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Leonardo Bargigli

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DISEI, Università degli Studi di Firenze
Via delle Pandette 9, 50127 Firenze (Italia) www.disei.unifi.it

How Large is Excess Volatility of the EUR/USD Exchange Rate? Evidence from a GAS Approach

Leonardo Bargigli *

Università di Firenze
Dipartimento di Scienze per L'Economia e l'Impresa

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Abstract

Using a novel Generalized Autoregressive Score (GAS) methodology applied to EUR/USD high-frequency interdealer data on price variations and net demand in 2016, this paper provides evidence of a substantial violation of market efficiency in the foreign exchange market. The analysis shows that endogenous factors amplified efficient price fluctuations by at least 46% on average, underscoring the importance of informational asymmetry and feedback trading in exchange rate dynamics. The key implication is that excess volatility of the EUR/USD exchange rate is not only sizeable but also structural, as it arises from mechanisms intrinsic to market functioning.

Keywords: excess volatility, foreign exchange, high frequency data, score-driven model, GARCH, SVAR.

JEL codes: G14, C32, C58, F31

*Address: via delle Pandette 9, 50127 Firenze, tel: +390552759694, mail: leonardo.bargigli@unifi.it

1 Introduction

The random walk hypothesis (RWH) of Fama (1965), in its simplest form, states that the fundamental information flow is unobservable but reflected in the innovations of the efficient price of assets. In this paper, I propose an adjustment to the simple RWH to account for the effects on price volatility of heterogeneity among investors, as well as between liquidity providers and takers. Specifically, the approach leverages the structural covariation between price and demand to derive a conditionally independent stochastic innovation of the efficient price which is free not only from microstructural noise but, more importantly, from the impact of investor heterogeneity that manifest itself through demand flows.

The simple RWH approach does not require reliance on standard stochastic discount rate models or on rational expectations. The contradiction in the subsequent joint-hypothesis framework of Fama (1976) is underscored by the argument of Cochrane (1991) that excess volatility tests only document time variation in discount rates, expected returns and risk premia that is not fully explained by discount rate models. In other terms, whenever excess volatility is detected, either markets are inefficient or asset pricing models fail to reflect the actual diversity of investor beliefs. But the latter suggests that these models could overestimate excess volatility, therefore undermining any evidence against market efficiency. In contrast, the simple RWH avoids this overestimation problem, making any detected excess volatility more robust. Moreover, the simple RWH approach does not necessitate restricting the analysis to specific news events. This is particularly appropriate given that, despite the recent explosion of available data, there are good reasons to believe that prices remain the most reliable manifestation of fundamental information. From this perspective, the approach of this paper might help to explain the apparent puzzle of "so few news and so many jumps" in prices (Aït-Sahalia *et al.*, 2024).

While evidence against the simple RWH is robust at low frequencies (Lo and MacKinlay, 1988; Liu and He, 1991), there are compelling arguments in favor of asset prices being unpredictable at high frequencies. This unpredictability, in turn, supports the notion that fundamental information is primarily reflected in prices. The limited deviations from the RWH observed at high frequencies, such as those reported in Cai and Zhang (2016) and Laurent and Shi (2022), can be attributed to market microstructural factors, including rounding errors from discrete price grids, temporary liquidity shocks, or inventory management by market makers. Microstructural effects are not the only source of non-fundamental volatility. Factors related to heterogeneity and informational asymmetry must also be taken into account. To assess these effects, I decompose volatility into separate endogenous and exogenous components within a properly specified statistical model, as detailed in the following sections.

The choice of the EUR/USD currency market for the empirical analysis is consistent with the general purpose of the paper to avoid the overestimation problem mentioned above. Indeed, the EUR/USD pair is exchanged on a highly traded market, in which liquidity shocks that could impact volatility are less likely to happen. Moreover, some forms of informational asymmetry like insider trading are less relevant on a liquid currency market than on, say, the stock market. Therefore, although the paper does not engage in a systematic analysis of different asset classes, there are reasons to believe that its results establish a realistic lower bound for excess volatility across markets. The main limitation of the proposed analysis resides in the lack of freely available demand data, which makes it more difficult to replicate its results. Therefore, extensions of the empirical analysis are left for future research.

The main finding of the paper is that excess volatility in the EUR/USD interdealer market was substantial in 2016. After discounting microstructural factors, the conditional volatility of efficient price variations would, on average, have been 46% lower if the time-varying response of liquidity providers to demand shocks had been absent. A natural interpretation of this result is that information is incorporated into asset prices through an informed component of demand. Prices thus reflect fundamental information because liquidity providers adjust rapidly to informed net order flows. The key implication is that excess volatility arises from the very mechanism through which financial markets operate.

The remaining of the paper is organized as follows. The following Section briefly reviews the relevant literature. Section 3 presents the time varying VAR model employed to gauge microstructural effects. Section 4 introduces the analysis of structural covariance by means of a combined GAS - GARCH model. Section 5 contains the empirical results on the EUR/USD interdealer market. Finally, section 6 concludes.

2 Literature Review

In a risk-neutral framework with full information, and under the absence of arbitrage, the volatility of an asset coincides with the volatility of the fundamental information flow (Ross, 1989). By contrast, the seminal work of Shiller (1981) showed that stock volatility substantially exceeded the restrictions implied by standard valuation models under rational expectations. This discrepancy, commonly referred to as excess volatility, sparked a broad debate on the efficiency of financial markets. Since then, excess volatility has been explained by models incorporating risk aversion, informational asymmetry, heterogeneity, or by frameworks that abandon the assumption of full rationality altogether (Thaler, 2005; Shiller, 2015). Although subsequent statistical refinements have weakened Shiller's original findings (Mankiw *et al.*, 1991; Cochrane, 1991; Cuthbertson and Hyde, 2002), the issue has re-emerged more recently with the "inelastic demand hypothesis" of Gabaix and Koijen (2021), which, according to Bouchaud (2022), aligns with the "order-driven view" advocated in the MM literature. Indeed, the latter has provided the primary inspiration of this paper, particularly through its applications to currency markets (King *et al.*, 2013), which emphasize the role of informational asymmetry as a driver of volatility. According to MM theory, the main channel for information transmission to the market is the order flow from informed traders, while uninformed traders respond to price innovations, generating a complex interdependence between price movements and demand.

Accurately capturing the diversity of views among market participants is a daunting task. Traders not only hold different information, but also rely on distinct and often imperfect models, pursue different objectives, and operate over varying time horizons. Even though more detailed data on the information flow are becoming available (Baker *et al.*, 2019), we still lack a solid theory of asset pricing. As a result, the same piece of information, received at the same time, may be interpreted in markedly different ways. Heterogeneity makes it difficult, if not impossible, to measure directly the fundamental flow of information to the market, and helps to explain why, despite a large body of literature documenting the significant impact of announcements and news on returns (Andersen *et al.*, 2003; Neely and Dey, 2010) and volatility (Maheu and McCurdy, 2004; Andersen *et al.*, 2003; Neely and Dey, 2011; Chan and Gray, 2018), as well as the influence of macroeconomic volatility on financial volatility (Engle and Rangel, 2008; Diebold and Yilmaz, 2008), studies such as Roll (1984), and more recently Filimonov and Sornette (2012) and Aït-Sahalia *et al.* (2024), emphasize the troubling disconnect between price and fundamental volatility when the latter is measured by the flow of public news [1].

The analysis of volatility in this paper builds on the identification through heteroscedasticity (IH) approach introduced by Rigobon (2003), which avoids the standard coefficient restrictions imposed in structural vector autoregression (S-VAR) models. In particular, I relax the restriction, first adopted by Hasbrouck (1991) and subsequently by much of the

[1] According to Djeutem and Kasa (2013), model uncertainty might be an important source of excess volatility on the foreign exchange market if agents attempt to construct forecasts that are robust to model misspecification. This type of uncertainty might explain the high sensitivity of currency markets to macroeconomic announcements.

MM literature, that price variations have no contemporaneous effect on net order flows. I also allow for the possibility that the sensitivity of price variations and demand to innovations is both time-varying and persistent (Berger *et al.*, 2009), by employing the generalized autoregressive score (GAS) framework (Creal *et al.*, 2011; Blasques *et al.*, 2023) to estimate a VAR model with time-varying coefficients. Ignoring this dimension risks overlooking part of excess volatility, since VAR models with fixed coefficients cannot capture the dynamic nature of microstructural effects. In this respect, the paper extends the common approaches used to quantify such effects, namely state-space model estimation (Menkveld *et al.*, 2007; Brogaard *et al.*, 2014) and VAR-based estimation (Hasbrouck, 1991).

To exploit heteroscedasticity for identification, it is necessary to specify the process governing the covariance matrices of the structural errors in a GAS-S-VAR model. Common choices include variance regimes (Rigobon, 2003), restricted or unrestricted multivariate ARCH/GARCH processes (Rigobon, 2002), and constant or dynamic conditional correlation processes (Weber, 2010)[2]. In this paper, I adopt a GARCH specification for structural volatility, since simulation studies show that a GARCH-based Gaussian ML approach performs particularly well when the underlying data-generating process is conditionally heteroscedastic, even if the distribution of the structural errors is misspecified[3]. The main methodological contribution of this paper relative to the existing literature is to combine the IH approach with a GAS process, thereby closing an apparent gap between unrestricted linear models such as Lanne *et al.* (2022) and restricted nonlinear models such as Primiceri (2005) and Berger *et al.* (2009).

[2]A detailed review is provided by Kilian and Lütkepohl (2017).

[3]See Herwartz *et al.* (2019), who employ skewed and leptokurtic distributions consistent with the properties of financial time series.

3 GAS-VAR Model

One common assumption in the MM literature is that the returns of an asset can be decomposed into a permanent component, embedding fundamental factors, and a transitory component, embedding non fundamental factors. Following the inspiration of Hasbrouck (1991), I choose to represent the latter through an autoregressive process involving both price variations and order flows. A process of this sort can account for market underreaction or overreaction to information which is generated by microstructural effects. In particular, according to the VAR framework, prices might incorporate information directly or indirectly, through the impact of informed order flows. Alternatively, state space models assume that efficient prices, reflecting fundamentals, are observed with a measurement error which is due to microstructural noise. In general, a VAR framework appears to be more consistent with the bivariate framework that I adopt, which is linked to the subsequent analysis of volatility explained in section 4.

