D) is played with probability 1, then we move to the following period. We proceed this way until we find a period, call it $\hat{t}$, in which (D, D) is not played with probability 1. We observe such a $\hat{t}$ must occur in at most m periods, if the memory contains D actions only. We now show that at period $\hat{t}$ the selected pair agents cannot play (H, H) with probability 1, which means that they play (H, D) or (D, H) with positive probability, and hence a convention is then reached with positive probability.

We first consider the case (i) in which $\min\{k_B, k_G\} < m$ holds, which means that at least for one population, say G, we have $k_G < m$. Suppose at time $(\hat{t} - 1)$ the green agent plays D with probability 1, and instead plays H with positive probability at time $\hat{t}$. It must be true that $n_G^{\hat{t}-1}/k_G < \alpha_G^* \leq n_G^{\hat{t}}/k_G$, with $n_G^{\hat{t}} = n_G^{\hat{t}-1} + 1$. At time $\hat{t}$ there is a positive probability to select a sample with $n_G^{\hat{t}-1} = n_G^{\hat{t}} - 1$ to whom action D is the best response for green agent, thus showing that (H, H) is not played with probability 1.

We now consider the case (ii) in which $q = m\alpha_G^*$ for some integer q (we have chosen G, without loss of generality), and we suppose again that the green agent takes action H

with positive probability. Since $n_G^{\hat{t}}/m < \alpha_G^*$, this means that $n_G^{\hat{t}+1} = n_G^{\hat{t}} + 1 = q$ (otherwise there would not exist an integer q such that $q = m\alpha_G^*$). Therefore, both D and H are best responses for the green agent, which implies that action D is chosen with positive probability, thus showing that (H, H) is not played with probability 1.

We finally consider the case (iii) in which there exists an integer number q such that $\alpha_G^* m < q < \alpha_B^* m$ (we have chosen $\alpha_G^* < \alpha_B^*$, without loss of generality). Since $n_G^{\hat{t}}/m < \alpha_G^*$, $n_B^{\hat{t}}/m < \alpha_B^*$, and $n_G^{\hat{t}} = n_B^{\hat{t}}$, if (D, D) is not played with probability 1, then the only possibility is that $n_G^{\hat{t}+1} = n_B^{\hat{t}+1} = n_G^{\hat{t}} + 1 = n_B^{\hat{t}} + 1 = q$, which implies that H is the unique best response for the green agent, and D is the unique best response for the green agent, thus showing that (H, H) is not played with probability 1.

We now show the "only if" part of the statement, by contraposition. The negation of conditions (i), (ii) and (iii) amounts to assuming that $k_B = k_G = m$, and there exists an integer number $q < m$ such that $q < \alpha_B^* < q + 1$, $q < \alpha_G^* < q + 1$. Assume that the state at time t is given by a history where (D, D) has been played for q times, and (H, H) has been played the remaining $m - q$ times. The pair of selected agents at time t plays (D, D) with probability 1, since $n_G^t = q < \alpha_G^*$ and $n_B^t = q < \alpha_B^*$. This implies that $n_G^{t+1} = q + 1 > \alpha_G^*$ and $n_B^{t+1} = q + 1 > \alpha_B^*$, and hence the pair of selected agents at time $t + 1$ plays (H, H) with probability 1, thus determining that $n_G^{t+2} = q = n_B^{t+2}$. Therefore, the cycle between a state with q occurrences of (D, D) in memory and a state with $q + 1$ occurrences continues forever, so that we will never have convergence to a convention.

□

Proof. Lemma 2

We assume $k_G < k_B$ hence we respect condition (i) of Lemma 1. Let G be a 2×2 coordination game with the corresponding conventions (strict Nash equilibria) (H, D) and (D, H), and corresponding absorbing states with history h_{HD} and h_{DH}. Assume that row players have sample size k_B and the column players have sample size k_G. For payoffs as in matrix 1 we define $\alpha^* = 1 - \frac{V}{C}$. Parameter α^* then refers to the necessary share of green (blue) players choosing strategy D in the sample of a blue (green) player to induce a shift in the best response play of a player to H.

