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The Optimal Mix of Quantity and Quality of Education

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Abstract

We study the roles of quantity and quality of public education in an OLG model in which the working adult cares for her child's education as well as for her elderly parent's consumption, and spends time providing assistance to her parent. A more educated agent has more ability, hence she earns more and provides better assistance. First, we identify the optimal quality-quantity mix that a parent chooses as a function of policy (school fees, income tax, pension). Then, we discuss in a steady-state framework the socially efficient determination of the policy tools. We find that school fees tend to reduce education quantity by boosting working time for both kids and adults, and generally favour the generation of working adults (even though they are the ones paying them), possibly at the expense of a negative impact on the well-being of the young and of the elderly. The opposite is true if education is mostly financed through the income tax. Overall quality is only moderately sensitive to changes in the policy mix.

Keywords: Education Policy, Redistributive taxation.

JEL Classification: H42, H52.

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I Introduction

It is now well-recognized that both quantity and quality of education play a crucial role not only in shaping individuals' human capital but also in boosting economic growth (Hanushek and Woessmann (2012)).

From a microeconomic viewpoint, previous literature has concentrated the attention on the relationship between quantity of education and individual earnings, but more recent works have stressed that nowadays the abilities which are needed in the labour market are more and more knowledge-based and individual earning capacities are also affected by the quality of education one has received both in terms of acquired cognitive and noncognitive abilities, i.e. self-control, self-esteem, perseverance, communication skills, team working abilities, etc. Hanushek and Wößmann (2008) survey a number of papers showing that individual earning advantages are closely associated to higher school quality, i.e. higher achievements on standardized tests. Also, Hanushek et al. (2015) use data available from the Programme for the International Assessment of Adult Competencies conducted by the OECD in 2011-2012 to estimate the earning returns to cognitive skills across 23 countries. However, not only cognitive skills but also noncognitive skills (Cuhna and Heckman (2007)) may be decisive on the labour market and, in this respect, Chetty et al. (2011) show that both test scores and future earnings are affected by noncognitive skills, such as effort, initiative, and lack of disruptive behaviour, which are built in early childhood and also depend on the quality of the classroom a student is assigned at the kindergarten (e.g. class size, teacher and peer characteristics).

Education quality has also been viewed as crucially important from a macroeconomic viewpoint, possibly more so than education quantity. Indeed, to study the impact of education on growth, cross-section analyses using quantitative measures of education, e.g. average years of schooling, do not take into account that the quality of educational systems vary a lot across countries and that education outcomes depend on both formal schooling and other nonschool factors such as family, peers, and culture (Hanushek and Wößmann (2012)). For these reasons, using the results of international tests to measure the quality of the education that the students have received has appeared to be decisive in assessing the impact of education on economic growth (Hanushek and Kimko (2000), Hanushek and Wößmann (2021)).[1]

[1] For example, PISA (Programme for International Student Assessment, testing math, science, and reading performance of 15-year-olds), PIRLS (Progress in International Reading Literacy Study, testing reading performance at primary school), and TIMSS (Trends in International Mathematics and Science Study, testing math

Given the economic relevance of education along its two main dimensions, it is clearly of the utmost importance to study the mix of quantity and quality of education that the parents choose for their kids in response to the economic environment in which they find themselves, which includes in our setting the array of policy tools predisposed by a benevolent policy-maker. We refer to an "array" of policy tools, because clearly public education policy is not usually decided in isolation with respect to other policies. There is indeed a field of research within the economic literature that has concentrated the attention on the interaction between education policies, tax policies and pension policies. For example, Pogue and Sgontz (1977), Konrad (1995), and Kemnitz (2000) have shown that education may have positive effects on pension benefits and this helps in motivating a political support for public expenditure on it. Along the same line, Soares (2006) adopts a political economy point of view to show that current middle-aged individuals may vote in favour of an education policy because they take into account that a more skilled labor force also implies a higher level of social security benefits. Kaganovich and Zilcha (1999) analyse the role of education and social security as intergenerational transfers and show that social security may improve growth and welfare, pointing out that, differently from parents, the government takes into account that education positively affects the human capital of children which in turn positively affects the parents' pension benefits via income taxation. Finally, by taking into account the role of nonmarket institutions, Rangel (2003) shows that social security plays a decisive role in supporting the optimal level of investment in education favouring future generations, while Glomm and Kaganovich (2003) analyse how human capital distribution depends on publicly financed education and social security.

To analyse the interaction between quantity and quality of education, we posit a steady-state overlapping generations model in which individuals make their decisions considering both the current period and the future. Decision makers are driven by a mixture of altruistic and self-interested motives – that is, they make their choices taking into account the well-being of their children (the education they receive) as well as that of their parents (the care they receive). Given this background, it makes perfect sense to consider the interaction between education policy and social security and how such an interaction may be affected by the agents' choices concerning the quantity and quality of education. In each period, there are three generations: the working adults, the children (who go to school but may also work), and the retired parents of the current adults. Education is supposed to affect a child's "ability" *via* the amount of

and science performance of eightgraders).

schooling as well as the quality of the schools they attend: in turn, the child's "ability" affects their earning potential once they reach adulthood as well as their ability to provide meaningful care for their old, retired parents. The latter point suggest that the individuals may care for the education of their kids for altruistic motives but also for a selfish reason: they know that investing in their kids' education will enable them, once old, to obtain a return in terms of better care.

