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Abstract

In an economy where graduate jobs are allocated by tournament, and some of the potential participants cannot borrow against their expected future earnings, the government can increase efficiency and ex ante equity by redistributing wealth or, if that is not possible, by borrowing wholesale and lending to potential participants. Both policies replace some of the less able rich with some of the more able poor and bring education investments closer to their first-best levels.

Key words: higher education, matching tournaments, credit.

JEL: , C78, D82, H42, I22, J24

1 Introduction

The present paper examines the effects and desirability of policy intervention in a situation where non-graduate jobs are allocated by a conventional market and graduate jobs are allocated by a matching tournament.[1] A tournament is a contest where heterogeneous participants compete for one or more prizes. In a matching tournament, there are two categories of participants (men and women, employers and employees, schools and students), and each member of each category seeks to form the match most advantageous to itself with a member of the other category (in other words, the "prize" is a match). An early example of matching tournament is provided by Becker (1973), where the participants are young men and women intent on marriage. Exploiting the result in Koopmans and Beckmann (1957) that an efficient location pattern associates the most productive economic activity with the most advantageous site, the second most productive activity with the second most advantageous site, etc., Becker shows that the most attractive man will marry the most desirable woman, the second

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[1] Some evidence of this is reported in, among others, Bratti et al. (2004) and Castagnetti and Rosti (2009).

most attractive man will marry the second most desirable woman, etc. ("positive assortative matching"). Gale and Shapley (1962) show that an efficient allocation can be reached by a ritualized search or "courting" routine where (i) each man proposes to his favourite woman; (ii) each proposed-to woman keeps the best suitor waiting and rejects all others; (iii) each rejected man proposes to his next favourite woman; (iv) steps (ii) and (iii) are repeated until either there are no rejected men or every rejected man exhausts the list of women. Such an allocation will be reached also if this male chauvinist routine is replaced by a liberated one, where the persons making marriage proposals are the women.[2] There are obvious parallels between these procedures and the exchanges of CVs and job offers that occur between graduates and potential employers.

The early matching literature abstracts from informational problems. The properties of job-worker matching tournaments with asymmetric information are studied by Hoppe et al. (2009) in a context where the number of jobs and the number of workers are finite, and by Hopkins (2012) in one where there is a continuum of both. In those two articles, jobs and workers are differentiated by a quality parameter, but workers may choose to further differentiate themselves by investing in education. The latter may be either a pure signal as in Spence (1973), or actually enhance the worker's capabilities. While the quality of each job and the educational level of each worker are common knowledge, the quality of each worker is private information. The tournament ranks workers on the basis of their educational level, and matches them with graduate jobs in such a way that the candidate with the highest educational level will get the highest quality job, the one with the second-highest educational level will get the second-highest quality job, and so on. A similar analysis is carried out in Fernandez and Galí (1999) with reference to the allocation of students to schools. There, however, students are differentiated by wealth as well as ability (which, in the circumstances, is to be interpreted as ability to learn), and the aim of the paper is to compare the performance of a tournament with that of a conventional market. The authors find that, if at least some of the students are effectively credit constrained, tournaments dominate conventional markets in terms of matching efficiency and possibly also aggregate consumption.

Like Hoppe et al. (2009) and Hopkins (2012), we are concerned with the matching of workers to jobs. Unlike those authors, however, we distinguish between graduate jobs, which require a university degree and are assigned by tournament, and non-graduate jobs, which do not require a university degree and are allocated by a conventional market. In our framework, therefore, education is more than a signal. As in Fernandez and Galí (1999), potential university students differ not only in their learning ability, but also in their initial wealth. In contrast with those authors, however, we are interested in how the government can improve the matching and bring individual educational investments closer to their efficient levels given the matching, rather than in whether markets or tournaments produce the better result. Furthermore and more crucially, we

[2] Cigno (1991, Ch. 1) shows, however, that there may be more than one efficient allocation and that, if this is the case, the male chauvinist and the liberated courting procedures will seek out different allocations.

assume that the number of graduate jobs is given, so that any policy facilitating access to higher education for the poor will restrict the number of graduate jobs available for the rich and thus affect the educational investment behaviour not only of the poor but also of the rich. In Fernandez and Galí, by contrast, the number of school places is infinitely expandable, and any such policy would affect only the behaviour of the poor.