Another well known fact of financial markets is that the impact of information on the market changes over time. In order to take into account this feature, I incorporate the additional flexibility of observation driven models into the VAR framework. For a given time series $y^T = \{y_1, \ldots, y_T\}$, the general form of observation driven models is as follows:

$$y_t \sim p(y^{t-1}, f_t, \theta) \tag{1}$$

$$f_t = \phi(y^{t-1}, f^{t-1}, \theta) \tag{2}$$

where p is the conditional distribution of y_t, $y^{t-1} = \{y_1, \ldots, y_{t-1}\}$, $f^{t-1} = \{f_{t-k}, \ldots, f_{t-1}\}$ with $k \geq 1$, and θ is a vector of parameters to estimate. One popular example in this class is given by score-driven (SD) models, also called generalized autoregressive score (GAS) models (Creal *et al.*, 2011), in which Eq.(2) takes the following form:

$$f_t = \theta_0 + \Theta_1 \, s_{t-1} + \Theta_2 \, f_{t-1} \tag{3}$$

$$s_{t-1} = S_{t-1}(f_{t-1}, \theta) \nabla_{t-1} \tag{4}$$

$$\nabla'_{t-1} = \frac{\partial \log p(y_{t-1} | f_{t-1}, \theta)}{\partial f'_{t-1}} \tag{5}$$

where S_{t-1} is a scaling matrix and $(\theta_0, \Theta_1, \Theta_2) \in \theta$. In order to introduce my approach, I adopt the standard representation for VAR models with p lags. A VAR(p) model with time-varying coefficients can be written in the following compact form:

$$z_t = c_t + A_t z_{t-1} + \xi_t \tag{6}$$

where $z_t = \left[y'_t, \ldots, y'_{t-p+1}\right]'$, $c_t = [\mu'_t \quad 0']'$, $\xi_t = [u'_t \quad 0']'$, and finally:

$$A_t = \begin{bmatrix} B_{1,t} & B_{2,t} & \ldots & B_{p-1,t} & B_{p,t} \\ I & 0 & \ldots & 0 & 0 \\ 0 & I & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & I & 0 \end{bmatrix} \tag{7}$$

where A_t is of size $np \times np$, $\xi_t \sim \mathcal{N}(0, \Sigma_t)$ with $\Sigma_t = \begin{bmatrix} H_t & 0 \\ 0 & 0 \end{bmatrix}$ and $u_t \sim \mathcal{N}(0, H_t)$ with $H_t = B_{0,t}^{-1} G_t B_{0,t}^{-1'}$ and G_t positive definite and diagonal[4]. The vector of time varying parameters to be estimated from the reduced form model is given by $f_t = [\mu'_t \; b'_t \; , h'_t]'$, where

$$b_t \equiv \text{vec}\left(\begin{bmatrix} B_{1,t} & B_{2,t} & \ldots & B_{p-1,t} & B_{p,t} \end{bmatrix}'\right) \tag{8}$$

[4] In principle all the arguments below can be extended to the case of a Student t distribution as in Creal *et al.* (2011). I avoid doing so for sake of the simplicity of exposition.

and $h_t \equiv \text{vech}(H_t)$. In order to close the model we need to specify the updating equations for the components for f_t in Eq. (3). In Appendix A.1 I derive the following specifications:

$$s_{\mu_t} = u_t \tag{9}$$

$$s_{b_t} = u_t \otimes z_{t-1} \tag{10}$$

where $\otimes$ stands for the Kronecker product. The scaling matrix $S_{b_t} = \|z_{t-1}\|^2 \, \mathcal{I}^{-1}_{b_t|t-1}$ is chosen to avoid that s_{b_t} is non linearly dependent on z_{t-1}, since this dependency would make it impossible to guarantee that the residuals u_t are stationary, as required to guarantee that f_t itself is stationary (see Blasques *et al.* (2023), lemma 1). One convenient feature of Eqs. (9)-(10) is that the r.h.s. does not depend on h_t. Therefore it is possible to estimate the parameters of the process (μ_t, b_t) independently from those of h_t. Instead, (μ_t, b_t) must be estimated jointly since they are connected through u_t.

4 GAS-S-MGARCH Model

As remarked in Sec. 1, I suppose that volatility depends partially on endogenous factors. This claim is supported by the MM literature which highlights the simultaneous impact of order flows on returns (Evans and Lyons, 2002; Daníelsson and Love, 2006). On the other hand, many contributions underline the ubiquitous role of feedback/ momentum trading across asset classes (Jegadeesh and Titman, 2001; Daníelsson and Love, 2006; Osler, 2011; Asness *et al.*, 2003). These results converge to support the assumption of a simultaneous feedback effect between returns, or price variations, and demand, whose causal direction is difficult to disentangle. Indeed, some papers find empirical support for these simultaneous effects (Daníelsson and Love, 2006; Bargigli and Cifarelli, 2023). In general, they will manifest themselves in the form of a positive correlation between price variations and demand, which is consequential to market models with asymmetric information (Kyle, 1985).

In order to connect the idea of endogenous volatility with the GAS-VAR model of the previous section, I define $u_t = B_{0,t}^{-1}\epsilon_t$, where $B_{0,t}$ is an invertible matrix with unit diagonal. Milunovich and Yang (2013) propose the following model for ϵ_t:

$$\epsilon_t|\mathcal{F}_{t-1} \sim Q(0, G_t) \qquad G_t = \omega\omega' + \beta G_{t-1}\beta' + a\,\text{diag}(\epsilon_{t-1})\,\text{diag}(\epsilon_{t-1})'\,a' \tag{11}$$

where G_t, ω, β, $\text{diag}(\epsilon_{t-1})$ and a are $n \times n$ diagonal matrices. In Appendix A.4 I show that, in the event that the distribution Q is a multivariate normal (or student) distribution, it's possible to derive the model above as a special case of the GAS framework of Creal *et al.* (2011). Following Milunovich and Yang (2013), model (11) can be rewritten as follows:

$$G_t = \omega\omega' + \beta G_{t-1}\beta' + \alpha\,(I_n \otimes \epsilon_{t-1})(I_n \otimes \epsilon'_{t-1})\,\alpha' \tag{12}$$

where $\alpha = \text{diag}(\alpha_1, \ldots, \alpha_n)$ is a $n \times n^2$ block diagonal matrix where $\alpha_i = (0, \ldots, a_i, \ldots, 0)$ is a $1 \times n$ vector with a_i as its sole nonzero element. Given that we assume that $\epsilon_t = B_{0,t}u_t$ and therefore $G_t = B_{0,t}H_tB'_{0,t}$, we obtain the following observable counterpart of model (11), which I label as GAS-S-BEKK model and is connected to the GAS-VAR model of the previous section through H_t:

$$H_t = \psi_t\psi'_t + \rho_t H_{t-1}\rho'_t + \phi_t(I_n \otimes u_{t-1})(I_n \otimes u'_{t-1})\phi'_t \tag{13}$$

where ρ_t and ψ_t are $n \times n$ matrices with ψ_t being lower triangular, while $\phi_t = \text{diag}(\phi_{1,t}, \ldots, \phi_{n,t})$ is a $n \times n^2$ matrix with $\phi_{i,t}$ being diagonal matrices. Althoug the parameters in (13) are time varying, it's possible to show that the model is globally identified under mild conditions (see Appendix A.5). The mapping from the structural parameters to the reduced form ones is

$$\psi_t \psi_t' = B_{0,t}^{-1} \omega \omega' B_{0,t}'^{-1} \qquad \rho_t = B_{0,t}^{-1} \beta B_{0,t-1} \qquad \phi_t = B_{0,t}^{-1} \alpha \left(I_n \otimes B_{0,t-1} \right) \tag{14}$$

For simplicity of notation the mapping (14) is not written as a function of the parameters driving the process for $B_{0,t}$, for which a GAS specification is provided below. Since we already know that these parameters are locally identified w.r.t. the process itself, we only need to check that the latter is locally identified by the mapping (14). This means, in practice, that we need to show that any change to $b_{0,t}$, which is the column vector containing the free parameters in $B_{0,t}$, has an independent impact on H_t. Since model (13) is globally identified, in order to prove that $(b_{0,t}, \omega, \alpha, \beta)$ is locally identifiable we can follow the arguments of Milunovich and Yang (2013) who rely on Theorem 6 in Rothenberg (1971). According to the latter, given a mapping from $(b_{0,t}, \omega, \alpha, \beta)$ to (ψ_t, ϕ_t, ρ_t), if $(b_{0,t}, \omega, \alpha, \beta)$ is a regular point of the Jacobian of the mapping[5], then it is locally identifiable if and only if the Jacobian has rank $n^2 + 2n$ (full column rank) [6].

The local identification conditions for the structural parameters can be summarized in the following proposition:

Proposition 1. *Suppose that all elements of* ω *are non-zero and that* $B_{0,t}$ *is invertible with unit diagonals. Then* $(b_{0,t}, \omega, \alpha, \beta)$ *is a regular point of the Jacobian of the mapping (14) which is locally identifiable.*

Proof. See Appendix A.6. □

The reduced form model (13) inherits the poor scaling properties of the original BEKK model of Engle and Kroner (1995), which are often contrasted with the better scaling

[5] A point is regular if there is a neighborhood of it in which the rank of the Jacobian does not change.

[6] Note that requiring local identifiability and regularity for $b_{0,t}$ provides a justification for a GAS specification of the process for $b_{0,t}$ since it guarantees that $\mathcal{I}(b_{0,t})^{-1}$ exists according to Theorem 1 in Rothenberg (1971).

properties of the DCC model of Engle (2002). Indeed, it would be more convenient to estimate the independent GARCH processes of the structural errors instead of the coupled BEKK processes of the reduced form errors. This is the approach followed by Weber (2010) in a linear context (i.e. one where $B_{0,t} = B_0$), which is extended to a time-varying framework by rewriting Eq. (12) as follows:

$$G_t = \omega\omega' + \beta G_{t-1}\beta' + \phi_t(I_n \otimes u_{t-1})(I_n \otimes u'_{t-1})\phi'_t \tag{15}$$

where

$$\phi_t = \alpha\,(I_n \otimes B_{0,t-1}) \tag{16}$$

Since G_t in Eq. (15) is a restriction of H_t in Eq. (13), model (15), which I label as GAS-S-MGARCH, is globally identified under the same conditions of the GAS-S-BEKK model. Therefore we can proceed along the same lines of Proposition 1 to establish the following:

Proposition 2. *Suppose that all elements of a are non-zero, and $B_{0,t}$ is invertible with unit diagonals. Then $(b_{0,t-1}, \alpha)$ is a regular point of the Jacobian of the mapping (16) which is locally identifiable.*

Proof. See Appendix A.8. □

Another advantage of the GAS-S-MGARCH model of Eq. (15) is that the score takes a much simpler form than the one of the GAS-S-BEKK model of Eq. (13) (for the latter see Appendix A.7). Indeed, from the results of Appendix A.9 it's possible to check that the elements of the gradient vector (A.9.4) take the following form:

$$\frac{\partial \mathcal{L}_t}{\partial b_{0,n,m,t}} = -\frac{\epsilon_{n,t}}{g_{n,t}} u_{m,t} \qquad \text{for} \quad n \neq m \tag{17}$$

In order to simplify further the estimation process, it's possible to multiply these terms by $g_{n,t}$. Therefore our final choice for the GAS specification of this model is:

$$s_{b_{0,n,m,t}} = -\epsilon_{n,t} u_{m,t} \qquad \text{for} \quad n \neq m \tag{18}$$

5 Empirical Analysis

5.1 Data Overview

The empirical analysis employs tick-by-tick interdealer transaction data, recorded on the EBS FX Spot trading platform and provided by NEX data[7]. The dataset spans the time interval from 3 January 2016, 5.55 pm, to 30 December 2016, 9.55 pm, and, for the purpose of the analysis, it is sampled at the 5 minutes frequency. Weekends, holidays and late-evening / night periods are excluded, in order to remove the main sources of daily and weekly seasonal patterns of volatility. Fig. 1 exhibits the EUR/USD exchange rate in first differences (Δp_t), together with net demand z_t measured in Million of Euros. The outcome of the Brexit referendum (June 24) and the first election of Donald Trump as US president (November 9), together with few other events, are marked with a thick vertical line in the graph. The largest price swings followed a controversial announcement by the ECB of a further expansion of Quantitative Easing on March 10 [8] and the announcement of weak US jobs data on June 3[9], both of which lead to a stark appreciation of the Euro. The largest demand swing instead was triggered by the Euro falling below the critical 1.04 $ benchmark for the first time in 14 years on December 15 [10].

It must be stressed that, since the FX market is highly decentralized, the interdealer

[7] According to the EBS Level 1 terminology, our time series refers to "deals" (i.e. transactions closed after a market order, called a "hit" in the EBS terminology) and doesn't include "quotes" (i. e. limit orders).

[8] https://www.ft.com/content/02ec97ea-e6d9-11e5-bc31-138df2ae9ee6

[9] https://www.ft.com/content/eb77d7d6-2937-11e6-8ba3-cdd781d02d89

[10] https://www.ft.com/content/25fba186-fc9f-3bf2-a020-82efe29f1f7b

segment represents only a single component of a larger and much opaque market infrastructure, where it is very difficult to detect insider trading activities (Batten *et al.*, 2021). The resulting informational asymmetries are likely reflected in the dynamics of the interdealer market. The main purpose of the methodology introduced in the previous section is to disentangle their impact on the volatility of the EUR/USD exchange rate.