The aforementioned array of policy tools includes an income tax rate as well as a school fee as sources of revenue: government expenditure includes instead education and pensions (modeled very simply as a lump-sum payment). We assume that the government maximises a social welfare function by optimally choosing these instruments.[2] We only look at the efficiency angle of optimality: all individuals in our model are identical, therefore equity is not an issue. Our main result shows, firstly, that if public expenditure is optimally financed primarily through the school fee rather than through the income tax, then there will be a positive effect on the adults willingness to work. This is mainly due to the simple fact that a relatively small income tax implies a moderate decrease of the shadow price of non-working hours. However, there will also be a negative, albeit equally moderate, impact both on the quantity of schooling and on the amount of time devote to caring for the old parents. Secondly, the quality of schooling seems to be, counterintuitively, not especially affected by the prevalence of the school fee over the income tax as revenue source or *viceversa*: as we shall explain in due course, this is due to the fact that the policy tools act asymmetrically on the agent's choice in terms of quality and quantity of education, since the latter has an opportunity cost that the former does not have. Thirdly, if school quantity goes down in the presence of a large school fee and quality is by and large unaffected, then we may expect a reduction in the child's ability, i.e. in their earning potential. Finally, if agents have a lower ability and work more, we expect that the overall care received, one period ahead, by the retired generation will be reduced – and we cannot establish whether this can be compensated by a larger pension (which would presumably require a higher tax rate and thus more distortion). Summing up, then, if the efficient mix of policy tools requires a relatively large school fee, this will create strong work incentives and boost the income of the working adults, but with a possibly negative side-effect in terms of reduced welfare for the

[2] See Balestrino *et al.* (2024) for a model where both quantity and quality of education are considered, and the aim is to understand the different role played by school fees and vouchers in affecting altruistic parents' welfare, on the one hand, and social welfare, on the other, when a positive externality of education is taken into acocunt.

other two generations (reduced care for the old and reduced earning potential for the kids). The opposite would be true if the optimal policy requires a relatively large income tax.

The plan of the paper is as follows. Section 1 describes the model; Section 2 discusses the efficient mix of quantity and quality of education from the societal viewpoint; Section 3 contains some concluding remarks.

II The model

We consider an overlapping generations model with three generations. In period 0, the agent is a kid, in period 1 she is an adult, and in period 2 she is retired. As a kid, the agent makes no decisions as her time allocation (she can work as well as attend school) is chosen by her parent, and her consumption is taken to be fixed. As an adult, the agent has a kid, works and provides family care. As an old person, the agent consumes her pension and enjoys the care provided by her adult child. The total endowment of time is normalised to unity both for the kid and the adult: S is the time the kid spends in school while $1 - S$ is the time she spends working, while l and h represent market work and home-production of family care for the adult, so that $l + h = 1$ To denote generations we use superscripts: "k" refers the generation of kids, "a" refers to the generation of adults and "o" to that of the old. However, to simplify the notation, in what follows we omit the superscript "a" for the adult generation whenever possible. Without loss of generality, we suppose there are no savings – Samuelson (1958).

We consider a model with a separable quasi-linear utility function (this simplifies the analysis considerably, without affecting the qualitative results); for an agent who is adult in period 1 this is

$$u = C_1 + \varphi(C_2) + d(C_1^o) + g(A^k), \tag{1}$$

where all the sub-utility functions are assumed to be concave and increasing in their arguments. C_1 denotes the consumption of an agent of the adult generation at period 1, and C_2 her consumption when she will be old at period 2. The adult agent, however, also cares for her parent and her child. Specifically, she cares about the consumption of her old parent, i.e. consumption of an agent of the old generation at period 1, C_1^o, and she derives utility from the abilities of her kid, A^k.[3] In particular, A^k is an index of the kid's cognitive and non-cognitive skills, which

[3] For the variable A the subscript 1 is not necessary given that A denotes the individual abilities that are acquired during childhood and last all life.

depends on the level of education of the child, more precisely on the quantity of education S (i.e. the time spent at school) and the quality of education Q (i.e. the expenditure on quality):

$$A^k = A(S^k, Q^k). \tag{2}$$

All education is assumed to be provided by the government. The function $A(\cdot)$ is supposed to be concave and increasing in all its arguments; Q is taken to vary within the range $(0, \overline{Q}]$ – we rule out the possibility that quality is zero simply to avoid complications in the maximisation problem below. For the time being, we do not make an assumption on whether quality and quantity are technological complements or substitutes (but see the appendix); certainly, they cannot be independent, i.e. the cross-derivatives cannot be zero.[4] Notice that in (1) the consumption of the previous generation when old is

$$C_1^o = Y + F, \tag{3}$$

with

$$F = bf\left(h^a, A^a\right). \tag{4}$$

Y is a lump-sum transfer that the parent receives from the government when she is old (basically, a pension) and F is the monetary equivalent of the care that she receives from her adult kid. In particular, (4) shows that the latter depends on both the time devoted by her adult kid to producing the service, i.e. parent's care, h^a, and on her adult kid abilities, A^a, which are the result of a choice made by the old parent when she was an adult. Further, in (4), b is the coefficient converting the service received from the parent into its monetary value, and $f(\cdot)$ is assumed to be concave and increasing in both its arguments. Notice that we are assuming that, besides the altruistic motive for educating one's children, there is also an egoistic one – the child's abilities represent an investment from the point of view of the parent, with a return in terms of the care received in her old age.

Notice also that in the way we built the utility function there is an assumption of both forward and backward altruism: the adult agent cares for her child, i.e. she would like her child to be smart, and cares for her parent, i.e. she would like her parent to have an adequate level of consumption when she is no longer able to work.

[4] Otherwise, we could have a child with positive ability but either zero schooling or zero quality (the latter is ruled out, as we just mentioned, but the principle still stands).

The individual per-period budget constraints are written as

$$C_1 = (1-t)\left[(1-h)\,w\left(S^a, Q^a\right) + \beta\left(1-S^k\right)\underline{w}\right] - p(Q)S^k; \tag{5}$$

$$C_2 = Y + F. \tag{6}$$

In the first constraint, $t \in (0,1)$ is the marginal income tax rate, $1-h$ is the labour supply, w is the adult's wage rate (as determined by her parent's decision about her education), $\beta \in [0,1]$ is the proportion of the child's income that goes to the parent, $1-S^k$ is the time devoted by the child to market work, $\underline{w}$ is the wage rate for an uneducated child, and finally $p = p(Q)$, is the school fee paid by the parent for the education of her kid. Notice that such school fee depends on the quality of education, so that we assume $p' > 0$, and $p'' \leq 0$. The second constraint, as already pointed out above when discussing the constraint of the adult's parent, shows that second period consumption is financed via a pension, Y, received from the government in period 2, and the monetary equivalent of the care that the kid when adult will give to her parent, F

The policy instruments to be considered here are the marginal income tax rate, t, the lump-sum transfer of period 2, Y, and the function to determine the price of education quality, p. In this respect, to keep things simple, we follow Glewwe (2002) by supposing that $p = p(Q)$ is a linear function:

$$p = \alpha Q, \tag{7}$$

where $\alpha \in [0,1]$ is the actual policy variable. Recall that Q is expenditure on quality of education, not quality as such: hence, the price of quality faced by the agent is, in general, a fraction of the expenditure, but, in extreme cases, can be either zero or equal to the total individual expenditure.