We find that, without policy intervention, at least some workers would be excluded from higher education and thus from graduate jobs not because they are insufficiently talented, but because they are insufficiently wealthy. That is undesirable not only on equity, but also on efficiency grounds. Assuming that initial wealth is uncorrelated with native talent, some graduate jobs will in fact be occupied by untalented but wealthy workers, and some non-graduate jobs by talented but unwealthy ones. Furthermore, the less wealthy will invest in their education less, and the more wealthy more, than would be efficient. We show that, were it possible to redistribute initial wealth, straight redistribution would raise efficiency as well as ex-ante equity, while an education price subsidy financed by a wealth tax could actually reduce efficiency. Alternatively, if wealth redistribution is unfeasible or excessively costly for informational or political reasons, the government can raise efficiency and ex-ante equity by borrowing on the wholesale money market and lending to students. Like wealth redistribution, student loans have the effect of replacing some of the less talented workers with some of the more talented ones in the performance of graduate jobs, and of bringing individual educational investments closer to their efficient levels. With wealth redistribution, graduate jobs of the same quality will go to graduates of the same learning ability and educational level. With student loans, graduate jobs of the same quality will still go to graduates with the same level of education, but the more wealthy among them will have lower learning ability than the less wealthy. This constitutes an important departure from the existing job-worker matching literature, and makes the results closer to our perception of reality. Given that personal ability can be inferred ex post, because the productivity of the match, and all the factors other than personal ability that contribute to this productivity, are observable, this leaves the door open for the possibility that competition among employers to secure the best workers will result in an ex-post wage improvement for the poor who have a higher productivity than the rich with same education level. This constitutes another departure from the existing literature.

2 Framework

The agents are school leavers. There is a continuum of them differentiated by learning ability, z, and wealth, y. Wealth takes only two values, $y \in \{0, \overline{y}\}$ where $\overline{y} > 0$. Learning ability is distributed over "poor" ($y = 0$) and "rich" ($y = \overline{y}$) agents with the same distribution function $G(z)$ and density function $g(z)$, such that $g(z|0) = g(z|\overline{y})\ \nabla z \in [0, \overline{z}]$. The Lebesgue measure of the rich is a proper fraction α of that of the total agent population, which we normalize to unity.[3] An agent can go into the labour market straight after leaving school, or after a period in higher education. There is also a continuum of graduate jobs differentiated by quality, $s \in [0, \overline{s}]$, with distribution function $H(s)$. We can think of s as an index of technological sophistication or entrepreneurial ability. The Lebesgue measure of graduate jobs is a fraction $\beta \leq \alpha$ of that of rich agents. Therefore, not all agents (possibly not even all the rich ones) can get a graduate job. Those who do not will take a non-graduate job, and earn a fixed wage w_0.[4] As our focus is on the allocation of graduate jobs, we assume that there are enough graduate and non-graduate jobs to occupy all school leavers, but nothing of substance changes if we allow for unemployment.