Assume that the blue and green player populations are currently in h_{HD}. Hence, a blue player B currently playing strategy $s_B = H$ will only change strategy to $s_B = D$ if there is a sufficient number of column players playing $s_G = H$ in his sample. Thus, there must be at least $\lceil (1 - \alpha^*) k_B \rceil$ players committing an error in subsequent periods, occurring with probability $\varepsilon^{(1-\alpha^*)k_B}$. For a column player G with $s_G = D$ to switch to strategy $s_G = H$ there must be a sufficient number of blue players playing $s_B = D$ in his sample. Hence, there must be again at least $\lceil \alpha^* k_G \rceil$ of these players in m, happening with probability of at least $\varepsilon^{\alpha^* k_G}$. Therefore, the minimum of $\lceil (1-\alpha^*)k_B \rceil$ and $\lceil \alpha^* k_G \rceil$ is the stochastic potential of h_{DH}. The proof for a shift from h_{DH} to h_{HD} is analogous.

We obtain $r_{DH} = \min\{\lceil (1-\alpha^*)k_B \rceil, \lceil \alpha^* k_G \rceil\}$ and $r_{HD} = \min\{\lceil \alpha^* k_B \rceil, \lceil (1-\alpha^*)k_G \rceil\}$. □

Proof. Proposition 1

(a) We have to prove that $r_{HD} \leq r_{DH}$.
Firstly, we show that $r_{HD} = \min\{\lceil \alpha^* k_B \rceil, \lceil (1-\alpha^*)k_G \rceil\} = \lceil (1-\alpha^*)k_G \rceil$. If $\alpha^* > 0.5$ and $k_B \geq k_G$, then $(1-\alpha^*)k_G < \alpha^* k_B$ and, thus, $\lceil (1-\alpha^*)k_G \rceil \leq \min\{\lceil (1-\alpha^*)k_B \rceil, \lceil \alpha^* k_G \rceil\}$.

(b) We find sufficient conditions for the convention h_{HD} to be the only stochastically stable convention. Given Theorem 1 in Young (1993), it suffices to show that $r_{HD} < r_{DH}$.
By point (a), $r_{HD} = \lceil (1-\alpha^*)k_G \rceil$. We thus require that $\lceil (1-\alpha^*)k_G \rceil < \lceil (1-\alpha^*)k_B \rceil$ and $\lceil (1-\alpha^*)k_G \rceil < \lceil \alpha^* k_G \rceil$, which is implied by:

$$\begin{aligned} (1-\alpha^*)k_G &\leq (1-\alpha^*)k_B - 1 \\ (1-\alpha^*)k_G &\leq \alpha^* k_G - 1. \end{aligned}$$

By rearranging terms, we obtain:

$$\begin{aligned} k_G &\leq k_B - \tfrac{1}{(1-\alpha^*)} \\ k_G &\geq \tfrac{1}{2\alpha^* - 1} \end{aligned}$$

□

Proof. Proposition 2
The proof is analogous to the proof of Proposition 1, once α^* is replaced by $1-\alpha^*$. □

Proof. Lemma 3
The proof is analogous to the proof of Lemma 2 with the exception that $\alpha^*_B \neq \alpha^*_G$. Assuming that $k_B + k_G \leq m$ and $k_G < k_B$ we respect condition (i) of Lemma 1.
Assume that the blue and green player populations are currently in h_{HD}. Hence, a blue player B currently playing strategy $s_B = H$ will only change strategy to $s_B = D$ if there is a sufficient number of column players playing $s_G = H$ in his sample. Thus, there must be at least $\lceil (1-\alpha^*_B)k_B \rceil$ players committing an error in subsequent periods, which occurs with probability $\varepsilon^{(1-\alpha^*_B)k_B}$. For a column player G with $s_G = D$ to switch to strategy $s_G = H$ there must be a sufficient number of blue players playing $s_B = D$ in his sample. Hence, there must be again at least $\lceil \alpha^*_G k_G \rceil$ of these players in m, which occurs with a probability of at

least $\varepsilon^{\alpha^*_G k_G}$. Therefore, the minimum of $\lceil(1-\alpha^*_B)k_B\rceil$ and $\lceil\alpha^*_G k_G\rceil$ is the stochastic potential of h_{DH}. The proof for a shift from h_{DH} to h_{HD} is analogous.