By using (2), (5), (**??**), and (7), we can re-write (1) as

$$\begin{aligned} u = (1-t)\left[(1-h)\,w\left(Q^a, S^a\right) + \beta\left(1-S^k\right)\underline{w}\right] - \alpha Q^k S^k + \varphi(Y + bf(h^k, A\left(S^k, Q^k\right)) + \\ + d(Y^o + bf\left(h, A\left(S^a, Q^a\right)\right)) + g\left(A\left(S^k, Q^k\right)\right). \end{aligned} \tag{8}$$

For simplicity, let

$$u_A \equiv g' + \varphi' b \frac{\partial f}{\partial A}, \tag{9}$$

denote the overall marginal utility of the child's ability, where the first term describes the altruistic component and the second term represents the egoistic one. Using (9), we can write the FOCs for the maximisation of (8) w.r.t. h, S^k, and Q^k as follows:

$$d'b\frac{\partial f}{\partial h} = (1-t)\,w; \tag{10}$$

$$u_A\frac{\partial A}{\partial S^k} = \alpha Q^k + (1-t)\,\beta \underline{w}; \tag{11}$$

$$u_A\frac{\partial A}{\partial Q^k} \geq \alpha S^k,\ Q \leq \overline{Q}, \tag{12}$$

plus complementary slackness.

The first FOC simply states the rule for the optimal time allocation, while the remaining two govern the optimal demand of quantity and quality of education, respectively. A comparison between the second and third FOC highlights an asymmetry, pointed out originally by Glewwe (2002), which will play a role in what follows. In both cases, the marginal benefit includes both a direct impact on the parent's utility as well as an indirect one through the second period's care; but, as far as the marginal costs are concerned, the cost of quantity includes both the fee (which depends on the chosen quality) and the foregone income from the child's labour, i.e. an opportunity cost, while the cost of quality only depends on the fee (and the chosen quantity). This implies that having $\alpha = 0$ is sufficient to induce the agent to acquire the maximum possible level of quality, as no other factor impinges on the choice of the quality level.

At this stage, we mention the comparative statics results, which are derived in full in the appendix. In the case of an interior solution for all variables, they are as follows.

$$\mathrm{d}h/\mathrm{d}w < 0; \tag{13}$$

$$\mathrm{d}S^k/\mathrm{d}w = \mathrm{d}Q^k/\mathrm{d}w = 0. \tag{14}$$

Starting from the wage rate, we find that the home-production family care is decreasing in the adult wage rate or, equivalently, that the labour supply is increasing in the adult wage rate; the educational choices of the parent with respect to the quantity and quality of education for her child are instead unaffected due to the separability of the utility function. Turning now to the policy tools, we see that, trivially, variations in Y have no effect on the equilibrium variables h, S and Q. As for the other two instruments, t and α, we have

$$\mathrm{d}h/\mathrm{d}t > 0; \tag{15}$$

$$\mathrm{d}S^k/\mathrm{d}t > 0; \tag{16}$$

$$\mathrm{d}Q^k/\mathrm{d}t < 0; \tag{17}$$

$$\mathrm{d}h/\mathrm{d}\alpha = 0; \tag{18}$$

whereas $\mathrm{d}S^k/\mathrm{d}\alpha$ and $\mathrm{d}Q^k/\mathrm{d}\alpha$ cannot be signed. Whenever the income tax rate goes up, the labour supply of both adults and children falls and consequently the time devoted to the care of the parents rises – see (15) – and the quantity of schooling rises while the quality of schooling declines – see (16) and (17): as the total cost of quality goes up with the increase in school time, the adults compensate in budget terms by choosing a lower quality school. The effect of an increase in the price of quality (parameter α) has no impact on the labour supply – see (18) – thanks to the separability of the utility function, and has an ambiguous effect on quality as well as quantity of education: such an increase makes in fact them both costlier and therefore the agents rearrange their consumption basket in a way that cannot be predicted *a priori*. It seems plausible to assume, however, that when α increases and thus makes both S^k and Q^k more expensive, they both decrease:

$$\mathrm{d}S^k/\mathrm{d}\alpha < 0; \tag{19}$$

$$\mathrm{d}Q^k/\mathrm{d}\alpha < 0. \tag{20}$$

The comparative statics results allow us to make an inference concerning the agent's choice of the mix of quality and quantity in education. We can state that, if education is financed primarily through the income tax, the mix is biased towards a larger role of quantity: this is because of the asymmetry we noticed above in the costs of quantity and quality. Indeed, we just noticed that if t rises and thus lowers the opportunity cost of going to school, quantity receives a direct boost while quality is reduced.

In the extreme case in which α equals zero and thus plays no role in financing the public expenditure, we saw that a corner solution for Q arises. In this situation, the optimal value of quality, i.e. the upper limit $\overline{Q}$, is insensitive to variations in all parameters except α: in particular, changes in the level of t have no impact (also changes in the level of w have no impact, but that would have been true anyway because of the assumption of quasi-separability – see above). An increase in α would trigger a reduction in Q.

III The efficient quantity-quality mix of education

We have seen that the way in which households decide their preferred quality-quantity mix in education depends, among other things, on the policy decisions of the government. Specifically, in each period, the government will make decisions that impact the life of i) the children, by providing the quality and quantity of education desired by the parents; ii) the adults, by setting the tax rate and the school fee; iii) the pensioners, by setting, indeed, their pension. Since we are in a steady-state economy, with no population growth and identical households, this can be represented by letting the government choose its own array of instruments by maximising the indirect utility of a representative household subject to a per-capita public budget constraint. Then, we can now omit the generation superscript.