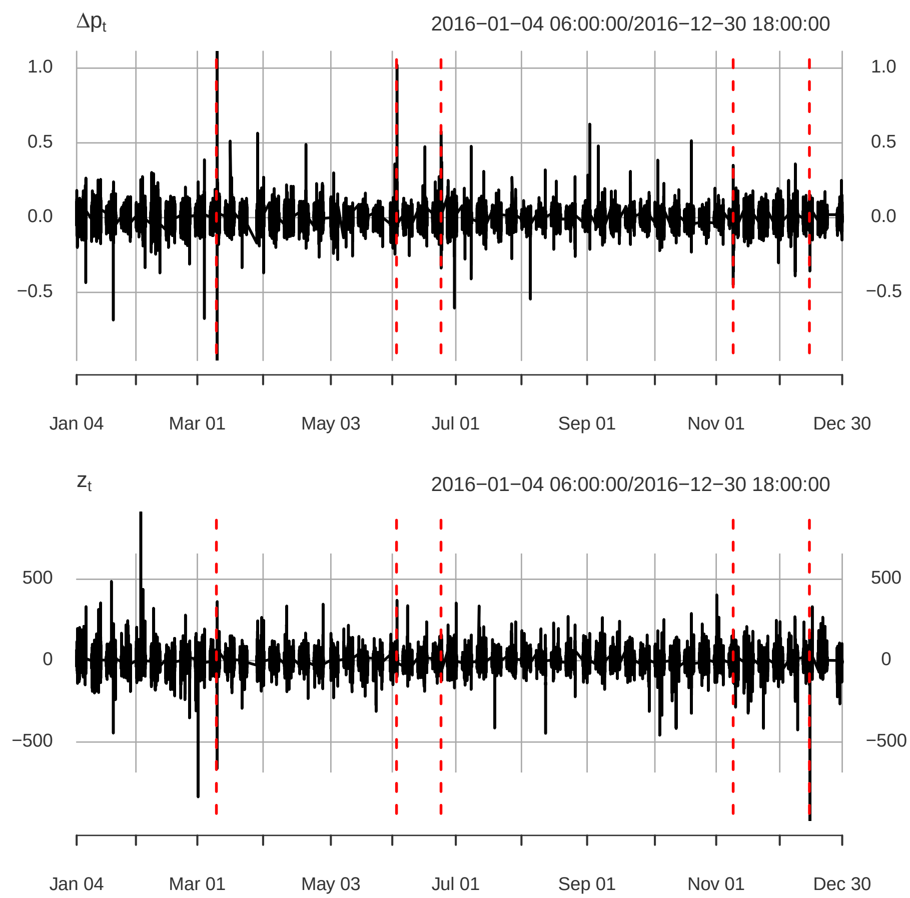

Figure 1: Time series plots for the EUR/USD exchange rate variation Δp_t (above) and net order flow z_t (below), both measured at the 5 minutes frequency, in 2016. The sequence of dashed vertical lines correspond to the following events: QE announcement from ECB (March 10); weak US jobs data (June 3); Brexit referendum (June 24); Trump election (November 9); EUR/USD passing the 1.04 threshold (December 15)

From Fig. 1, we see that large demand swings do not necessarily correspond to large price swings and vice versa. The explanation is twofold. On the one hand, information can be incorporated directly into the exchange rate. On the other hand, the EUR/USD market is liquid enough to absorb large demand imbalances without large price effects. At any rate, the Pearson correlation coefficient between Δp_t and z_t is 0.305. The source of this correlation can be twofold. On the one hand, it might depend on interactions between market participants in a context where fundamental innovations only affect Δp_t. On the other hand, it might additionally depend on the exogenous flow of fundamental innovations affecting Δp_t and z_t at the same time. As we will see, the empirical analysis lends more support to the latter option.

5.2 GAS-VAR Analysis

The first step of the analysis is to quantify the temporary microstructural effects by means of the GAS-VAR model given by Eq. (6) with $y_t = (\Delta p_t, z_t)'$[11]. For the sake of clarity I rewrite the system (6) more explicitly as follows:

$$\Delta p_t = \mu_{1,t} + \sum_{s=1}^{p} b_{s,t}^0 y_{t-s} + u_{1,t} \tag{19}$$

$$z_t = \mu_{2,t} + \sum_{s=1}^{p} b_{s,t}^1 y_{t-s} + u_{2,t} \tag{20}$$

The model is completed by Eqs. (9)-(10), which specify the s_{μ_t} and s_{b_t} terms in the GAS equation (4) for $\mu_t = (\mu_{1,t}, \mu_{2,t})'$ and the entries of the matrices $B_{1,t} = \begin{bmatrix} b_{1,t}^0 \\ b_{1,t}^1 \end{bmatrix}, \ldots, B_{p,t} = \begin{bmatrix} b_{p,t}^0 \\ b_{p,t}^1 \end{bmatrix}$. In other terms, I set $f_t = (\mu_t', b_t')'$ where b_t is defined as in Eq. (8). An ordinary VAR model is also estimated as term of comparison. The lags of both models are selected as the largest ones among those which minimize the set of information criteria reported in Tab. 1. The restrictions implied by the VAR model are strongly rejected from the data. Therefore, the GAS-VAR model provides evidence that microstructural effects are

[11]The reader can observe that, since all the coefficients are time varying, the system (19)-(20) can be rewritten in terms of returns simply by rescaling the coefficient themselves.

better captured by a model which takes into account the time varying conditions of the market. There is also a marked improvement w.r.t. VAR in terms of explained volatility as measured by the coefficient of determination of the two equations of the model. Under this respect, there is also an improvement w.r.t. the non-parametric approach suggested by Aslanidis and Casas (2013) and Casas and Fernandez-Casal (2019) for time-varying regression estimation, which is reported in the last line of the table under the label NP-VAR[12]. On the other hand, according to unreported results, the GAS-VAR and NP-VAR models do not improve the low forecasting power of the VAR model. Thus, the time varying coefficients of the two models do not appear to convey information about future movements of the exchange rate, which supports their interpretation as microstructural effects.

Model(lags)	$\mathcal{L}$	AIC	BIC	HQIC	LR	$R^2(\Delta p_t)$	$R^2(z_t)$
VAR(24)	- 37,692	75,580	76,414	75,845	1,389.03***	0.005	0.019
GAS-VAR(24)	- 36,995	74,578	76,080	74,451	-	0.031	0.028
NP-VAR(24)						0.008	0.023

*** significant at the 1% level. Note: $\mathcal{L}$ = likelihood; $AIC = -2 \times \mathcal{L} + 2 \times d$; $BIC = -2 \times \mathcal{L} + \ln T \times d$; $HQIC = -2 \times \mathcal{L} + 2 \times \ln \ln T \times d$; LR = loglikelihood ratio test statistics. The GAS-VAR model is given by Eqs. (19)-(20) with further specifications given by Eqs. (3)-(4) and (9)-(10). The NP-VAR model is the nonparametric approach of Aslanidis and Casas (2013) and Casas and Fernandez-Casal (2019).

Table 1: Comparison of VAR, GAS-VAR and NP-VAR models

In order to quantify the impact of microstructural effects, I adopt a relative excess volatility indicator as defined in Tab. 2, which measures the relative deviation of the volatility of observed price variations Δp_t from the volatility of efficient price variations $\Delta \tilde{p}_t$. The definition of the latter is model dependent. For the VAR, NP-VAR and GAS-VAR models, it is given by the standard deviation of the estimation residuals. For the state space estimation of the efficient price, it is given by the standard deviation of the state variable. From Tab. 2 we see that the GAS-VAR estimation is closer to the estimate of the SS model than the VAR and NP-VAR alternatives. At any rate, the comparative

[12]In this case the number of lags is set to be equal to those of the VAR model.

analysis confirms that microstructural effects have a limited bearing on price volatility.

	(1) VAR(24)	(2) GAS-VAR(23)	(3) NP-VAR(24)	(4) SS
$\frac{\sigma(\Delta p)}{\sigma(\Delta \tilde{p})} - 1$	0.003	0.016	0.004	0.028

The GAS-VAR model is given by Eqs. (19)-(20) with further specifications given by Eqs. (3)-(4) and (9)-(10). The NP-VAR model is the nonparametric approach of Aslanidis and Casas (2013) and Casas and Fernandez-Casal (2019). The SS model is efficient price estimation by means of a state space model. For models (1)-(3), the volatility of efficient price variations $\sigma(\Delta \tilde{p})$ is given by the standard deviation of the estimation residuals. For model (4), it is given by the standard deviation of the state variable.

Table 2: Relative excess volatility of price variations due to microstructural effects, comparison of models

5.3 GAS-S-MGARCH Analysis

The second step of the analysis is to quantify the endogenous components of volatility, represented by the terms in $B_{0,t}$, given that $\epsilon_t = B_{0,t} u_t$. In this case the system (6) can be rewritten as follows:

$$\Delta p_t = \mu'_{1,t} + b_{0,1,2,t} z_t + \sum_{s=1}^{p} b^{0*}_{s,t} y_{t-s} + \epsilon_{1,t} \tag{21}$$

$$z_t = \mu'_{2,t} + b_{0,2,1,t} \Delta p_t + \sum_{s=1}^{p} b^{1*}_{1,s} y_{t-s} + \epsilon_{2,t} \tag{22}$$

Here $b_{0,1,2,t}$ and $b_{0,2,1,t}$ represent simultaneous reaction coefficients, which might be related to the interplay between liquidity providers and liquidity takers on the market, and $B^*_{s,t} = B_{0,t} B_{s,t}$ for $s = 1, \ldots, p$.

Setting $\epsilon_t \sim \mathcal{N}(0, G_t)$ with $G_t = \text{diag}(g_t)$, $g_t = (g_{1,t}, g_{2,t})'$ and $b_{0,t} = (-b_{0,1,2,t}, -b_{0,2,1,t})'$, according to Eqs. (15)-(16) we have that $f_t = (b'_{0,t}, g'_t)'$. The model is completed by Eq. (18), which provides the $s_{b_{0,t}}$ term in the GAS equation (4), whereas from Eq. (15) g_t

follows an ordinary GARCH(1,1) process. Making some adjustments to the notation of the previous paragraphs, the model of Eqs. (15)-(16) is estimated through the following recursions:

$$\epsilon_{1,t-1} = u_{1,t-1} - b_{0,1,2,t-1}u_{2,t-1} \tag{23}$$

$$b_{0,1,2,t} = \theta_{0,1} + \theta_{1,1}\epsilon_{1,t-1}u_{2,t-1} + \theta_{2,1}b_{0,1,2,t-1} \tag{24}$$

$$g_{1,t} = \omega_1 + \alpha_1\epsilon^2_{1,t-1} + \beta_1 g_{1,t-1} \tag{25}$$

$$\epsilon_{2,t-1} = u_{2,t-1} - b_{0,2,1,t-1}u_{1,t-1} \tag{26}$$

$$b_{0,2,1,t} = \theta_{0,2} + \theta_{1,2}\epsilon_{2,t-1}u_{1,t-1} + \theta_{2,2}b_{0,2,1,t-1} \tag{27}$$

$$g_{2,t} = \omega_2 + \alpha_2\epsilon^2_{2,t-1} + \beta_2 g_{2,t-1} \tag{28}$$

Eqs. (23)-(25) and (26)-(28) refer respectively to Eq. (21) and to Eq. (22), and their parameters can be estimated separately. Table 3 reports the maximum likelihood estimates of the parameters of Eqs. (23)-(25) for the EUR/USD dataset in 2016, which are compared with two alternatives. The first one is an ordinary GARCH(1,1) model, which is obtained from Eqs. (23)-(25) by setting $\theta_{0,1} = \theta_{1,1} = \theta_{2,1} = 0$. The second one is a S-GARCH(1,1) model with time invariant $b_{0,1,2,t} = \theta_{0,1}$, obtained by setting $\theta_{1,1} = \theta_{2,1} = 0$. All the coefficient estimates in Table 3 are highly significant, and the restrictions implied by the GARCH(1,1) and the S-GARCH(1,1) models are rejected from the data. Table 4 reports instead the results for Eq.(26)-(28). In this case I need to resort to variance targeting because of the near unit root behavior of Eq. (28). The coefficients are all 1% significant with the only exception of $\theta_{2,2}$ which is non significant. Again, the restriction implied by GARCH(1,1) and the S-GARCH(1,1) models are rejected.