Let x denote the educational level achieved by an agent who attended university. We can think of this as either a degree level (e.g., BA, MA, Ph.D.) or a degree mark, or both. The output produced by a graduate with learning ability z and education x, employed in a job of quality s, is $\pi(s, x, z)$, with $\pi_s > 0$, $\pi_x > 0$, $\pi_z > 0$, $\pi_{xx} < 0$, $\pi_{zz} < 0$, $\pi_{sx} = 0$, $\pi_{zs} > 0$, and $\pi_{zx} = 0$. The sixth of these assumptions is required for integrability. The seventh says that graduate job quality and graduate worker's ability are complements in production and, together with the eighth, is required for stability of the matching equilibrium. Where π_{zx} is concerned, stability requires only nonnegativity, but we set it equal to zero to simplify the algebra. The cost of acquiring x units of education for an agent of ability z is $c(x, z)$, with $c(0, .) = 0$, $c_x > 0$, $c_z < 0$, $c_{xx} = 0$, $c_{zz} \leq 0$ and $c_{xz} < 0$. We further assume that $\pi_x(0, 0, 0) > c_x(0, 0)$. Therefore, z has a dual role. First, it reduces the cost of x. Second, it directly increases output. One way to justify the last of these two roles is to say that x correctly measures the work capacity of the newly appointed worker, but the worker's future ability to learn from work experience and adapt to changing circumstances is positively dependent on z. If we use this justification, we must interpret π as the present value of an output stream. As a degree is necessary to carry out a graduate job, the function $\pi(.)$ is defined only for $x \geq x_0$, where $x_0 > 0$ is the minimum level of education required for such a job (say, a BA with a low graduation mark). Without loss of generality, we set x_0 equal to the efficient level of education for an agent of learning ability $z = 0$ employed in a graduate job of quality $s = 0$,

$$x_0 = \arg\max \pi(0, x, 0) - c(x, 0).$$

The utility of an agent endowed with wealth y and ability z, who buys x units of university education and gets a job of quality s, is

$$u(s, x, z) = y + w(s, x, z) - c(x, z), \tag{1}$$

where $w(s, x, z)$ is the worker's wage; obviously, $w(s, x, z) \leq \pi(s, x, z)$. The utility of an agent who does not invest in education is $y + w_0$. If we interpret π

[3] If the number of agents were finite, we would be saying that the number of rich agents may be different from the number of poor agents.

[4] It would be more realistic to assume that the non-graduate wage increases with z or with some other index of individual ability, but this would make no difference of substance to the results.

as the present value of an output stream, we must similarly interpret w as the present value of a wage stream.

3 First best

In first best (FB), s, x, y and z are common knowledge. The policy maker prescribes educational investments to agents and assigns graduate jobs to graduates so as to maximize the social surplus

$$\int_z \int_s \left[\pi\left(s,x,z\right)-c\left(x,z\right)\right]dsdz$$

subject to the resource constraint

$$\int_z \left[\overline{y}-2c\left(x,z\right)\right]g(z)dz \geq 0.$$

Koopmans and Beckmann (1957) demonstrate that this maximization implies assortative matching in (s,z), and that there will be a value of x for each value of z in the maximizing allocation. There will then be a threshold value of z, $\widetilde{z}>0$, defined by

$$G(\widetilde{z})=1-\beta, \tag{2}$$

such that all agents with $z\geq\widetilde{z}$ will attend university independently of their y. This subpopulation of agents is distributed with distribution function $\frac{G(z)-(1-\beta)}{\beta}$, and density function $\frac{g(z)}{\beta}$. The first-best level of university education for a school leaver of ability $z\geq\widetilde{z}$ matched with a job of quality s, will then be

$$x_{FB}(s,z)=\arg\max\left[\pi\left(s,x,z\right)-c\left(x,z\right)\right],$$

and will thus satisfy

$$\pi_x\left(s,x,z\right)-c_x\left(x,z\right)=0. \tag{3}$$

Given that $\widetilde{z}>0$ it follows from the assumptions on $c(x,z)$ and $\pi(s,x,z)$ that $x_{FB}(s,z)>x_0\ \forall\ z\geq\widetilde{z}$.

Given that, in FB, the distribution of the surplus is independent of resource allocation, we say nothing on the matter. Our interest here is just to characterize an efficient allocation.