To begin with, we write the indirect utility of each agent which obtains as

$$v = v\left(t, Y, \alpha; w, \underline{w}, \beta, b\right), \tag{21}$$

where the first three terms represent the policy variables. By deriving (21) wrt t, Y, and α we obtain the following:

$$\frac{\partial v}{\partial t} = -\left[\left(1-h\right)w + \left(1-S\right)\beta\underline{w}\right] < 0; \tag{22}$$

$$\frac{\partial v}{\partial Y} = \varphi' + d' > 0; \tag{23}$$

$$\frac{\partial v}{\partial \alpha} = -SQ < 0. \tag{24}$$

Clearly, utility is increasing in the pension, Y, and decreasing in both the tax rate, t, and the fee, α.

Let us now move to the revenue constraint. In each period, the government's revenue comes from the income tax and the school fee while the government's expenditure includes the pension and the cost of providing quantity and quality of education (recall that all education is provided by the government):

$$t\left[\left(1-h\right)w\left(Q,S\right) + \underline{w}\left(1-S\right)\right] + \alpha QS = Y + Q + \gamma S. \tag{25}$$

Since Q expresses the expenditure on the quality of education, we have chosen to indicate the expenditure on quantity separately as γS, with $\gamma > 0$, that is we have assumed a separable cost function with constant average and marginal costs – this is however for convenience of exposition only.

The objective function of the government is given by (21); this has to be maximised by choosing t, Y and α subject to the revenue constraint (25). We focus on interior solutions for t while allowing for corner solutions as far as α is concerned; we have therefore to add a non-negativity constraint as well as an upper limit for α. Letting μ and λ denote the Lagrange multipliers, the policy problem is as follows:

$$\begin{aligned} \max \quad & v(t, Y, \alpha) \\ \text{s.t.} \quad & t\left[(1-h)\,w(Q,S) + \underline{w}(1-S)\right] + \alpha QS = Y + Q + \gamma S; \quad (\mu) \\ & \alpha \geq 0; \\ & 1 - \alpha \geq 0. \qquad (\lambda) \end{aligned} \tag{26}$$

Here, μ can be interpreted as the social marginal value of public funds, while λ is the social marginal value of the school fee. In order to simplify the interpretation of the results, let us introduce a few definitions. First, letting w_S and w_Q denote the derivatives of the wage rate as a function of the quantity and quality of education, respectively, we define

$$B_S(\alpha) \equiv t\left[(1-h)\,w_S - \underline{w}\right] + \alpha Q - \gamma, \tag{27}$$

as the net marginal social benefit of providing quantity of education, expressed as a function of the fee for reasons that will become clear later: B_S is basically the change in the budget associated with the provision of one extra unit of quantity of schooling and includes positive terms, i.e. an increase in the tax base, $t(1-h)\,w_S$, plus the revenue from the school fee, αQ, as well as negative terms, i.e. the lost revenue from the kids' income, $-t\underline{w}$ and of course the cost in terms of provision, γ. Notice that (27) is in principle ambiguous in sign, depending, among other things, on the magnitude of the effect of quantity of education on the wage rate w_S relative to the cost of provision γ. However, as we are considering the level of α which maximizes social welfare, we can take it to be positive at the margin.[5] Similarly, let us define

$$B_Q(\alpha) \equiv t(1-h)\,w_Q + \alpha S - 1, \tag{28}$$

as the net marginal social benefit of providing quality of education given by an increase in revenue, $t(1-h)\,w_Q + \alpha S$, counterbalanced by the cost of provision, 1. Notice that $\alpha S < 1$, since $\alpha \leq 1$ and $S < 1$; again, as we are considering the level of α which maximizes social welfare, we take B_Q to be positive. This implies that α is set at such a level that w_Q is large enough to more than compensate the difference between the cost of quality provision and the revenue from the school fee, $1 - \alpha S > 0$.

Now, letting

$$\psi \equiv (1-h)\,w(Q,S) + (1-S)\,\beta\underline{w} \tag{29}$$

[5] In other words, we are excluding the case where there does not exist a level of $0 \leq \alpha \leq 1$ such that $B_S \leq 0$.

denote the parent's tax base of t and using (27) and (28), we can write the FOCs of the maximization problem (26) with respect to t, Y, and α as:

$$-\psi + \mu \left\{ \psi - tw\frac{\partial h}{\partial t} + B_S\frac{\partial S}{\partial t} + B_Q\frac{\partial Q}{\partial t} \right\} = 0; \tag{30}$$

$$\varphi' + d' - \mu = 0; \tag{31}$$

$$-SQ + \mu \left\{ -tw\frac{\partial h}{\partial \alpha} + SQ + B_S\frac{\partial S}{\partial \alpha} + B_Q\frac{\partial Q}{\partial \alpha} \right\} - \lambda \leq 0 \text{ and } \alpha \geq 0; \tag{32}$$

$$1 - \alpha \geq 0 \text{ and } \lambda \geq 0; \tag{33}$$

plus complementary slackness for the last two FOCs.

The second FOC says simply that efficiency requires that the marginal utility of second-period consumption ($\partial v/\partial Y = \varphi' + d'$) should equal the marginal value of public funds. The first and third FOCs require a little elaboration to be interpreted and to this end we distinguish three cases. We will use a "hat" to denote the optimised values of the policy tools.

Case 1. $0 < \widehat{\alpha} < 1$ *and* $\lambda = 0$

Let us start with the case in which also α has an interior solution (and thus $\lambda = 0$).

Then, using the comparative statics results (18) (23) and (24), the FOC (31), as well as the quasi-linearity and separability of the utility function, we can re-write FOCs (30) and (32) as follows:

$$\frac{1}{\varphi' + d'} = 1 - \frac{\widehat{t}w}{\psi}\frac{\partial h}{\partial t} + \frac{B_S}{\psi}\frac{\partial S}{\partial t} + \frac{B_Q}{\psi}\frac{\partial Q}{\partial t}; \tag{34}$$

$$\frac{1}{\varphi' + d'} = 1 + \frac{B_S}{SQ}\frac{\partial S}{\partial \alpha} + \frac{B_Q}{SQ}\frac{\partial Q}{\partial \alpha}. \tag{35}$$

In turn, this implies

$$\frac{-\widehat{t}w\frac{\partial h}{\partial t} + B_S\frac{\partial S}{\partial t} + B_Q\frac{\partial Q}{\partial t}}{\psi} = \frac{B_S\frac{\partial S}{\partial \alpha} + B_Q\frac{\partial Q}{\partial \alpha}}{SQ}. \tag{36}$$