The range of the reaction coefficients $b_{0,1,2,t}$ and $b_{0,2,1,t}$ of Eqs.(24) and (27) is remarkably wide (see Fig. 2). These coefficients occasionally turned negative in the period of study (respectively in the 0.049% and 0.019% of periods). More importantly, sometimes they

Parameter	GARCH(1,1)	S-GARCH(1,1)	GAS-S-GARCH(1,1)
$\theta_{0,1}$	-	0.3840	0.0348
		(0.0058)	(0.0020)
$\theta_{1,1}$	-	-	0.0521
			(0.0088)
$\theta_{2,1}$	-	-	0.9109
			(0.0052)
ω_1	0.0123	0.0110	0.0110
	(0.0005)	(0.0005)	(0.0005)
α_1	0.1715	0.1732	0.1741
	(0.0056)	(0.0057)	(0.0058)
β_1	0.7879	0.7860	0.7852
	(0.0058)	(0.0061)	(0.0061)
$\mathcal{L}$	-19,382	-17,257	-17,237
LR	4,250.7***	39.8***	-

*** significant at the 1% level. Notes: Standard errors in parentheses; $\mathcal{L}$ = likelihood; LR = loglikelihood ratio test statistics with respect to the model on the next column The GARCH(1,1) model is obtained from Eqs. (23)-(25) by setting $\theta_{0,1} = \theta_{1,1} = \theta_{2,1} = 0$. The S-GARCH(1,1) model is obtained from Eqs. (23)-(25) by setting $\theta_{1,1} = \theta_{2,1} = 0$.

Table 3: Maximum likelihood estimates for the GAS-S-GARCH model of Eqs. (23)-(25) and two alternative models (see notes)

Parameter	GARCH(1,1)	S-GARCH(1,1)	GAS-S-GARCH(1,1)
$\theta_{0,2}$	-	0.2360	0.2360
		(0.0038)	(0.0073)
$\theta_{1,2}$	-	-	0.0530
			(0.0102)
$\theta_{2,2}$	-	-	0.0093
			(0.0262)
α_2	0.1594	0.1581	0.1584
	(0.0032)	(0.0010)	(0.0010)
β_2	0.7827	0.7831	0.7828
	(0.0045)	(0.0003)	(0.0003)
$\mathcal{L}$	-11,139	-9,215	-9,204
LR	3,848.5***	22.8***	-

*** significant at the 1% level. Notes: Variance targeting adopted; Standard errors in parentheses; $\mathcal{L}$ = likelihood; LR = loglikelihood ratio test statistics with respect to the model on the next column; $\overline{g}_2$ = unconditional volatility of ϵ_2. The GARCH(1,1) model is obtained from Eqs. (26)-(28) by setting $\theta_{0,2} = \theta_{1,2} = \theta_{2,2} = 0$. The S-GARCH(1,1) model is obtained from Eqs. (26)-(28) by setting $\theta_{1,2} = \theta_{2,2} = 0$.

Table 4: Maximum likelihood estimates for the GAS-S-GARCH model of Eqs. (26)-(28) and two alternative models (see notes)

exceeded unity when positive, leading to dynamic instability (respectively in the 0.025% and 0.001% of periods). From Fig. 2 we see that jumps in the coefficients coincide with some of the events highlighted in Fig. 1. In particular, the dynamics of $b_{0,2,1,t}$ captures with a big jump upwards the peculiarity of the EUR/USD exchange rate passing the 1.04 threshold on December 15. Indeed, at that juncture, there were probably many limit and stop orders set at the 1.04 threshold, which made demand extremely sensitive to price crossing it. On the other hand, big informational surprises (like the US job announcement of June 3 which changed the interest rate outlook for the US economy) make price extremely reactive to demand variations, with a jump upwards of $b_{0,1,2,t}$. This result could be interpreted as liquidity providers becoming more fearful of adverse selection in these moments. But, since the reactivity coefficient of demand to price variations $b_{0,2,1,t}$ becomes larger as well, it is likely that also uninformed liquidity takers are reacting more intensely to the information shock at this juncture.

Fig. 2 shows the large movements of endogenous volatility of price variations as measured by the relative excess conditional volatility $\left(\frac{h_{1,t}}{g_{1,t}}\right)^{1/2} - 1$. This quantity was on average of 46%, but reached a maximum of 1008% on September 2, when US jobs data disappointed, and a minimum of -86% on March 1, when $b_{0,1,2,t}$ briefly turned negative. Indeed, the relative excess volatility was negative in 12% of the periods. As explained in Sec. 5.5 below, a negative $b_{0,1,2,t}$ can occurr as a combination of lower risk aversion of liquidity providers and of lower reactivity of liquidity takers to price variations.

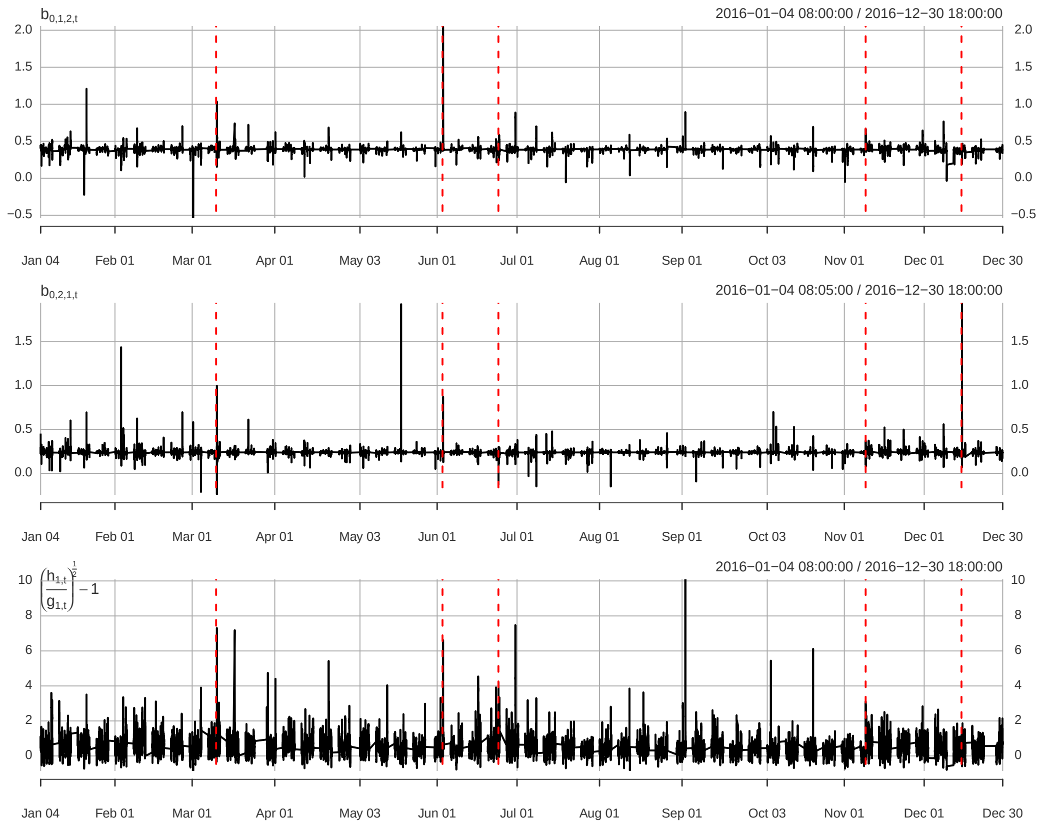

Figure 2: Time series plots for $b_{0,1,2,t}$, $b_{0,2,1,t}$, from Eqs. (24) and (27), and relative excess volatility of price variations $\left(\frac{h_{1,t}}{g_{1,t}}\right)^{1/2} - 1$. For the sequence of dashed vertical lines see Fig. 1.

5.4 Excess volatility, Information and Jumps

Excess volatility should become more intense when informational surprises occur. In order to analyze more systematically the relationship between the variables in Fig. 2 and the flow of information, I employ a sample of 1,219 standardized surprises referring to U.S. fixed-time macroeconomic announcements collected from the economic calendar of Bloomberg[13]. The focus is on news related to the U.S. economy because these are likely to impact most the EUR/USD exchange rate. I compute the cross-correlation functions between the absolute value of these surprises at t and the values of $b_{0,1,2,t+h}$, $b_{0,2,1,t+h}$ and $\left(\frac{h_{1,t+h}}{g_{1,t+h}}\right)^{1/2} - 1$ for $h \in [-100, 100]$. From Fig. 3 we see that informational surprises have a positive impact on relative excess volatility, with a delay of 2 periods aka 10 minutes (bottom panel). This effect is linked to the increased demand reactivity of liquidity takers $b_{0,1,2,t}$ (top panel), while demand reactivity to price variations $b_{0,2,1,t}$ appears completely disconnected from informational surprises. This asymmetry speaks in favor of the "order-driven view" mentioned in Sec. 2, underlining that information is embedded into the price mainly through trade.

This conclusion is corroborated by the analysis of price jumps, which are detected with the approach of Lee and Mykland (2008) as implemented in Boudt *et al.* (2022). According to the stated methodology, the dataset contains 50 jumps at the 5 minutes frequency. Fig. 4 depicts the average value of $b_{0,1,2,t+h}$, $b_{0,2,1,t+h}$ and $\left(\frac{h_{1,t+h}}{g_{1,t+h}}\right)^{1/2} - 1$ when a price jump occurs at t for $h \in [-100, 100]$. From the bottom panel we see that relative excess volatility increases on average from 23% to 115% one period after the jump. From the top panel we see that this increase is connected with a significant increase of $b_{0,1,2,t+h}$, while $b_{0,2,1,t+h}$ is not reactive. The alignment with the previous results is not surprising since jumps and informational surprises are correlated (Neely and Dey, 2011). Overall, the

[13] A surprise is nothing but a z-score, where the mean and the standard deviation are computed from the expectations of a sample of experts.

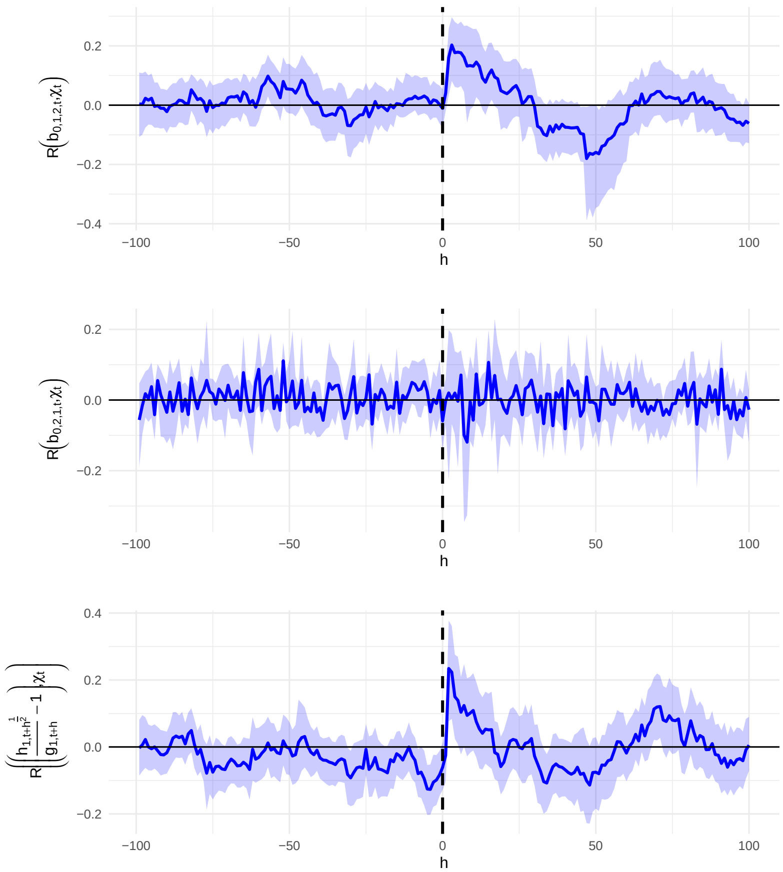

Figure 3: Cross correlation function R between the absolute U.S. news surprise χ_t at t and $b_{0,1,2,t+h}$, $b_{0,2,1,t+h}$ and the relative excess volatility of price variations $\left(\frac{h_{1,t+h}}{g_{1,t+h}}\right)^{1/2}-1$ at $t+h$ for $h \in [-100, 100]$. News surprises refer to a sample of 1,219 macroeconomic prescheduled announcements, and are obtained from the economic calendar of Bloomberg. The shaded area represents 95% confidence intervals obtained with bootstrap estimation.