We obtain $r_{DH} = \min\{\lceil(1-\alpha^*_B)k_B\rceil, \lceil\alpha^*_G k_G\rceil\}$ and $r_{HD} = \min\{\lceil\alpha^*_B k_B\rceil, \lceil(1-\alpha^*_G)k_G\rceil\}$. □

Proof. Proposition 3.

We proceed by dividing the (α^*_B, α^*_G) plane into four different areas characterized by different values of $\min\{\alpha^*_G, (1-\alpha^*_G), \alpha^*_B, (1-\alpha^*_B)\}$. From Lemma 3 recall that $r_{DH} = \min\{\lceil(1-\alpha^*_B)k_B\rceil\ ,\ \lceil\alpha^*_G k_G\rceil\}$ and $r_{HD} = \min\{\lceil\alpha^*_B k_B\rceil\ ,\ \lceil(1-\alpha^*_G)k_G\rceil\}$, furthermore recall that we assume $k_B > k_G$.

- The first area is characterized by $\min\{\alpha^*_G, (1-\alpha^*_G), \alpha^*_B, (1-\alpha^*_B)\} = \alpha^*_G$.
  In this area $\alpha_G k^*_G = \min\{\alpha^*_G k_G, (1-\alpha^*_G)k_G, \alpha^*_B k_B, (1-\alpha^*_B)k_B\}$, from Lemma 3 $r_{DH} \leq r_{HD}$ and then from Theorem 1 in (Young, 1993) we find that h_{DH} is stochastically stable;

- The second area is characterized by $\min\{\alpha^*_G, (1-\alpha^*_G), \alpha^*_B, (1-\alpha^*_B)\} = (1-\alpha^*_G)$.
  In this area $(1-\alpha^*_G)k_G = \min\{\alpha^*_G k_G, (1-\alpha^*_G)k_G, \alpha^*_B k_B, (1-\alpha^*_B)k_B\}$, from Lemma 3, $r_{HD} \leq r_{DH}$ and then from Theorem 1 in (Young, 1993) we find that h_{HD} is stochastically stable;

- The third area is characterized by $\min\{\alpha^*_G, (1-\alpha^*_G), \alpha^*_B, (1-\alpha^*_B)\} = \alpha^*_B$.
  The condition that characterize this area can be rewritten as $\max\{\alpha^*_G, (1-\alpha^*_G), \alpha^*_B, (1-\alpha^*_B)\} = (1-\alpha^*_B)$. Since $k_B > k_G$, we have $\alpha^*_G k_G < (1-\alpha^*_B)k_B$, and from Lemma 3 we obtain $r_{DH} = \lceil\alpha^*_G k_G\rceil$.
  If $\alpha^*_G < 0.5$ and $\alpha^*_G k_G < \alpha^*_B k_B$, then $r_{DH} \leq r_{HD}$ and for Theorem 1 in (Young, 1993) the convention h_{DH} is stochastically stable; if $\alpha^*_G < 0.5$ and $\alpha^*_G k_G > \alpha^*_B k_B$, then $r_{HD} \leq r_{DH}$ and for Theorem 1 in (Young, 1993) the convention h_{HD} is stochastically stable;
  If instead $\alpha^*_G > 0.5$ we have that $\alpha^*_G k_G > \min\{(1-\alpha^*_G)k_G, \alpha^*_B k_B\}$ resulting in $r_{HD} \leq r_{DH}$. From Lemma 3 and Theorem 1 in (Young, 1993), we find that h_{HD} is stochastically stable;