In order to interpret (36), it is useful to transform it in such a way that the impacts of the policy instruments are expressed in elasticity terms. A little manipulation yields:

$$\frac{-\widehat{t}wh\varepsilon_{ht} + SB_S\varepsilon_{St} + QB_Q\varepsilon_{Qt}}{\widehat{t}\psi} = \frac{SB_S\varepsilon_{S\alpha} + QB_Q\varepsilon_{Q\alpha}}{\widehat{\alpha}SQ}, \tag{37}$$

where $\varepsilon_{ht} \equiv (t/h)/(\partial h/\partial t)$ is the elasticity of adult home time relative to the tax rate and the other elasticities are defined similarly.[6] At the numerator on the LHS, we have a measure of the

[6] That is:

$$\varepsilon_{St} \equiv \frac{t}{S}\frac{\partial S}{\partial t}, \varepsilon_{Qt} \equiv \frac{t}{Q}\frac{\partial Q}{\partial t}, \varepsilon_{S\alpha} \equiv \frac{\alpha}{S}\frac{\partial S}{\partial \alpha}, \varepsilon_{Q\alpha} \equiv \frac{\alpha}{Q}\frac{\partial Q}{\partial \alpha}.$$

total marginal impact of the revenue-raising instrument $\widehat{t}$ on the budget, while at the numerator on the RHS, we have a measure of the total marginal impact of the revenue-raising instrument $\widehat{\alpha}$; both effects are weighed by the share of revenue attributable to $\widehat{t}$ and $\widehat{\alpha}$, respectively, and are, as we mentioned, expressed in elasticity terms. At the optimum, the variations in revenue induced by a change in the policy tools must be equalised at the margin.

Notice that $\varepsilon_{S\alpha} < 0$ and $\varepsilon_{Q\alpha} < 0$, while $\varepsilon_{ht} > 0$, $\varepsilon_{St} > 0$ and $\varepsilon_{Qt} < 0$ – see (16) and (17) – while B_S and B_Q are positive. Then, the RHS of (37) is negative. On the LHS, both the first and the third terms are negative, while the second is positive; hence, the latter must be small enough not to compensate the other two. This implies that when quality is only partially financed by the school fee, at the optimum we have either a relatively low S or a small ε_{St} (or both). That is, kids do not go to school for long and/or the time they spend in school is not very responsive to variations in the tax rate.

Also, (37) can be further manipulated to yield

$$B_S = B_Q \frac{Q\left(\widehat{t}\psi\varepsilon_{Q\alpha} - \widehat{\alpha}SQ\varepsilon_{Qt}\right)}{\widehat{\alpha}S\left(SQ\varepsilon_{St} - \widehat{t}\psi\varepsilon_{S\alpha}\right)} + \frac{\widehat{\alpha}SQtwh\varepsilon_{ht}}{\widehat{\alpha}S\left(SQ\varepsilon_{St} - \widehat{t}\psi\varepsilon_{S\alpha}\right)}. \tag{38}$$

For (38) to hold, the RHS must be positive. A sufficient condition for this is that

$$\widehat{t}\psi\left|\varepsilon_{Q\alpha}\right| < \widehat{\alpha}SQ\left|\varepsilon_{Qt}\right|, \tag{39}$$

i.e., roughly, the impact on the public budget of a change in the provision of quality induced by a marginal variation of the tax rate is larger, in absolute value, than the corresponding impact of a marginal variation of the school fee. Or, in simpler terms, that quality is more responsive to variations in the tax rate than to variations in the school fee.

Case 2. $\widehat{\alpha} = 1$ *and* $\lambda \geq 0$

The net marginal social benefits of education are now re-defined as

$$B_S(1) \equiv t\left[(1-h)w_S - t\underline{w}\right] + Q - \gamma; \tag{40}$$

$$B_Q(1) \equiv t(1-h)w_Q + S - 1, \tag{41}$$

and again taken as positive. We can now proceed as before, and re-write the FOCs (30) and (32) as

$$\frac{1}{\varphi' + d'} = 1 - \frac{\widehat{t}w}{\psi}\frac{\partial h}{\partial t} + \frac{B_S}{\psi}\frac{\partial S}{\partial t} + \frac{B_Q}{\psi}\frac{\partial Q}{\partial t}; \tag{42}$$

$$\frac{1}{\varphi' + d'} = 1 + \frac{B_S}{SQ}\frac{\partial S}{\partial \alpha} + \frac{B_Q}{SQ}\frac{\partial Q}{\partial \alpha} - \frac{\lambda}{SQ}. \tag{43}$$

Hence:

$$-\frac{\widehat{t}w}{\psi}\frac{\partial h}{\partial t}+\frac{B_S}{\psi}\frac{\partial S}{\partial t}+\frac{B_Q}{\psi}\frac{\partial Q}{\partial t}=\frac{B_S}{SQ}\frac{\partial S}{\partial \alpha}+\frac{B_Q}{SQ}\frac{\partial Q}{\partial \alpha}-\frac{\lambda}{SQ}, \tag{44}$$

that is, in elasticity terms:

$$\frac{-\widehat{t}wh\varepsilon_{ht}+SB_S\varepsilon_{St}+QB_Q\varepsilon_{Qt}}{\widehat{t}\psi}\leq\frac{SB_S\varepsilon_{S\alpha}+QB_Q\varepsilon_{Q\alpha}}{SQ}. \tag{45}$$

If $\lambda=0$, then (45) holds as an equality and we are in the same situation as when α is at an interior solution. For the case in which $\lambda>0$, instead, the arbitrage condition requires that the impact of $\widehat{\alpha}$ on the budget expressed in elasticity terms, on the RHS, exceeds that of $\widehat{t}$, on the LHS. However, the RHS is still negative and again the second term at the numerator, on the LHS, must be small. Consequently, it must be the case that at the optimum kids do not go to school for long and/or the time they spend in school is not very responsive to variations in the tax rate.