GAS-S-GARCH(1,1) approach highlights the connection between fundamental surprises and excess volatility. According to these results, endogenous and exogenous volatility are complements rather than substitutes.

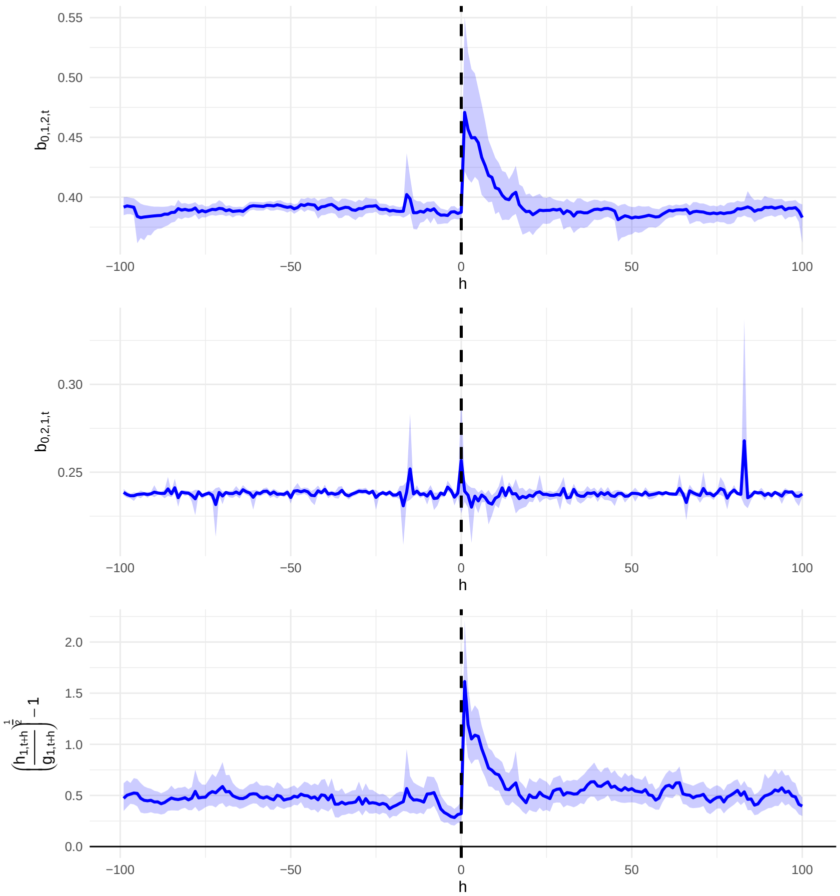

Figure 4: Average value of $b_{0,1,2,t}$, $b_{0,2,1,t}$ and of relative excess volatility of price variations $\left(\frac{h_{1,t+h}}{g_{1,t+h}}\right)^{1/2} - 1$ at $t+h$ when a price jump occurs at t for $h \in [-100, 100]$. The shaded area represents 95% confidence intervals obtained with bootstrap estimation.

5.5 Discussion

In order to provide an economic interpretation of the GAS-S-MGARCH analysis of Sec. 5.3, it's possible to sketch two simple models where liquidity takers and providers interact under conditions of informational asymmetry. The first model, described in Appendix A.2, devises a condition of milder, possibly less realistic, asymmetry of information, whereby in particular the liquidity provider is perfectly informed on fundamentals and on demand conditions, while the liquidity taker has imperfect information on the price charged by the liquidity provider. Under these assumptions, efficient price variations (which coincide the residuals $u_{1,t}$ of Eq. (19)) are correlated with net demand shocks (i.e. $u_{2,t}$ of Eq. (20)) but the structural errors ϵ_t are independent of each other. Instead, if we assume that the information held by liquidity takers as a whole convey fundamental innovations to the market and that liquidity providers are imperfectly informed on fundamentals, like in the model of Appendix A.3, we obtain that the ϵ_t themselves are correlated, representing time varying linear combinations of two independent factors, which are respectively the fundamental information process $\eta_{1,t}$ and the fundamental noise process $\eta_{2,t}$.

Although, as a consequence of general results like proposition 2 in Rigobon (2003), the resulting empirical model lacks identification (see Eqs.(A.3.8)-(A.3.9) in Appendix A.3), it is nevertheless possible to test for the (conditional) correlation of the ϵ_t. A rejection of the hypothesis of no correlation would suggest that the model of Appendix A.3 fits better the data than the model of Appendix A.2. The most important implication of a rejection would be that the true excess volatility on the EUR/USD market is even larger than the one devised in the empirical analysis above. In order to proceed, I rely on the Dynamic Conditional Correlation (DCC) model of Engle (2002), whereby we have (see Eq. (A.3.4)):

$$A_t = D_t R_t D_t \tag{29}$$

$$D_t = \mathrm{diag}(\sqrt{g_t}) \tag{30}$$

$$R_t = \mathrm{diag}(Q_t)^{-\frac{1}{2}} Q_t \mathrm{diag}(Q_t)^{-\frac{1}{2}} \tag{31}$$

$$Q_t = (1-a-b)\overline{Q} + a\left((D_{t-1})^{-1}B_{0,t-1}u_{t-1}\right)\left((D_{t-1})^{-1}B_{0,t-1}u_{t-1}\right)' + bQ_{t-1} \tag{32}$$

where a, b are scalars and g_t is still given by Eqs. (15)-(16). We cannot estimate the volatilities of $(\eta_{1,t}, \eta_{2,t})$ with this approach, since it relaxes the condition of independence of structural errors at the cost of normalizing the variance of $\eta_{0,t}$ and $\eta_{1,t}$ to $\tilde{g}_{1,t} = \tilde{g}_{2,t} = 1$. Therefore, as explained above, the sole purpose of the estimation is to test the hypothesis of no conditional correlation of $\epsilon_{1,t}$ and $\epsilon_{2,t}$. The results are summarized in the following particolarization of eq. (32):

$$Q_t = \Big(1 - \underset{(0.002126)}{0.002781} - \underset{(0.059939)}{0.865727}\Big) \begin{bmatrix} 1.000003 & -0.296153 \\ -0.296153 & 1.148114 \end{bmatrix} + \\ + \underset{(0.002126)}{0.002781} \ \left(D_{t-1}^{-1} B_{0,t-1} u_{t-1}\right) \left(D_{t-1}^{-1} B_{0,t-1} u_{t-1}\right)' + \underset{(0.059939)}{0.865727} \ Q_{t-1} \tag{33}$$

A loglikelihood ratio test for the restrictions implied by setting $A_t = I$ in Eq. (A.3.4), as required by the model of Appendix A.2, is strongly rejected by the data[14]. This result states that the model of Appendix A.3 provides a better explanation of the empirical results of Sec. 4, and corroborates the conjecture that our previous estimate of excess volatility provides a lower bound for the true excess volatility on the market.

6 Conclusions

This paper studies excess volatility in the EUR/USD exchange rate, using a GAS framework (Creal *et al.*, 2011) coupled with identification through heteroscedasticity (IH) (Primiceri, 2005), with the main aim to establish a realistic lower bound for excess volatility on financial markets. This is accomplished by adopting a broad definition of fundamental information flow as a properly derived conditionally independent stochastic innovation of price measured at the intra-daily frequency. This definition requires no commitment to a

[14] In pratice, I compare the loglikelihood of the DCC model ($\mathcal{L}_{DCC} = -25.735$) with the sum of the loglikelihoods in Tabs. 3 and 4.

specific asset pricing model and no assumptions on rationality or homogeneity of market participants. By decomposing volatility into endogenous (non fundamental) and exogenous (fundamental) components, this study aims to provide a more nuanced understanding of the forces driving exchange rate fluctuations. The empirical analysis, based on high-frequency transaction data of the EUR/USD interdealer market in 2016, highlights the significant role of informational asymmetry and feedback trading in amplifying exogenous price volatility. Market participants react to both public information and private signals according to their own trading strategies, creating a complex interdependency between price movements and demand dynamics. The empirical findings of the previous sections suggest that, even in a highly liquid market such as the EUR/USD pair, a notable portion of volatility arises from endogenous mechanisms rather than from fundamental shocks.

According to results of Sec. 5.3, excess volatility depends on the reactivity of price variations to demand shocks, which becomes higher when fundamental shocks occur. The simple theoretical model of Appendix A.3 explains the larger reactivity at these times in terms of liquidity providers becoming more risk adverse. This interpretation is consistent with the assumption, in that model, that liquidity providers learn fundamental information from liquidity takers, and therefore are more likely to charge a wider bid-ask spread when they are less informed than the latter. Indeed, the main implication of the model of Appendix A.3 is that the structural innovations of price variations and demand are themselves correlated. The empirical evidence presented in Sec. 5.5 favors this implication, lending further support to the idea that information is incorporated into asset prices through an informed component of demand, as claimed by the MM literature. While the increase of the bid-ask spread on the interdealer market implied by my results is consistent with standard adverse selection models, it does not contradict the evidence that dealers decrease bid-ask spreads for their over-the-counter informed clients (Osler, 2011). Instead, it supports the standard explanation of this behavior, i.e. that dealers try to capture informed order flow to then use the information extracted in subsequent interdealer trading.

The main implication of the results above is that excess volatility on the foreign exchange market is related to fundamental volatility. As mentioned in the introduction, this conclusion is similar in spirit to the one of Wehrli and Sornette (2022), who explain excess volatility by a combination of exogenous and endogenous factors, where the latter act as amplifiers. Instead, it is rather distant from the "inelastic demand hypothesis" of Gabaix and Koijen (2021). In the first place, these authors adopt a different identification strategy which relies on instrumental variables, with all their potential pitfalls. In the second place, they adopt a narrow definition of fundamental information for stocks, by relying on dividends. In the third place, they don't take into account the possibility that endogenous and exogenous volatility may interact. Lastly, they admit no role for asymmetry of information.

This last point deserves further comments. Indeed, the evidence provided by this paper, which support the view that asymmetry of information is extremely relevant on the EUR/USD market, align with the recent work of Ranaldo and Somogyi (2021), which refers to a large set of currency pairs and years and allow for agent heterogeneity. The main differences from their work are that these authors sideline the issue of interdependency between price and demand, which is at the heart of the GAS-IH approach, that they do not focus specifically on volatility, and that their study involves all segments of the market, while mine only regards the interdealer market and requires no subdivision of demand components. Therefore, we might see one relevant advantage of the combined GAS-IH methodology in the fact that it makes it possible to consistently detect excess volatility, on a single market platform, whenever aggregate demand data are available.