4 Laissez faire

In laissez faire (LF), s and x are common knowledge, and π is observable ex post, but y and z are private information. As lenders do not observe z, we assume like Fernandez and Galí (1999) that agents cannot borrow.[5] Employers

[5] Stiglitz and Weiss (1981) show that, if the characteristics of would-be borrowers are private information, credit may be rationed.

do not observe z either, but can infer it in a separating equilibrium. Following Hopkins (2012), we represent the equilibrium process as a two-stage game. At the first (non-cooperative) stage, the agents choose whether and how much to invest in education subject to the liquidity constraint

$$y - c(x, z) \geq 0. \tag{4}$$

At the second (cooperative) stage, graduate jobs are allocated by a matching tournament, and the product of each match is shared between the parties in such a way that the matching scheme will be stable.

As the minimum educational investment required to carry out a graduate job is positive, and given that $c(x, .)$ is positive for any positive x, (4) is binding for all the poor, who will consequently invest $x = 0$ and be excluded from the tournament. By contrast, (4) may be slack for some of the rich. Without loss of generality, we assume that there are as many graduate jobs as there are rich agents $(\alpha = \beta)$, so that all the rich can participate in the tournament if it is to their advantage. As all the rich have then the opportunity of taking graduate jobs, the support of the ability distribution of graduate workers is wider than in FB, where it includes only agents with $z \geq \widetilde{z}$. The line of reasoning is analogous to that of Hopkins (2012), which in turn draws on Mailath (1987). The only difference is that it now applies only to a subset of the population. In this section, therefore, we will limit ourselves to summarizing the results of that article, namely that there exists a unique separating equilibrium.[6] In subsequent sections, we will see how the properties of this equilibrium are modified by government intervention. Given that in the separating equilibrium z is revealed by x, the observation of $\pi(s, x, z)$ after the match is made will not bring any additional information.

In the separating laissez-faire equilibrium, all rich agents adopt a symmetric, differentiable and strictly increasing investment strategy $x_{LF}(z)$ yet to be specified. Let $F(x)$ be the distribution function of x induced by the distribution of z, $G(z)$ and by $x_{LF}(z)$. The rank position $F(x(z_i))$ of an agent of ability $z_i \in [0, \overline{z}]$ buying $x_{LF}(z_i)$ will then be equal to this agent's rank $G(z_i)$ in the ability distribution. The only stable matching is the positive assortative one, whereby a worker buying $x_i = x_{LF}(z_i)$ is matched with a job of quality $s_i \in [0, \overline{s}]$, such that

$$F(x_i) = G(z_i) = \phi(G(z_i)) = H(s_i), \tag{5}$$

where ϕ: $[0, 1] \to [0, 1]$ is the matching function.[7] This defines the function

$$s(z) = H^{-1}(G(z)),$$

which associates a job of quality s with an agent of ability z. The first derivative of this function is

$$s'(z) = \frac{g(z)}{h(s(z))}.$$

[6] There also exist pooling equilibria in which wages reflect average productivity.

[7] This function is measure-preserving and one-to-one on $\phi([0, 1])$. See Hopkins (2012) for details.

Notice that some graduate jobs are filled by (rich) graduates of ability $z < \widetilde{z}$.

The equilibrium utility of an agent of ability z is

$$u\left(s\left(z\right), x\left(z\right), z\right) = y + w\left(s\left(z\right), x\left(z\right), z\right) - c\left(x\left(z\right), z\right). \tag{6}$$

Hopkins (2012) considers both the transferable utility case, where wages are bargained between employers and employees, and the nontransferable utility one, where wages are sticky.[8] The second of these assumptions seems more appropriate for non-graduate wages than for graduate ones. As our tournament concerns only graduate jobs, we will then restrict our attention to the transferable utility case. For the equilibrium to be stable, the x chosen by the employee must be such, and the product of the match must be divided between employer and employee in such a way (i.e., the wage schedule must be such), that the match in question is not dominated by any other. The stability conditions are then

$$w\left(s\left(z+\varepsilon\right), x, z+\varepsilon\right) + \pi\left(s\left(z\right), x, z\right) - w\left(s\left(z\right), x, z\right) \geq \pi\left(s\left(z\right), x, z+\varepsilon\right) \tag{7}$$

and

$$w\left(s\left(z\right), x+\varepsilon, z\right) + \pi\left(s\left(z\right), x, z\right) - w\left(s\left(z\right), x, z\right) \geq \pi\left(s\left(z\right), x+\varepsilon, z\right). \tag{8}$$

These are the stability conditions under complete information. Having assumed $\pi_{sx} = 0$, however, the wage schedule implied by these conditions does not depend on the functional form of $x_{LF}\left(.\right)$, and will thus be the same under incomplete information.