As before, when $\lambda>0$, we can transform (45) into a condition that highlights the role of the social marginal benefits of quality and quantity:

$$B_S<B_Q\frac{Q\left(\widehat{t}\psi\varepsilon_{Q\alpha}-SQ\varepsilon_{Qt}\right)}{S\left(SQ\varepsilon_{St}-\widehat{t}\psi\varepsilon_{S\alpha}\right)}+\frac{SQtwh\varepsilon_{ht}}{S\left(SQ\varepsilon_{St}-\widehat{t}\psi\varepsilon_{S\alpha}\right)}, \tag{46}$$

which can be interpreted in a similar way as (37), in the sense that i) it is still necessary that the RHS is positive, and ii) (39) is sufficient for the latter to be true. However, now it must also be the case that, in some sense, the marginal social benefit of school quantity in terms of revenue, B_S, is "small" relative to the marginal social benefit of school quality in terms of revenue, B_Q.

Case 3. $\widehat{\alpha}=0$ *and* $\lambda=0$

The net marginal social benefits of quantity and quality are once again re-defined, now as

$$B_S(0) \equiv t(1-h)w_S-t\underline{w}-\gamma; \tag{47}$$

$$B_Q(0) \equiv t(1-h)w_Q-1, \tag{48}$$

and once again taken to be positive. We have that (30) and (32) can be re-written as

$$\frac{1}{\varphi'+d'} = 1-\frac{tw}{\psi}\frac{\partial h}{\partial t}+\frac{B_S}{\psi}\frac{\partial S}{\partial t}+\frac{B_Q}{\psi}\frac{\partial Q}{\partial t}; \tag{49}$$

$$\frac{1}{\varphi'+d'} > 1+\frac{B_S}{SQ}\frac{\partial S}{\partial \alpha}+\frac{B_Q}{SQ}\frac{\partial Q}{\partial \alpha}, \tag{50}$$

where however $\partial Q/\partial t = 0$ because we are at a corner solution and $Q = \overline{Q}$. A trasformation into an elasticity format cannot be employed here, because it would involve a division by zero. We can however use the comparison between the social marginal benefits of quality and quantity to gain some insight into the situation. Starting from the fact that

$$-\frac{tw}{\psi}\frac{\partial h}{\partial t} + \frac{B_S}{\psi}\frac{\partial S}{\partial t} > \frac{B_S}{S\overline{Q}}\frac{\partial S}{\partial \alpha} + \frac{B_Q}{S\overline{Q}}\frac{\partial Q}{\partial \alpha} \tag{51}$$

would be required at the optimum, we can show that this is equivalent to

$$B_S > B_Q \frac{-\frac{\partial Q}{\partial \alpha}/\frac{\partial S}{\partial t}}{tw\left(\frac{\partial h}{\partial t}/\frac{\partial S}{\partial t}\right) + \frac{\psi^2}{S\overline{Q}}\left(\frac{\partial S}{\partial \alpha}/\frac{\partial S}{\partial t}\right)}. \tag{52}$$

Notice that the term on the RHS is negative if

$$tw < \frac{\psi^2}{S\overline{Q}}\left|\frac{\partial S/\partial \alpha}{\partial h/\partial t}\right|, \tag{53}$$

in which case (52) is necessarily satisfied. If, however, the sign is reversed in (53), then (52) can still hold as long as the marginal social benefit of school quantity in terms of revenue, B_S, is, in some sense, "large" relative to the marginal social benefit of school quality in terms of revenue, B_Q. This is the opposite of what was required in a situation where $\widehat{\alpha} = 1$ (see above). Notice that for (53) to fail, there must be a relatively high tax rate, which is necessarily the case here, since there is no school fee.

To sum up, we can state the following

Proposition

i) When the marginal social benefit of quantity in terms of revenue, B_S, is small relative to the marginal social benefit of quality in terms of revenue, B_Q, then $\widehat{\alpha} \in (0, 1]$, $\widehat{t}$ tends to be relatively small and kids devote relatively little time to education (quantity of education is relatively low);

ii) when the marginal social benefit of quantity in terms of revenue, B_S, is large relative to the marginal social benefit of quality in terms of revenue, B_Q, then $\widehat{\alpha} = 0$, $\widehat{t}$ is necessarily relatively large and kids devote relatively much time to education (quantity of education is relatively high).

Intuitively, when the school fee does not raise revenue, the whole public expenditure (quality and quantity of education, and the pension) must be financed through the income tax. It might then be necessary to distort the choice of educational quantity to a large extent. However, as the fee increases and covers progressively a larger share of the expenditure on education,[7] then the income tax can become more lenient and its distortionary impact on educational quantity can be reduced in order to optimally balance the policy instruments. Correspondingly, the social marginal benefit of quantity in terms of revenue becomes less and less important as kids devote less time to school when the income tax rate falls.

What are the wider implications of this? First, we can notice that the impact on the quality of education is ambiguous because on the one hand it costs more ($\widehat{\alpha}$ goes up) but on the other hand there are more available resources as educational quantity decreases – which opens up some space for increasing the expenditure on educational quality ($\widehat{t}$ goes down). It is difficult to be precise at the present level of generality, but one can expect a minor variation in overall quality, since the two forces pull in opposite directions. This is somewhat surprising, as one would expect that a fee on quality would discourage parents from actually investing in quality itself, i.e. from choosing "good" schools. This does not turn out to be necessarily the case, because the main effect of the fee is mediated by the public budget constraint, *via* its impact on the optimal tax rate – which in turn acts directly on the quantity but not on the quality of education, as we already noticed more than once.

Second, as we move towards a situation in which the expenditure on education is financed primarily *via* the school fee and therefore the tax rate tend to be lower, adults become more and more encouraged to work: just as kids go to school for a shorter period of time due to the relatively minor distortion imposed by the tax rate, so the adults spend less time at home.

In turn, the two effects outlined above have consequences for the care of the elderly. If the time available for the domestic production of care by the adults is reduced, then F is negatively affected along that dimension; furthermore, the educational quantity decreases and so do the children's abilities A. Unless this is somehow compensated by a sufficiently large increase in educational quality (but we saw that the impact on quality is ambiguous and possibly minor), then we expect a likely decrease in the care of the elderly. This is not necessarily matched by a more generous public pension, as we cannot say, at this level of generality, whether $\widehat{t}$ remains large enough as $\widehat{\alpha}$ increases: the pension will presumably become a relatively more important source of income for the old, but we cannot establish how the overall consumption level changes.