- The fourth area is characterized by $\min\{\alpha^*_G, (1-\alpha^*_G), \alpha^*_B, (1-\alpha^*_B)\} = (1-\alpha^*_B)$.
  The condition that characterize this area can be rewritten as $\max\{\alpha^*_G, (1-\alpha^*_G), \alpha^*_B, (1-\alpha^*_B)\} = \alpha^*_B$. Since $k_B > k_G$, we have $(1-\alpha^*_G)k_G < \alpha^*_B k_B$, and from Lemma 3 we obtain $r_{HD} = \lceil(1-\alpha^*_G)k_G\rceil$.
  If $\alpha^*_G > 0.5$ and $(1-\alpha^*_G)k_G > (1-\alpha^*_B)k_B$, then $r_{DH} \leq r_{HD}$ and by Theorem 1 in

(Young, 1993) the convention h_{DH} is stochastically stable; if $\alpha_G^* > 0.5$ and $(1-\alpha_G^*)k_G < (1-\alpha_B^*)k_B$, then $r_{HD} \leq r_{DH}$ and by Theorem 1 in (Young, 1993) the convention h_{HD} is stochastically stable;
If instead $\alpha_G^* < 0.5$ we have that $(1-\alpha_G^*)k_G > \min\{(1-\alpha_B^*)k_B, \alpha_G^* k_G\}$ resulting in $r_{DH} \leq r_{HD}$, Lemma 3, and then from Theorem 1 in (Young, 1993) we find that h_{DH} is stochastically stable;

The result follows by putting together the regions in which the two conventions are stochastically stable (see Figure 2). □

Proof. Proposition 4
We proceed by dividing the (α_B^*, α_G^*) plane into four different areas characterized by different values of $\min\{\alpha_G^*, (1-\alpha_G^*), \alpha_B^*, (1-\alpha_B^*)\}$. From Lemma 3 recall that $r_{DH} = \min\{\lceil(1-\alpha_B^*)k_B\rceil\ ,\ \lceil\alpha_G^* k_G\rceil\}$ and $r_{HD} = \min\{\lceil\alpha_B^* k_B\rceil\ ,\ \lceil(1-\alpha_G^*)k_G\rceil\}$, furthermore recall that we assume $k_B > k_G$.

We start from the proof of part (a). To find sufficient conditions under which the unique stochastically stable convention is h_{DH}, we have to identify a region of the plane (α_B^*, α_G^*) in which $r_{HD} < r_{DH}$.

- Firstly notice that from Proposition 3 if $\min\{\alpha_G^*, (1-\alpha_G^*), \alpha_B^*, (1-\alpha_B^*)\} = \alpha_G^*$, or $\min\{\alpha_G^*, (1-\alpha_G^*), \alpha_B^*, (1-\alpha_B^*)\} = (1-\alpha_B^*)$ with $\alpha_G^* < 0.5$, the convention h_{DH} is stochastically stable then, trivially, the convention h_{HD} cannot be the unique stochastically stable convention.

- In the area characterized by $\min\{\alpha_G^*, (1-\alpha_G^*), \alpha_B^*, (1-\alpha_B^*)\} = (1-\alpha_G^*)$, we have $r_{HD} = \lceil(1-\alpha_G^*)k_G\rceil$. If $\lceil(1-\alpha_G^*)k_G\rceil < \lceil(1-\alpha_B^*)k_B\rceil$ and $\lceil(1-\alpha_G^*)k_G\rceil < \lceil\alpha_G^* k_G\rceil$ we obtain $r_{HD} < r_{DH}$. This condition is assured by the system:

$$\begin{cases}(1-\alpha_G^*)k_G < (1-\alpha_B^*)k_B - 1\\ (1-\alpha_G^*)k_G < \alpha_G^* k_G - 1.\end{cases}$$

  By rearranging we obtain:

$$\begin{cases}\alpha_G^* > \alpha_B^* \frac{k_B}{k_G} + \frac{1+k_G-k_B}{k_G}\\ \alpha_G^* > \frac{k_G+1}{2k_G}.\end{cases}$$