Let $\underline{w}_{LF}$ denote the lowest graduate wage. Above that, the wage schedule is

$$w\left(s\left(z\right), x, z\right) = \int_{0}^{z} \pi_z\left(s\left(t\right), x_0, t\right) dt + \int_{x_0}^{x} \pi_x\left(s\left(z\right), t, z\right) dt + \underline{w}_{LF}, \tag{9}$$

where t is a running variable.[9] At $w = \underline{w}_{LF}$, $w_x = \pi_x$. In contrast with Hopkins (2012), where all jobs are assigned by tournament and $x_0 = 0$, $\underline{w}_{LF}$ cannot be set arbitrarily, but must satisfy

$$\underline{w}_{LF} \geq w_0 + c\left(x_0, 0\right).$$

Competition among graduates will ensure that this constraint is satisfied as an equation (i.e., that the lowest paid graduate will be indifferent between investing in education and getting a graduate job, or going straight into the non-graduate labour market).

In a separating equilibrium, it is unprofitable for an agent of ability z to choose the level of x appropriate for an agent of ability $z' \neq z$. Exploiting this

[8] Clark (2006) establishes conditions for the existence of a unique stable matching in this case.

[9] The marginal conditions on the wage schedule, that is the argument of (9), are obtained by taking the limit ε to zero in (7) and (8).

(incentive-compatibility) condition and (9), Hopkins demonstrates that, for all participating agents other than those with $z = 0$,

$$x'_{LF}(z) = \frac{\pi_z (s(z), x, z)}{c_x (x, z) - \pi_x (s(z), x, z)}, \tag{10}$$

where $c_x(x, z)$ is greater than $\pi_x (s(z), x, z)$ for all $z > 0$, and thus all agents of ability higher than $z = 0$ invest more than would be efficient given the job allocation. Those of ability $z = 0$, assigned to jobs of quality $s(0) = 0$, will choose x so that $c_x(x, 0) = \pi_x (s(0), x, z)$, i.e., they invest x_0. Integrating (10) from $z = 0$ upwards, gives us the LF equilibrium investment strategy $x_{LF}(z)$ we were looking for. This equilibrium is inefficient for two reasons. First, because all graduate jobs other than those of quality $s(\overline{z})$ are occupied by graduates of lower ability than in FB. Second, because the denominator of (10) must be positive, and x is thus inefficiently high for all $z > 0$. The former derives from the fact that the agents excluded from the tournament are the poor rather than the less able. The latter reflects the fact that, as graduate workers are ranked according to their educational level, all rich agents other than the marginal ones (those who are indifferent between going to university or straight into the labour market) have an incentive to invest more in order to make a better match. In Appendix A, we show that, for all $z \in [\widetilde{z}, \overline{z})$, $s_{LF}(z) > s_{FB}(z)$, implying that all rich agents except those with the highest ability level have a better match in laissez faire than in first best.

5 Wealth redistribution and price subsidies

If the government observes y, it can improve the allocation by redistributing wealth. Provided that wealth redistribution (WR) can be carried far enough to leave every agent with the same amount of wealth and that this amount of wealth is sufficient to relax the liquidity constraint for those with $z \geq \widetilde{z}$, all these agents will invest in education (though not, as we will see, at the efficient level) and participate in the matching tournament. As in the LF case we use the results of Hopkins (2012), in particular the one that the equilibrium allocation is characterized by positive assortative matching.