[7] Notice that even if $\widehat{\alpha} = 1$, we cannot say whether this is enough to cover the whole public expenditure on education, because it still may be that $\widehat{\alpha}QS < Q + \gamma S$ at the policy optimum. Conversely, it might happen that $\widehat{\alpha}QS = Q + \gamma S$ even if $\widehat{\alpha} < 1$.

We can then infer that when the policy mix tends towards a larger role of the school fee and a correspondingly smaller role for the income tax rate, we risk a worsening of the situation for the old due to the ensuing reduction of school time for the kids and increase of working time for the adults.

The general conclusion is then that, as the mix of policy tools moves from a situation in which $\widehat{\alpha} = 0$ towards a situation in which more and more public expenditure on educational quality is financed through $\widehat{\alpha}$ itself, and therefore $\widehat{t}$ is progressively reduced, the agent will definitively work more, send her child to school for a shorter period and care less for her old parent. It is also possible, although not certain, that the abilities of the child as well as the consumption level of the parent will be reduced. Thus, school fees as a means of financing education are good in terms of work incentives and generally favour the generation of working adults, but this may come at the expense of a negative impact on the well-being of the young and of the elderly.

IV Concluding remarks

This paper has analysed the provision of public education, in particular, the efficient mix of education quantity and quality in an overlapping generations model with three generations. The amount of schooling and the quality of attended schools affect a child's ability which, in turn, affects her future earnings but also both the type of care that old parents may receive from their kids and social security benefits through income taxation. As far as the educational choices of the working adults are concerned, we show that when education is financed primarily through an income tax rather than a school fee, the mix of quantity and quality of education is biased towards a larger role of quantity: this happens because the opportunity cost of quantity, that is the kid's forgone wage, is reduced in the presence of a relatively large income tax (quality has no opportunity costs).

From a social point of view, the government is supposed to maximize a social welfare function (given by the indirect utility function of a representative individual) by optimally choosing policies on taxation, education, and social security. Our main result shows how the efficient mix of education quantity and quality changes depending on whether education is financed primarily through school fees or taxation. A larger role for the former implies a stronger incentive to work for adults and children, and therefore a reduction in school quantity and in the amount of time devoted to the care of the elderly. Since quality seems not to be particularly affected by variations in the mix of optimal policy tools, the above effects suggest a likely reduction in the children's ability as well as in the overall care for the elderly. We are not able to say whether the latter reduction can be compensated by a larger pension.

The fact that school fees tend to benefit the working adults, that is those who actually pay them, is somewhat unexpected, and it shows the importance of assessing the impact of a policy tool always in the context of the whole array of instruments available to the government. In this case, it is the interaction with the income tax rate that drives the result.

Our analysis has been performed in a set-up with only public education and a benevolent government. Possible extensions include a set-up where the parents may choose private education for their kids, the government may provide scholarships and social decisions are made by taking a political economy perspective.

Appendix

Consider the adult's maximization problem in the case with interior solutions for all variables. Let us first derive (9) to obtain

$$u_{AS} = \left(g'' + b^2 \varphi'' \left(\frac{\partial f}{\partial A} \right)^2 + b\varphi' \frac{\partial^2 f}{\partial A^2} \right) \frac{\partial A}{\partial S^k} < 0; \tag{54}$$

$$u_{AQ} = \left(g'' + b^2 \varphi'' \left(\frac{\partial f}{\partial A} \right)^2 + b\varphi' \frac{\partial^2 f}{\partial A^2} \right) \frac{\partial A}{\partial Q^k} < 0, \tag{55}$$

where the signs follow from the assumption of concavity of the various functions. Let us now rewrite the FOCs of the agent's maximisation problem as follows:

$$-(1-t)\,w + d'b\frac{\partial f}{\partial h} = 0; \tag{56}$$

$$-\left(\alpha Q^k + (1-t)\,\beta \underline{w}\right) + u_A \left(\partial A / \partial S^k\right) = 0; \tag{57}$$

$$-\alpha S^k + u_A \left(\partial A / \partial Q^k\right) = 0. \tag{58}$$

Total differentiation (considering only the policy instruments and the adult wage rate) yields:

$$\left(d'b\frac{\partial^2 f}{\partial h^2}+d''\left(b\frac{\partial f}{\partial h}\right)^2\right)\mathrm{d}h+(0)\,\mathrm{d}S^k+(0)\,\mathrm{d}Q^k-(1-t)\,\mathrm{d}w+(w)\,\mathrm{d}t+(0)\,\mathrm{d}Y+(0)\,\mathrm{d}\alpha=0; \tag{59}$$

$$(0)\,\mathrm{d}h+\left(u_{AS}\frac{\partial A}{\partial S^k}+u_A\frac{\partial^2 A}{(\partial S^k)^2}\right)\mathrm{d}S^k+\left(u_{AQ}\frac{\partial A}{\partial S^k}+u_A\frac{\partial^2 A}{\partial S\partial^k Q^k}\right)\mathrm{d}Q^k+$$
$$+(0)\,\mathrm{d}w+(\beta\underline{w})\,\mathrm{d}t+(0)\,\mathrm{d}Y+\left(-Q^k\right)\mathrm{d}\alpha=0; \tag{60}$$

$$(0)\,\mathrm{d}h+\left(u_{AS}\frac{\partial A}{\partial Q^k}+u_A\frac{\partial^2 A}{\partial Q^k\partial S^k}\right)\mathrm{d}S^k+\left(u_{AQ}\frac{\partial A}{\partial Q^k}+u_A\frac{\partial^2 A}{(\partial Q^k)^2}\right)\mathrm{d}Q^k+$$
$$+(0)\,\mathrm{d}w+(0)\,\mathrm{d}t+(0)\,\mathrm{d}Y+\left(-S^k\right)\mathrm{d}\alpha=0. \tag{61}$$