- In the area characterized by $\min\{\alpha_G^*, (1-\alpha_G^*), \alpha_B^*, (1-\alpha_B^*)\} = (1-\alpha_B^*)$ with $\alpha_G^* > 0.5$, we have $r_{HD} = \lceil(1-\alpha_G^*)k_G\rceil$. As above, if $\lceil(1-\alpha_G^*)k_G\rceil < \lceil(1-\alpha_B^*)k_B\rceil$ and $\lceil(1-\alpha_G^*)k_G\rceil < \lceil\alpha_G^* k_G\rceil$ we obtain $r_{HD} < r_{DH}$. This condition is assured by the system:

$$\begin{cases}(1-\alpha^*_G)k_G < (1-\alpha^*_B)k_B - 1\\(1-\alpha^*_G)k_G < \alpha^*_G k_G - 1.\end{cases}$$

By rearranging we obtain:

$$\begin{cases}\alpha^*_G > \alpha^*_B \frac{k_B}{k_G} + \frac{1+k_G-k_B}{k_G}\\\alpha^*_G > \frac{k_G+1}{2k_G}.\end{cases}$$

- In the area characterized by $\min\{\alpha^*_G, (1-\alpha^*_G), \alpha^*_B, (1-\alpha^*_B)\} = \alpha^*_B$, we have $r_{DH} = \lceil \alpha^*_G k_G \rceil$. To have $r_{HD} < r_{DH}$ we need $\lceil \alpha^*_B k_B \rceil < \lceil \alpha^*_G k_G \rceil$ or $\lceil (1-\alpha^*_G)k_G \rceil < \lceil \alpha^*_G k_G \rceil$:

$$\begin{aligned}\alpha^*_G k_G - 1 &> \alpha^*_B k_B\\\alpha^*_G k_G - 1 &> (1-\alpha^*_G)k_G.\end{aligned}$$

  Then h_{HD} is the unique stochastically stable convention if at least one of the following condition holds:

$$\begin{aligned}\alpha^*_G &> \frac{\alpha^*_B k_B + 1}{k_G}\\\alpha^*_G &> \frac{k_G+1}{2k_G}\end{aligned}$$

The result of part (a) follows by putting together the regions in which h_{HD} is the unique stochastically stable convention (see Figure 3).
It is possible to prove part (b) of the Proposition analogously by finding sufficient conditions to have $r_{DH} < r_{HD}$. □

Proof. Proposition 5

(a) As already argued in the proof of point (a) of Proposition 1, $r_{HD} = \lceil (1-\alpha^*)k_G \rceil$. In fact, if $\alpha^* > 0.5$ and $k_B > k_G$, $\lceil (1-\alpha^*)k_G \rceil \leq \lceil \alpha^* k_B \rceil$. Now we require that $\lceil (1-\alpha^*)k_G \rceil < \lceil (1-\alpha^*)k_B \rceil$ and $\lceil (1-\alpha^*)k_G \rceil < \lceil \alpha^* k_G \rceil$. The first inequality is implied by:

$$(1-\alpha^*)k_G \leq (1-\alpha^*)k_B - 1.$$

By rearranging terms, we obtain:

$$k_G \leq k_B - \frac{1}{(1-\alpha^*)}$$

The second inequality is always verified if $\alpha^* > 0.5$ and k_G is an even number.

(b) The proof is analogous to the proof of point (a) considering that $\alpha^* < 0.5$.

□

Proof. Proposition 6.

The proof is analogous to the proof of Proposition 4. We proceed by dividing the (α_B^*, α_G^*) plane into four different areas characterized by different values of $\min\{\alpha_G^*, (1-\alpha_G^*), \alpha_B^*, (1-\alpha_B^*)\}$. From Lemma 3 recall that $r_{DH} = \min\{\lceil(1-\alpha_B^*)k_B\rceil\ ,\ \lceil\alpha_G^* k_G\rceil\}$ and $r_{HD} = \min\{\lceil\alpha_B^* k_B\rceil\ ,\ \lceil(1-\alpha_G^*)k_G\rceil\}$, furthermore recall that we assume $k_B > k_G$.

We start with the proof of part (a). To find sufficient conditions under which the unique stochastically stable convention is h_{DH} we have to identify a region of the plane (α_B^*, α_G^*) in which $r_{HD} < r_{DH}$.