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A Appendices

A.1 GAS-VAR model

In order to specify the SD-VAR model, we need to pin down the score function and the information matrix for the time-varying likelihood. In the following, we make use of the standard notation for VAR models detailed in Sec. 3. The general form of SD models (Creal *et al.*, 2011) is given by Eqs. (1) and (3)-(5). In our case we have that $f_t = [\mu_t' \;\; b_t' \;\; h_t']'$ as detailed in Sec. 3, and the first differential of the normal loglikelihood function reads as follows (Abadir and Magnus, 2005):

$$d\mathcal{L}_t = \frac{1}{2}\mathrm{tr}\left(\left(\mathrm{d}\Sigma_t\right)\Sigma_t^{+}\left(\xi_t\xi_t' - \Sigma_t\right)\Sigma_t^{+}\right) + \mathrm{d}\left(A_t z_{t-1}\right)'\Sigma_t^{+}\xi_t + \mathrm{d}c_t'\Sigma_t^{+}\xi_t \tag{A.1.1}$$

where Σ_t^{+} denotes the Moore-Penrose inverse of the matrix Σ_t. In order to specify the process for μ_t according to the SD model we need to compute the score function of Eq. (5), which only depends on the third additive term on the rhs of (A.1.1):

$$\nabla_{\mu_t} = P_{np\times n}'\Sigma_t^{+}\,\xi_t \tag{A.1.2}$$

where

$$\frac{\partial\, c_t}{\partial\left(\mathrm{vec}\,\mu_t\right)'} = \begin{bmatrix} I_n \\ 0_{n(p-1)\times n} \end{bmatrix} \equiv P_{np\times n} \tag{A.1.3}$$

and where $0_{i\times j}$ is an empty matrix of shape $i \times j$. In general, it is convenient to normalize the score function by the information matrix, since this choice makes the estimation of the parameters of (μ_t, b_t) independent from the estimation of the parameters of h_t. In the case of μ_t the information matrix is as follows:

$$\mathcal{I}_{\mu_t|t-1} \equiv E_{t-1}[\nabla_{\mu_t}\nabla_{\mu_t}'] = P_{np\times n}'\Sigma_t^{+}P_{np\times n} \tag{A.1.4}$$

Therefore we can write s_{μ_t} as follows:

$$s_{\mu_t} = P'_{np\times n}\xi_t = u_t \tag{A.1.5}$$

with $S_{\mu_t} = \mathcal{I}^{-1}_{\mu_t|t-1}$ as scaling matrix.

In order to specify the process for $b_t \equiv \text{vec}\left([B_{1,t}\ B_{2,t} \dots B_{p-1,t}\ B_{p,t}]'\right)$, we need to compute a score function which only depends on the second additive term on the rhs of (A.1.1) :

$$\nabla_{b_t} = P'_{n^2p^2\times n^2p}\,\text{vec}\left(z_{t-1}\,\xi_t'\,\Sigma_t^+\right) \tag{A.1.6}$$

where

$$\frac{\partial\,\text{vec}(A_t')}{\partial\,(\text{vec}\,B_t)'} = \begin{bmatrix} I_{n^2\,p} \\ 0_{n^2\,p\,(p-1)\times n^2p} \end{bmatrix} \equiv P_{n^2p^2\times n^2p} \tag{A.1.7}$$

The information matrix is as follows:

$$\mathcal{I}_{b_t|t-1} \equiv E_{t-1}[\nabla_{b_t}\nabla'_{b_t}] = P'_{n^2p^2\times n^2p}\left(\Sigma_t^+ \otimes z_{t-1}\,z'_{t-1}\right)P_{n^2p^2\times n^2p} \tag{A.1.8}$$

where the symbol $\otimes$ stands for the Kronecker product. Taking into account (A.1.6) and (A.1.8), we can write s_{b_t} as follows:

$$s_{b_t} = \|z_{t-1}\|^2\,P'_{n^2p^2\times n^2p}\text{vec}\left(\left(z'_{t-1}z_{t-1}\right)^{-1}z_{t-1}\xi_t'\right) = u_t \otimes z_{t-1} \tag{A.1.9}$$

with $S_{b_t} = \|z_{t-1}\|^2\,\mathcal{I}^{-1}_{b_t|t-1}$ as scaling matrix. We remark that the processes μ_t and b_t depend on each other through u_t. Therefore, they must be jointly estimated.

A.2 A model of endogenous volatility with a perfectly informed liquidity provider

Let's consider the following system:

$$B_{0,t}y_t^* = \epsilon_t \tag{A.2.1}$$

with $y_t^* = (\Delta p_t - v_{1,t}^*, z_t - v_{2,t}^*)'$, where Δp_t and z_t represent respectively price variations and net demand for a zero-yield asset at t and $v_{1,t}^*$, $v_{2,t}^*$ represent (reduced form) microstructural factors affecting the two variables. Therefore $\Delta\tilde{p}_t \equiv \Delta p_t - v_{1,t}^*$ is the efficient price variation whose volatility is analyzed in Tab. 2. Furthermore, $\epsilon_t \sim \mathcal{N}(0, G_t)$ and G_t is diagonal. In this appendix I show how it is possible to derive a system like (A.2.1) from a very simple economic model involving heterogeneous liquidity takers (LT), which are not perfectly informed on price variations occurring at t (i.e. they need to infer the trading price in advance of submitting market orders), and a monopolistic liquidity provider (LP) which is perfectly informed on z_t and on fundamental innovations. In the following, assume that all random variables involved are normal and distinguish the conditional expectations of the econometrician $E_t[\cdot]$ from those of LT of types $i = 1, \ldots, S$, which are denoted as $E_{it}[\cdot]$.

Assume that price variations charged by the LP reflect fundamental information, which we identify with $\epsilon_{1,t}$ in Eq. (A.2.1), plus a spread variation $\delta_t(z_t) = s_t(z_t) - s_{t-1}(z_{t-1})$, where s_t is the source of profits for the LP at t, and microstructural factors $v_{1,t}$ which are independent of $\epsilon_{1,t}$. If we let $v_{1,t}$ absorb s_{t-1}, we may write:

$$\Delta p_t = s_t(z_t) + \epsilon_{1,t} + v_{1,t} \tag{A.2.2}$$

LT are of different types regarding expectations, and they are endowed with CARA preferences and different levels of risk aversion D_i^{-1}. The profit at t of the LT of type i on her net trade at $t-1$ is given by:

$$\Pi_{i,t} = (p_t - p_{t-1})z_{i,t-1} \tag{A.2.3}$$

From the latter we derive

$$E_{i,t}\left[\Pi_{i,t+1}\right] = \left(E_{i,t}\left[p_{t+1}\right] - p_t\right) z_{i,t} \tag{A.2.4}$$

$$V_{i,t}\left[\Pi_{i,t+1}\right] = V_{i,t}\left[p_{t+1}\right] z_{i,t}^2 \tag{A.2.5}$$

Taking into account Eqs. (A.2.4) and (A.2.5), and letting $V_{i,t}\left[p_{t+1}\right]$ be absorbed by $D_{i,t}^{-1} = V_{i,t}\left[p_{t+1}\right]/D_i$, we may write the objective for LT as follows:

$$\max_{z_{i,t}} \left\{ E_{i,t}[\Delta p_{t+1}]\, z_{i,t} - \frac{z_{i,t}^2}{2D_{i,t}} \right\} \tag{A.2.6}$$

We solve the FOC for $z_{i,t}$ to obtain the optimal demand of a generic LT of type i:

$$z_{i,t} = D_{i,t}E_{i,t}[\Delta p_{t+1}] \tag{A.2.7}$$

Then the total market demand is

$$z_t = \sum_{i=1}^{S} D_{i,t}E_{i,t}[\Delta p_{t+1}]N_{i,t} \tag{A.2.8}$$

where $N_{i,t}$ is the number of LT of type i at t. Introduce the following specification for $E_{i,t}[\Delta p_{t+1}]$:

$$E_{i,t}[\Delta p_{t+1}] = g_i E_{i,t}\left[\Delta p_t\right] \tag{A.2.9}$$

where the g_i are type specific fixed coefficients (not necessarily positive), measuring the impact of the expected price increment at t on the expectation the next period's price increment for LTs of type i. Eq. (A.2.9) reflects the fact that the execution price for the market orders of LTs at t is uncertain. Substituting (A.2.9) in (A.2.8) we obtain

$$z_t = \sum_{i=1}^{S} D_{i,t} N_{i,t} g_i E_{i,t} [\Delta p_t] \tag{A.2.10}$$

LTs of type i receive a noisy signal $\xi_{i,t} = \Delta p_t + \epsilon'_{i,t}$, where the $\epsilon'_{i,t}$ are normal with zero mean, reciprocally independent as well as independent from $\epsilon_{1,t}$. Supposing that the prior of all LT for Δp_t is null, from the distributional assumptions we get that $E_{i,t} [\Delta p_t] = \rho_{i,t} \Delta p_t + \rho_{i,t} \epsilon'_{i,t}$, where $\rho_{i,t} = \frac{\tau_{\epsilon'_{i,t}}}{\tau_{\epsilon'_{i,t}} + \tau_{\Delta p_t}}$ and $\tau_{\epsilon'_{i,t}} \equiv 1/\sigma^2_{\epsilon'_{i,t}}, \tau_{\Delta p_t} \equiv 1/\sigma^2_{\Delta p_t}$. Further suppose that $\epsilon'_{i,t}$ is composed by two independent noise components, i.e. $\epsilon'_{i,t} = \nu_{1,i,t} + \nu_{2,i,t}$. The latter originate from uncertainty referring respectively to $\epsilon_{1,t}$ (fundamental related uncertainty) and to $\delta_t(z_t) + v_{1,t}$ (non fundamental related uncertainty) but are independent of these variables. Then define

$$\begin{cases} \gamma_t \equiv \sum_{i=1}^{S} D_{i,t} N_{i,t} g_i \rho_{i,t} \\ \epsilon_{2,t} \equiv \sum_{i=1}^{S} D_{i,t} N_{i,t} g_i \rho_{i,t} \nu_{1,i,t} \\ v_{2,t} \equiv \sum_{i=1}^{S} D_{i,t} N_{i,t} g_i \rho_{i,t} \nu_{2,i,t} \end{cases} \tag{A.2.11}$$

Finally, we obtain that market demand is a time varying function of current price variations:

$$z_t = \gamma_t \Delta p_t + \epsilon_{2,t} + v_{2,t} \tag{A.2.12}$$

According to the hypothesis of perfect information and assuming that the LP is risk neutral but that it is costly to maintain an inventory of the asset, her profit is specified as $\Pi_{d,t} = s_t z_t - \frac{\omega_t}{2} z_t^2$. Maximizing $\Pi_{d,t}$ with respect to s_t and substituting into Eq. (A.2.2), we obtain

$$\Delta p_t = \left(\omega_t - \frac{1}{\gamma_t} \right) z_t + \epsilon_{1,t} + v_{1,t} \tag{A.2.13}$$

from which, we obtain Eq. (A.2.1) with $B_{0,t} = \begin{bmatrix} 1 & -\left(\omega_t - \frac{1}{\gamma_t}\right) \\ -\gamma_t & 1 \end{bmatrix}$ and $v_t^* = B_{0,t}^{-1} v_t$ where $v_t = (v_{1,t}, v_{2,t})'$.

A.3 A model of endogenous volatility with an imperfectly informed liquidity provider

In the following, still assume that all random variables involved are normal, but now distinguish between the conditional expectations of the econometrician (which are equal to those of the public), denoted by $E_t[\cdot]$, those of a monopolistic liquidity provider (LP), denoted by $E_t^d[\cdot]$, and those of a set of liquidity takers (LT) of types $i = 1, \ldots, S$, denoted by $E_{it}[\cdot]$. Still, LTs are of different types regarding expectations, all of them are endowed with CARA preferences, and their profit is given by Eq. (A.2.3). Therefore, assuming (A.2.9), we get to Eq.(A.2.10). Then we define

$$\begin{cases} \gamma_t \equiv \sum_{i=1}^{S} D_{i,t} N_{i,t} g_i \\ \eta_{2,t} \equiv \sum_{i=1}^{S} D_{i,t} N_{i,t} g_i E_{i,t}\left[\Delta \tilde{p}_t\right] - \gamma_t E_t\left[\Delta p_t\right] \\ v_{2,t} \equiv \sum_{i=1}^{S} D_{i,t} N_{i,t} g_i E_{i,t}\left[v_{1,t}\right] \\ \epsilon_{1,t} \equiv \Delta p_t - E_t[\Delta p_t] \end{cases} \tag{A.3.1}$$

We still obtain Eq. (A.2.12) but with some differences since now

$$\epsilon_{2,t} = \eta_{2,t} - \gamma_t \epsilon_{1,t} \tag{A.3.2}$$

Eq. (A.3.2) states that the statistical error on z_t depends on the private information of LTs, represented by $\eta_{2,t}$, which is channeled to the market through their trading activity. Since $E_t[\eta_{2,t}] = E_t[\epsilon_{1,t}] = 0$, from (A.2.12) we obtain that $\frac{\partial E_t[z_t]}{\partial s_t} = \gamma_t$[15].