Given the policy, the support of the ability distribution of participating agents is $[\widetilde{z}, \overline{z}]$ as in FB, and thus narrower than in LF. Again as in FB, the distribution function is $\frac{G(z)-(1-\beta)}{\beta}$, and an agent of ability z is assigned to a job of quality

$$s_{WR}(z) = H^{-1}\left(\frac{G(z) - (1-\beta)}{\beta}\right),$$

even if the matching is now made on the basis of the chosen level of x, so that for an agent of rank z_i matched to a firm of rank s_i, x_i satisfies

$$F(x_i) = \frac{G(z_i) - (1-\beta)}{\beta} = \phi\left(\frac{G(z_i) - (1-\beta)}{\beta}\right) = H(s_i).$$

Consequently, for all $z \in [\widetilde{z}, \overline{z})$, $s_{WR}(z) = s_{FB}(z) < s_{LF}(z)$. Using stability conditions analogous to (7) and (8), we can derive the wage schedule

$$w(s(z), x, z) = \int_{\widetilde{z}}^{z} \pi_z(s(t), x_0, t)\, dt + \int_{\widetilde{x}}^{x} \pi_x(s(z), t, z)\, dt + \underline{w}_{WR},$$

where t is a running variable. At $z = \widetilde{z}$, $w = \underline{w}_{WR}$, and $w_x = \pi_x$, where $\underline{w}_{WR}$ satisfies

$$\underline{w}_{WR} = w_0 + c(\widetilde{x}, \widetilde{z}),$$

and $\widetilde{x} > x_0$ is the education level chosen by agents of ability $\widetilde{z}$,

$$\widetilde{x} = \arg\max[\pi(0, x, \widetilde{z}) - c(x, \widetilde{z})].$$

These agents have no interest in buying more than the efficient amount of x because they have nothing to signal. Those with $z > \widetilde{z}$, by contrast, have an interest in signaling that their z is higher than the minimum and will thus adopt an investment strategy different from FB. The educational investment of these agents, again derived from stability and incentive-compatibility conditions, satisfies

$$x'_{WR}(z) = \frac{\pi_z(z, s_{WR}(z), x)}{c_x(x, z) - \pi_x(z, s_{WR}(z), x)}. \tag{11}$$

Integrating this equation from $\widetilde{x}$, we find the WR equilibrium investment strategy $x_{WR}(z)$.

Condition (11) implies that, for $z > \widetilde{z}$, $x_{WR}(z)$, is higher than $x_{FB}(z)$. In comparison with the FB, there is then overinvestment. However, $x_{WR}(z)$ is lower than $x_{LF}(z)$. In Figure 1, the LF curve represents the investment strategy of the participating agents in the LF equilibrium (i.e., the investment behaviour of the rich). The WR curve represents that of the participating agents in the WR equilibrium (all those of $z \geq \widetilde{z}$). As we show in Appendix B, the WR curve lies everywhere above the FB curve and below the LF curve. As it starts from a higher z than the LF, poor agents with ability $z < \widetilde{z}$ invest the same amount ($x = 0$) in both regimes, but all rich agents invest less, and poor agents of ability $z \geq \widetilde{z}$ more, in WR than in LF. Aggregate educational investment may then be higher or lower than in LF. Given, however, that those who would have overinvested in LF (the rich) now invest less, and those who would have underinvested (the high-ability poor) now invest more, wealth redistribution raises efficiency. There is an equity gain too, because every agent of ability z will have the same utility,

$$u_{WR}(s_{WR}(z), x_{WR}(z), z) = y_{WR} + w_{WR}(s_{WR}(z), x_{WR}(z), z) - c(x_{WR}(z), z),$$

irrespective of y.

As an alternative to simply redistributing wealth, the government could use a wealth tax to finance an education price subsidy. As this would not affect

$\widetilde{z}$, the matching scheme would be the same as under WR, but all participants would now invest more because the policy would reduce the private marginal cost of x. Therefore, the efficiency gain would be lower than under WR, and could actually be negative. The gain is even lower in practice, where education price subsidies are usually financed, at least in part, by distortionary taxes.