- Firstly notice that from Proposition 3 if $\min\{\alpha_G^*, (1-\alpha_G^*), \alpha_B^*, (1-\alpha_B^*)\} = \alpha_G^*$, or $\min\{\alpha_G^*, (1-\alpha_G^*), \alpha_B^*, (1-\alpha_B^*)\} = (1-\alpha_B^*)$ given $\alpha_G^* < 0.5$, the convention h_{DH} is stochastically stable then, trivially, the convention h_{HD} cannot be the unique stochastically stable convention.

- In the area characterized by $\min\{\alpha_G^*, (1-\alpha_G^*), \alpha_B^*, (1-\alpha_B^*)\} = (1-\alpha_G^*)$, we have $r_{HD} = \lceil(1-\alpha_G^*)k_G\rceil$. If $\lceil(1-\alpha_G^*)k_G\rceil < \lceil(1-\alpha_B^*)k_B\rceil$ and $\lceil(1-\alpha_G^*)k_G\rceil < \lceil\alpha_G^* k_G\rceil$ we obtain $r_{HD} < r_{DH}$. Notice that if $(1-\alpha_G^*) < \alpha_G^*$ then $\alpha_G^* > 0.5$. Since k_G is an even number and $\alpha_G^* > 0.5$, $\lceil(1-\alpha_G^*)k_G\rceil < \lceil\alpha_G^* k_G\rceil$ is always verified. The other condition is assured by:

$$(1-\alpha_G^*)k_G < (1-\alpha_B^*)k_B - 1$$

  By rearranging we obtain:

$$\alpha_G^* > \alpha_B^* \frac{k_B}{k_G} + \frac{1 + k_G - k_B}{k_G}$$

- In the area characterized by $\min\{\alpha_G^*, (1-\alpha_G^*), \alpha_B^*, (1-\alpha_B^*)\} = (1-\alpha_B^*)$ with $\alpha_G^* > 0.5$, we have $r_{HD} = \lceil(1-\alpha_G^*)k_G\rceil$. As above, if $\lceil(1-\alpha_G^*)k_G\rceil < \lceil(1-\alpha_B^*)k_B\rceil$ and $\lceil(1-\alpha_G^*)k_G\rceil < \lceil\alpha_G^* k_G\rceil$ we obtain $r_{HD} < r_{DH}$. Since k_G is an even number and $\alpha_G^* > 0.5$, $\lceil(1-\alpha_G^*)k_G\rceil < \lceil\alpha_G^* k_G\rceil$ is always verified. The other condition is assured by:

$$(1-\alpha_G^*)k_G < (1-\alpha_B^*)k_B - 1$$

  By rearranging we obtain:

$$\alpha_G^* > \alpha_B^* \frac{k_B}{k_G} + \frac{1 + k_G - k_B}{k_G}$$

- In the area characterized by $\min\{\alpha_G^*, (1-\alpha_G^*), \alpha_B^*, (1-\alpha_B^*)\} = \alpha_B^*$, we have $r_{DH} = \lceil \alpha_G^* k_G \rceil$. To have $r_{HD} < r_{DH}$ we need $\lceil \alpha_B^* k_B \rceil < \lceil \alpha_G^* k_G \rceil$ or $\lceil (1-\alpha_G^*) k_G \rceil < \lceil \alpha_G^* k_G \rceil$:

$$\begin{aligned} \alpha_G^* k_G - 1 &> \alpha_B^* k_B \\ \alpha_G^* k_G &> (1-\alpha_G^*) k_G. \end{aligned}$$

  Then h_{HD} is the unique stochastically stable convention if at least one of the following condition holds:

$$\begin{aligned} \alpha_G^* &> \tfrac{\alpha_B^* k_B + 1}{k_G} \\ \alpha_G^* &> \tfrac{1}{2} \end{aligned}$$

The result of part (a) follows by putting together the regions in which h_{HD} is the unique stochastically stable convention.
In an analogous way, it is possible to prove part (b) of the Proposition by finding sufficient conditions that ensure $r_{DH} < r_{HD}$.

□

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