[15]For simplicity we have assumed that γ_t is an exogenous, non random, variable. If it was a random

Assuming that $\eta_{2,t} \sim \mathcal{N}(0, \tilde{g}_{2,t})$ represents the flow of fundamental information to the market, price variations charged by the LP become as follows:

$$\Delta p_t = s_t \left(E^d \left[z_t\right]\right) + E_t^d[\eta_{2,t}] + v_{1,t} \tag{A.3.3}$$

The price equation now reflects the uncertainty of the LP regarding net demand z_t and fundamental information $\eta_{2,t}$. Eq. (A.3.3) takes into account that the LP cannot observe $\eta_{2,t}$ directly. Instead, she receives a noisy signal ξ_t on fundamentals which is correlated with the private information of LTs: $\xi_t = \eta_{2,t} + \eta_{1,t}$, where $\eta_{1,t} \sim \mathcal{N}(0, \tilde{g}_{1,t})$ is independent from η_2. From the distributional assumptions we get that $E_t^d\left[\eta_{2,t}\right] = \rho_t \xi_t$, where $\rho_t = \frac{\tau_{t,\eta_1}}{\tau_{t,\eta_1} + \tau_{t,\eta_2}}$ and $\tau_{t,\eta_1} \equiv \tilde{g}_{1,t}^{-1}$, $\tau_{t,\eta_2} \equiv \tilde{g}_{2,t}^{-1}$.

In order to proceed further, we need to conjecture that the solution for ϵ_t is of the following form, which satisfies the assumption $E_t[\epsilon_t] = 0$:

$$\epsilon_t = Q_t \eta_t \tag{A.3.4}$$

Furthermore, we assume (and later verify) that the elements of Q_t do not depend on s_t. Then we can compute

$$\begin{cases} E_t^d[z_t] = E[z_t] + E_t^d[\epsilon_{2,t}] = E[z_t] + (1 - \gamma_t q_{1,2,t})\rho_t \xi_t \\ E_t^d[z_t^2] = E_t[z_t]^2 + 2E_t[z_t](1 - \gamma_t q_{1,2,t})\rho_t \xi_t + g_{2,t} \end{cases} \tag{A.3.5}$$

where $q_{1,2,t}$ is to be determined later and $g_{2,t} = q_{2,1,t}\tilde{g}_{1,t} + q_{2,2,t}\tilde{g}_{2,t}$.

The LP is again risk neutral but subject to inventory costs. She maximizes her expected profits $E_t^d\left[\Pi_{d,t}\right] = s_t E_t^d[z_t] - \frac{\omega_t}{2} E_t^d\left[z_t^2\right]$ with respect to s_t. Since by hypothesis $g_{2,t}$ does not

variable, the same results would follow by substituting γ_t with its conditional expectation value according to the public, provided that the latter is independent from p_t. This assumption is consistent with the literature, since γ_t cannot covariate with price, and liquidity γ_t^{-1} cannot covariate with demand, if liquidity based speculation is to be excluded (Huberman and Stanzl, 2004).

depend on s_t, we obtain

$$\Delta p_t = \left(\omega_t - \frac{1}{\gamma_t}\right) z_t + \epsilon_{1,t} + v_{1,t} \tag{A.3.6}$$

where

$$\epsilon_{1,t} = \rho_t \xi_t + \left(\omega_t - \frac{1}{\gamma_t}\right) [(1 - \gamma_t q_{1,2,t})\rho_t \xi_t - \epsilon_{2,t}] \tag{A.3.7}$$

We can solve the system formed by Eqs. (A.3.2) and (A.3.7) to obtain an explicit solution of the form (A.3.4) where Q_t depends only on $(\omega_t, \gamma_t, \rho_t)$, none of which depends on s_t.

Since $\eta_t = Q_t^{-1} B_{0,t} u_t$, in order to bring the model to the data with the aspiration to estimate $\tilde{G}_t = \text{diag}(\tilde{g}_{1,t}, \tilde{g}_{2,t})$, we need to adapt the model of Eq. (12) as follows:

$$\tilde{G}_t = \omega\omega' + \beta\tilde{G}_{t-1}\beta' + \phi_t(I_n \otimes u_{t-1})(I_n \otimes u'_{t-1})\phi'_t \tag{A.3.8}$$

where now

$$\phi_t = \alpha\,(I_n \otimes Q_{t-1}^{-1} B_{0,t-1}) \tag{A.3.9}$$

It is easy to check that the jacobian of the mapping (A.3.9) w.r.t $(a_1, a_2, \gamma_{t-1}, \omega_{t-1})$, where (a_1, a_2) are the non zero elements in α, is not of full column rank. Therefore the model is not identified according to Theorem 6 in Rothenberg (1971). This result is consistent also with Proposition 2 in Rigobon (2003).

A.4 Proof that (11) is a special case of Eq. (3)

For a multivariate normal distribution we have that

$$\nabla_{g_t} = \frac{1}{2}\Psi_t' D_n' G_{t\otimes}^{-1}\text{vec}(\epsilon_t\,\epsilon_t' - G_t) \tag{A.4.1}$$

$$\mathcal{I}_{g_t|t-1} = \frac{1}{2}\Psi_t' D_n' G_{t\otimes}^{-1} D_n \Psi_t \tag{A.4.2}$$

$$s_{g_t} = \Psi_t^+ D_n^+ \text{vec}(\epsilon_t\,\epsilon_t' - G_t) = \Psi_t^+ \text{vech}(\epsilon_t\,\epsilon_t' - G_t) \tag{A.4.3}$$

where $G_{t\otimes} \equiv G_t \otimes G_t$, $\Psi_t \equiv \frac{\partial \text{vech}(G_t)}{\partial g_t'}$ is a $n(n+1)/2 \times n$ matrix and $g_t = (g_1, \ldots, g_n)'$; D_n is the $n^2 \times n(n+1)/2$ duplication matrix, such that, for a symmetric matrix A, $\text{vec}(A) = D_n \text{vech}(A)$. Given our assumptions, Ψ_t is an orthogonal matrix, which implies $\Psi_t^+ = \Psi'$. Therefore, we can simplify the expression for s_{g_t} in (A.4.3) to obtain the following:

$$s_{g_t} = (\epsilon_{t\odot} - g_t) \tag{A.4.4}$$

where $\odot$ stands for the Hadamard product and $\epsilon_{t\odot} \equiv \epsilon_t \odot \epsilon_t$. If we finally assume, using the notation of (3), that $\theta_0 = \omega_\odot$, $\Theta_1 = \text{diag}(a_\odot)$ and $\Theta_2 = \text{diag}(a_\odot + \beta_\odot)$, we obtain the vectorized equivalent of (11).

A.5 Proof that the model (13) is globally identified

The BEKK formulation, proposed in Engle and Kroner (1995), developed a general quadratic form for the conditional covariance equation. The model (13) has terms of the form

$$h_{l,l,t} = (\psi_t\psi_t')_{l,l} + \sum_i \sum_j \rho_{i,l,t}\rho_{j,l,t}h_{i,j,t-1} + \sum_i \sum_j \phi_{i,l,t}\phi_{j,l,t}u_{i,t-1}u_{j,t-1} \tag{A.5.1}$$

$$h_{l,m,t} = (\psi_t\psi_t')_{l,m} + \sum_i \sum_j \rho_{i,l,t}\rho_{j,m,t}h_{i,j,t-1} \qquad l \neq m \tag{A.5.2}$$

We can apply the arguments of Engle and Kroner (1995), Proposition 2.1, to Eqs.(A.5.1)-(A.5.2) in order to verify that the coefficients appearing in them are identified if (1) ψ_t is lower triangular with positive diagonals, (2) $\rho_{1,1,t} > 0$ and (3) $\phi_{1,1,t} > 0$.

A.6 Proof of Proposition 1

The Jacobian J_t of the mapping (14) has the following block-form:

$$J_t = \begin{bmatrix} J_\omega^{\psi_t} & J_{b_{0,t}}^{\psi_t} & 0 & 0 \\ 0 & J_{b_{0,t}}^{\phi_t} & J_a^{\phi_t} & 0 \\ 0 & J_{b_{0,t}}^{\rho_t} & 0 & J_\beta^{\rho_t} \end{bmatrix} \tag{A.6.1}$$

The sub-jacobians $J_\omega^{\psi_t}$, $J_{b_{0,t}}^{\psi_t}$, $J_a^{\phi_t}$ are the same of Milunovich and Yang (2013), apart from the time indices in $B_{0,t}$ and $B_{0,t-1}$:

$$J_\omega^{\psi_t} = C(\psi_t)^{-1} \left[D_n' \, B_{0,t \otimes} \, D_n\right]^{-1} \, 2D_n' S_n \, \omega \tag{A.6.2}$$

$$J_{b_{0,t}}^{\psi_t} = C(\psi_t)^{-1} \left[D_n' \, B_{0,t \otimes} \, D_n\right]^{-1} \, 2D_n' \, (I_n \otimes \omega\omega' B_{0,t}'^{-1}) K_n R_n \tag{A.6.3}$$

$$J_a^{\phi_t} = (I_n \otimes B_{0,t-1}') S_n \tag{A.6.4}$$

where $C(\psi_t)$ is the invertible Jacobian of the mapping from vech(ψ_t) to vech$(\psi_t \psi_t')$. For this mapping to be unique, it is required that ψ_t has a positive diagonal, which also ensures that the model (13) is globally identified. In the previous equations, $K_n = [I_n \otimes I_{n,\cdot 1}, \ldots, I_n \otimes I_{n,\cdot n}]$ is the $n^2 \times n^2$ commutation matrix such that, for any matrix A, $\text{dvec}\, A' = K_n \, \text{dvec}\, A$, and $S_n = \frac{\partial \text{vec}(\omega)}{\partial(\omega_1,\ldots,\omega_n)} = \frac{\partial \text{vec}(\alpha)}{\partial(a_1,\ldots,a_n)}$ is a $n^2 \times n$ block diagonal matrix as follows:

$$S_n = \begin{bmatrix} I_{n,\cdot 1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & I_{n,\cdot n} \end{bmatrix} \tag{A.6.5}$$

where $I_{n,\cdot i}$ is the ith column of I_n. The remaining sub-Jacobians are instead slightly different:

$$J_{\beta}^{\rho_t} = (B'_{0,t-1} \otimes B_{0,t}^{-1})S_n \tag{A.6.6}$$

$$J_{b_{0,t}}^{\rho_t} = -(B'_{0,t-1} \otimes B_{0,t}^{-1})(\beta B_{0,t}'^{-1} \otimes I_n)R_n \tag{A.6.7}$$

$$J_{b_{0,t}}^{\phi_t} = -V_n'(\phi_t' \otimes I_n)R_n = 0 \tag{A.6.8}$$

where $V_n = \begin{bmatrix} I_n \otimes I_{n,\cdot 1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & I_n \otimes I_{n,\cdot n} \end{bmatrix}$ is a $n^3 \times n^2$ matrix such that $\text{dvec}(\phi_t) = V_n \left[\phi_{1,t}, \dots, \phi_{n,t}\right]'$ and $R_n = \frac{\partial \text{vec}(B_t)}{\partial b'_{0,t}}$ is a $n^2 \times n(n-1)$ block diagonal matrix as follows:

$$R_n = \begin{bmatrix} I_{n,-1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & I_{n,-n} \end{bmatrix} \tag{A.6.9}$$

where $I_{n,-i}$ is the submatrix of I_n with the i^{th} column removed. Finally we have

$$J_t = \begin{bmatrix} C(\psi_t)^{-1} \; [D_n' \, B_{0,t\otimes} \, D_n]^{-1} \; 2D_n' & 0 & 0 \\ 0 & (I_n \otimes B_{0,t-1}') & 0 \\ 0 & 0 & (B_{0,t-1}' \otimes B_{0,t}^{-1}) \end{bmatrix}$$

$$\begin{bmatrix} S_n \, \omega & (I_n \otimes \omega\omega' B_{0,t}'^{-1}) K_n R_n & 0 & 0 \\ 0 & 0 & S_n & 0 \\ 0 & -(\beta \, B_{0,t}'^{-1} \otimes I_n) R_n & 0 & S_n \end{bmatrix} \quad \text{(A.6.10)}$$