Let us now write the Hessian matrix:

$$[H]=\begin{bmatrix} d'b\frac{\partial^2 f}{\partial h^2}+d''\left(b\frac{\partial f}{\partial h}\right)^2 & 0 & 0 \\ 0 & u_{AS}\frac{\partial A}{\partial S^k}+u_A\frac{\partial^2 A}{(\partial S^k)^2} & u_{AQ}\frac{\partial A}{\partial S^k}+u_A\frac{\partial^2 A}{\partial S^k\partial Q^k} \\ 0 & u_{AS}\frac{\partial A}{\partial Q^k}+u_A\frac{\partial^2 A}{\partial Q^k\partial S^k} & u_{AQ}\frac{\partial A}{\partial Q^k}+u_A\frac{\partial^2 A}{(\partial Q^k)^2} \end{bmatrix}, \tag{62}$$

and its determinant

$$\det(H)=\left[d'b\frac{\partial^2 f}{\partial h^2}+d''\left(b\frac{\partial f}{\partial h}\right)^2\right]\Psi, \tag{63}$$

where

$$\Psi\equiv\left[\left(u_{AS}\frac{\partial A}{\partial S^k}+u_A\frac{\partial^2 A}{(\partial S^k)^2}\right)\left(u_{AQ}\frac{\partial A}{\partial Q^k}+u_A\frac{\partial^2 A}{(\partial Q^k)^2}\right)\right]$$
$$-\left[\left(u_{AQ}\frac{\partial A}{\partial S^k}+u_A\frac{\partial^2 A}{\partial S^k\partial Q^k}\right)\left(u_{AS}\frac{\partial A}{\partial Q^k}+u_A\frac{\partial^2 A}{\partial Q^k\partial S^k}\right)\right]. \tag{64}$$

We see that

$$d'b\frac{\partial^2 f}{\partial h^2}+d''\left(b\frac{\partial f}{\partial h}\right)^2<0; \tag{65}$$

hence, it follows that a sufficient condition for the Hessian determinant to be negative, as we assume a well-behaved maximisation problem, is that $\Psi>0$. Notice that we have not determined the sign of the cross-derivatives $\partial^2 A/\partial S^k\partial Q^k=\partial^2 A/\partial Q^k\partial S^k$: however, if we look at the two terms in brackets in (64), we expect in general both to be positive. So $\Psi>0$ only obtains if the first term in brackets is larger than the second: roughly, this corresponds to a case in which the cross-effects are small – no matter their sign.

We can now proceed to the comparative statics, ignoring Y, whose variations, as can be seen from the results of the total differentiation above, have no effect on the equilibrium variables:

$$|H|\begin{vmatrix} \frac{\mathrm{d}h}{\mathrm{d}w} \\ \frac{\mathrm{d}S^k}{\mathrm{d}w} \\ \frac{\mathrm{d}Q^k}{\mathrm{d}w} \end{vmatrix} = \begin{vmatrix} 1-t \\ 0 \\ 0 \end{vmatrix} ; \ |H|\begin{vmatrix} \frac{\mathrm{d}h}{\mathrm{d}t} \\ \frac{\mathrm{d}S^k}{\mathrm{d}t} \\ \frac{\mathrm{d}Q^k}{\mathrm{d}t} \end{vmatrix} = \begin{vmatrix} -w \\ -\beta \underline{w} \\ 0 \end{vmatrix} ; \ |H|\begin{vmatrix} \frac{\mathrm{d}h}{\mathrm{d}\alpha} \\ \frac{\mathrm{d}S^k}{\mathrm{d}\alpha} \\ \frac{\mathrm{d}Q^k}{\mathrm{d}\alpha} \end{vmatrix} = \begin{vmatrix} 0 \\ Q^k \\ S^k \end{vmatrix} \tag{66}$$

Let us start from the wage rate:

$$\mathrm{d}h/\mathrm{d}w = \frac{1-t}{d'b\frac{\partial^2 f}{\partial h^2} + d''\left(b\frac{\partial f}{\partial h}\right)^2} < 0; \tag{67}$$

$$\mathrm{d}S^k/\mathrm{d}w = \mathrm{d}Q^k/\mathrm{d}w = 0. \tag{68}$$

The labour supply is increasing in the adult wage rate; educational choices are instead unaffected due to the separability of the utility function. As for the other two instruments, using eq. (64), we have

$$\mathrm{d}h/\mathrm{d}t = -\frac{w}{d'b\frac{\partial^2 f}{\partial h^2} + d''\left(b\frac{\partial f}{\partial h}\right)^2} > 0; \tag{69}$$

$$\mathrm{d}S/\mathrm{d}t = -\frac{\beta \underline{w}}{\Psi}\left(u_{AQ}\frac{\partial A}{\partial Q^k} + u_A\frac{\partial^2 A}{\left(\partial Q^k\right)^2}\right) > 0; \tag{70}$$

$$\mathrm{d}Q/\mathrm{d}t = \frac{\beta \underline{w}}{\Psi}\left(u_{AS}\frac{\partial A}{\partial Q^k} + u_A\frac{\partial^2 A}{\partial Q^k \partial S^k}\right) < 0; \tag{71}$$

$$\mathrm{d}h/\mathrm{d}\alpha = 0; \tag{72}$$

$$\mathrm{d}S/\mathrm{d}\alpha = \frac{Q^k}{\Psi}\left(u_{AQ}\frac{\partial A}{\partial Q^k} + u_A\frac{\partial^2 A}{\left(\partial Q^k\right)^2}\right) - \frac{S^k}{\Psi}\left(u_{AQ}\frac{\partial A}{\partial S^k} + u_A\frac{\partial^2 A}{\partial S^k \partial Q^k}\right); \tag{73}$$

$$\mathrm{d}Q/\mathrm{d}\alpha = \frac{S^k}{\Psi}\left(u_{AS}\frac{\partial A}{\partial S^k} + u_A\frac{\partial^2 A}{\left(\partial S^k\right)^2}\right) - \frac{Q^k}{\Psi}\left(u_{AS}\frac{\partial A}{\partial Q^k} + u_A\frac{\partial^2 A}{\partial Q^k \partial S^k}\right). \tag{74}$$

The signs follow from eqs. (54), (55), the concavity of the functions involved and our assumption that $\Psi > 0$. Concerning this latter point, notice that, had we not assumed that the cross-price effects are "small" exactly in order to sign Ψ, we could have argued that for relatively large and positive values of the cross-derivatives, then eqs. (73) and (74) could have been signed – but this route has been, as we said, precluded. The above results are reported in the main text as eqs. (15)-(20).

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