The system $Jv = 0$ can be simplified to

$$\begin{bmatrix} S_n \, \omega v_1 + (I_n \otimes \omega\omega' B_{0,t}'^{-1}) K_n R_n v_2 \\ S_n v_3 \\ (\beta \, B_{0,t}'^{-1} \otimes I_n) R_n v_2 + S_n v_4 \end{bmatrix} = 0 \quad \text{(A.6.11)}$$

Eq. (A.6.11) implies immediately that $v_3 = 0$ since S_n is full rank. For the same reason, $v_4 = 0$ if $v_2 = 0$ in the third equation. Therefore we only need to check that the first equation in (A.6.11) has a solution only if $[v_1', v_2']' = 0$. Using Sylvester's inequality, together with the fact that $B_{0,t}$ is invertible by assumption and R_n and K_n are of full column rank, we obtain the following lower bound

$$\text{rank}\left[(I_n \otimes \omega\omega' B_{0,t}'^{-1}) K_n R_n\right] \geq n(n_\omega + n - n) + n(n-1) - n^2 = n\,(n_\omega - 1) \quad \text{(A.6.12)}$$

where n_ω is the number of non zero elements in ω. This inequality proves that, if $n_\omega = n$, then the ranks of $J_\omega^{\psi_t}$ and $J_{b_{0,t}}^{\psi_t}$ are respectively n and $n(n-1)$. Moreover, the columns of $J_\omega^{\psi_t}$ and $J_{b_{0,t}}^{\psi_t}$ are independent from each other according to Lemma 2 in Milunovich and Yang (2013). We finally need to check that $(B_{0,t}, \omega, \alpha, \beta)$ is a regular point of J_t in order to satisfy the assumption of Theorem 6 in Rothenberg (1971). This immediately follows from the fact that, under the assumptions of the proposition, the rank of J_t never changes. This completes the proof.

A.7 SD-S-BEKK specification for $B_{0,t}$

We start from the reduced form normal loglikelihood:

$$\mathcal{L}_t(H_t) = -\frac{1}{2}\left[n\log(2\pi) + \log|H_t| + u_t' H_t^{-1} u_t\right] \tag{A.7.1}$$

Its differential is

$$\begin{aligned} \mathrm{d}\mathcal{L}_t = -\frac{1}{2}\left[\mathrm{tr}(H_t^{-1}\mathrm{d}H_t) + u_t'(\mathrm{d}H_t^{-1})u_t\right] = \\ = \frac{1}{2}\left[u_t' H_t^{-1}(\mathrm{d}H_t)H_t^{-1}u_t - \mathrm{tr}(H_t H_t^{-1}\mathrm{d}H_t H_t^{-1})\right] \end{aligned} \tag{A.7.2}$$

where we use $\mathrm{d}H_t^{-1} = -H_t^{-1}(\mathrm{d}H_t)H_t^{-1}$ and the cyclic property of trace. Using the vec operator and the identities $\mathrm{vec}(abc) = (c' \otimes a)\mathrm{vec}(b)$ and $\mathrm{tr}(AB) = \mathrm{vec}(A)'\mathrm{vec}(B)$ where A is symmetric, applying the vec operator to both sides of (A.7.2) we can rewrite it as follows

$$\mathrm{d}\mathcal{L}_t = \frac{1}{2}\left[u_{t\otimes}' - (\mathrm{vec}\,H_t)'\right] H_{t\otimes}^{-1}\,\mathrm{dvec}\,H_t \tag{A.7.3}$$

We are interested in $\mathcal{L}_t$ as a function of B_t. Since $H_t = B_{0,t}^{-1} G_t B_{0,t}'^{-1}$, we have that

$$\mathrm{d}H_t = (\mathrm{d}B_{0,t}^{-1})G_t B_{0,t}'^{-1} + B_{0,t}^{-1} G_t\,(\mathrm{d}B_{0,t}^{-1})' \tag{A.7.4}$$

Applying the vec operator to both sides of (A.7.4) we obtain (see Abadir and Magnus (2005), p. 364)

$$\mathrm{dvec}\,H_t = (I_{n^2} + K_n)\,(B_{0,t}^{-1} G_t \otimes I_n)\,\mathrm{dvec}\,B_{0,t}^{-1} \tag{A.7.5}$$

where K_n is the commutation matrix defined in Appendix A.6. Substituting (A.7.5) into (A.7.3), and noting that $\mathrm{d}B_{0,t}^{-1} = -B_{0,t}^{-1}(\mathrm{d}B_{0,t})B_{0,t}^{-1}$ implies $\mathrm{dvec}\, B_{0,t}^{-1} = -\left(B_{0,t}'^{-1} \otimes B_{0,t}^{-1}\right) \mathrm{dvec}\, B_{0,t}$, we obtain:

$$\mathrm{d}\,\mathcal{L}_t = -\left[u_{t\otimes}' - (\mathrm{vec}\, H_t)'\right] H_{t\otimes}^{-1} N_n (B_{0,t}^{-1} G_t \otimes I_n) \left(B_{0,t}'^{-1} \otimes B_{0,t}^{-1}\right) \mathrm{dvec}\, B_{0,t} =$$

$$= -\mathrm{vec}\,(u_t u_t' - H_t)'\, H_{t\otimes}^{-1}\, N_n\, (H_t \otimes B_{0,t}^{-1})\, \mathrm{dvec}\, B_{0,t} \quad \text{(A.7.6)}$$

where the properties of $N_n \equiv \frac{1}{2}\left(I_{n^2} + K_n\right)$ can be found in Abadir and Magnus (2005), pp. 299-317. Given the constraints on the diagonal elements of $B_{0,t}$, we have that

$$\frac{\partial \mathrm{vec}\, B_{0,t}}{\partial b_{0,t}'} = R_n = \begin{bmatrix} I_{n,-1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & I_{n,-n} \end{bmatrix} \quad \text{(A.7.7)}$$

where $b_{0,t} = (b_{0,2,1,t}, \dots, b_{0,n-1,n,t})'$ contains only the off diagonal terms in $B_{0,t}$ and $I_{n,-i}$ is the sub-matrix of I_n with the ith column deleted. Using the properties of N_n we obtain

$$\nabla_{b_{0,t}} = -R_n' \left(I_n \otimes B_{0,t}'^{-1} H_t^{-1}\right) \mathrm{vec}\,(u_t u_t' - H_t) \quad \text{(A.7.8)}$$

$$\mathcal{I}_{b_{0,t|t-1}} = 2n R_n' (H_t \otimes B_{0,t}'^{-1}) N_n H_{t\otimes}^{-1} N_n (H_t \otimes B_{0,t}^{-1}) R_n \quad \text{(A.7.9)}$$

A.8 Proof of Proposition 2

The Jacobian J_t of the mapping (16) has the following block-form:

$$J_t = \begin{bmatrix} J_{b_{0,t-1}}^{\phi_t} & J_a^{\phi_t} \end{bmatrix} \quad \text{(A.8.1)}$$

where

$$J^{\phi_t}_{b_{0,t-1}} = \begin{bmatrix} a_1 \ (I_n \otimes I_{n,})' \, R_n \\ \vdots \\ a_n \ (I_n \otimes I_{n,})' \, R_n \end{bmatrix} \tag{A.8.2}$$

and

$$J^{\phi_t}_{a} = (I_n \otimes B'_{0,t-1}) S_n \tag{A.8.3}$$

Following Milunovich and Yang (2013), lemma 1, the mapping (16) ensures that rank$(J^{\phi_t}_a) = n$ and rank $\left(J^{\phi_t}_{b_{0,t-1}}\right) = n_a(n-1)$, where n_a is the number of non zero elements in a. Since under the assumptions the rank of J_t don't change, $(b_{0,t-1}, \alpha)$ is a regular point for J_t and therefore the Theorem 6 in Rothenberg (1971) applies.

A.9 GAS-S-MGARCH specification for $B_{0,t}$

The likelihood function of model (15) is

$$\mathcal{L}_t = -\frac{1}{2}\left(\log|G_t| + u'_t B'_{0,t} G_t^{-1} B_{0,t} u_t\right) \tag{A.9.1}$$

Then the following holds (Abadir and Magnus, 2005), p. 357:

$$\text{dvec}\,\mathcal{L}_t = -\frac{1}{2} u'_{t\otimes} \text{dvec}\left(B'_{0,t} G_t^{-1} B_{0,t}\right) \tag{A.9.2}$$

Using the identities vec$(abc) = (c'b' \otimes I)$vec $a = (I \otimes ab)$vec c we obtain:

$$\begin{gathered} \text{dvec}\left(B'_{0,t} G_t^{-1} B_{0,t}\right) = \\ = \text{vec}\left(dB'_{0,t} G_t^{-1} B_{0,t}\right) + \text{vec}\left(B'_{0,t} G_t^{-1} dB_{0,t}\right) = \\ = \left(B'_{0,t} G_t^{-1} \otimes I_n\right) \, Q_t \, \text{dvec} B_{0,t} \end{gathered} \tag{A.9.3}$$

where $Q_t = K_n + \left(G_t \otimes B'_{0,t}\right)\left(B'_{0,t} \otimes G_t\right)^{-1}$ and K_n is the commutation matrix defined in Appendix A.6. From the expressions above we can derive the following:

$$\nabla_{b_{0,t}} = -\frac{1}{2} R'_n\, Q'_t\, \left(G_t^{-1} B_{0,t} \otimes I_n\right)\, u_{t\otimes} \tag{A.9.4}$$

In order to obtain $\mathcal{I}_{b_{0,t}|t-1}$ we must compute the expectation $E\left[(u_t u'_t \otimes u_t u'_t)\right]$. Since $u_t = B_{0,t}^{-1}\, G_t\, v_t$ where $v_t \sim \mathcal{N}(0,1)$, it is easier to compute $E\left[(v_t v'_t \otimes v_t v'_t)\right]$ which is given by the matrix Z with elements

$$Z[(i-1)\cdot k + l, (j-1)\cdot k + m] = \delta_{ij}\delta_{lm} + \delta_{il}\delta_{jm} + \delta_{im}\delta_{il} \tag{A.9.5}$$

where δ_{ij} is the Kronecker delta (see Creal *et al.* (2011), p. 562). Making some simplifications, we obtain:

$$\mathcal{I}_{b_{0,t}|t-1} = \frac{1}{4} R'_n Q'_t \left(I_n \otimes B_{0,t}^{-1} G_t\right)\, Z\, \left(I_n \otimes G_t B_{0,t}'^{-1}\right) Q_t R_n \tag{A.9.6}$$

It is possible to check that the elements of the gradient vector (A.9.4) take the following form:

$$\frac{\partial \mathcal{L}_t}{\partial b_{0,n,m,t}} = -\frac{\epsilon_{n,t}}{g_{n,t}} u_{m,t} \qquad \text{for} \quad n \neq m \tag{A.9.7}$$