6 Student loans

If y is imperfectly observable, or lump-sum redistribution is not feasible, WR may be out of the government's reach. Provided, however, that the government (unlike individual agents) can borrow wholesale against its future tax revenue, and that the interest rate is lower than the return to educational investment for at least the more talented agents, the government can still raise equity and efficiency by lending to students at stage 1 of the game, and recovering the credit at stage 2. The liquidity constraint is now

$$y + b - c(x, z) \geq 0, \tag{12}$$

where b denotes the loan. It may be that the government can borrow unlimited amounts from the wholesale money market, and that the maximum it will then lend to each potential student is then determined by ability-to-repay considerations only. But, it may also be the case that overall public debt management considerations or wholesale money market conditions dictate a lower ceiling. We will now show that the value of b will determine which of the following situations applies. The demonstrations are in Appendix C.

$b = 0$	SL=LF separating equilibrium exists
$0 \leq b < \widetilde{b}$	Either SL=LF or no SL equilibrium
$\widetilde{b} \leq \overline{b} = b < b_0$	Partially separating SL equilibrium including rich and poor students with $z \geq \widetilde{z}$; all poor students with $z < \overline{z}$ are liquidity constrained
$\widetilde{b} \leq \overline{b} < b < b_0$	Partially separating SL equilibrium including rich and poor students with $z \geq \widetilde{z}$; poor students with $\widetilde{z} < \underline{z} < z < \overline{z}$ are not liquidity constrained
$b \geq b_0$	SL separating equilibrium with same matching pattern and investment profile as WR, but without wealth redistribution

Let b_0 denote the value of b that makes (12) slack for all $z \geq \widetilde{z}$. Let $\widetilde{b}$ denote the value of b that allows poor students of ability $\widetilde{z}$ to buy $\widetilde{x}$. For $b \geq b_0$, an equilibrium with student loans (SL) exists and coincides with WR. An equilibrium will exist also if $b = 0$, but it will then coincide with LF. For $b_0 > b \geq \widetilde{b}$ an SL equilibrium may exist. If such an equilibrium exists, it will allow poor and rich agents with $z \geq \widetilde{z}$ to participate in the tournament, and will be only partially separating. Graduate jobs of the same quality will be assigned to graduates with the same educational level, but different abilities (a lower one for the rich, and a higher one for the poor). Equilibrium beliefs will reflect true ability values and out-of-equilibrium beliefs will satisfy the intuitive criterion (Cho and Kreps, 1987).[10] For $0 < b < \widetilde{b}$ a partially separating SL equilibrium does not exist.

For $b = \widetilde{b}$, poor agents with $z = \widetilde{z}$ can buy their optimal level of education $\widetilde{x}$. This, however, is not sufficient for an SL equilibrium to exist. In order to have an SL equilibrium it is necessary that at least the poor with the highest ability level, $z = \overline{z}$, are not liquidity constrained in their choice of x. Let $\widetilde{b} \leq \overline{b} < b_0$ denote the value of b for which not only the poor with ability $z = \widetilde{z}$, but also those with $z = \overline{z}$, are not liquidity constrained. For $b = \overline{b}$, a poor of ability $\overline{z}$ will buy the same amount of x, let us call it $\overline{x}$ as an equally talented rich, and a poor of ability $\widetilde{z}$ will buy the same amount of x, let us call it $\widetilde{x}$, as an equally talented rich. A poor of ability $\widetilde{z} < z < \overline{z}$ will buy less than a rich of the same ability. For each $\widetilde{x} \leq x < \overline{x}$, there will then be two levels of z, a lower one if the agent is rich, and a higher one if the agent is poor.

Graduates and jobs will be matched on the basis of their observable characteristics (x for the former, s for the latter), and the matching will be positively assortative. Given that jobs with the same s are assigned at random among graduates with the same x, those among these graduates who are liquidity constrained will have have higher z and thus produce a larger π than those who are not liquidity constrained. Then, when π is observed, the employer will infer z. We will argue that employers offer employees a contract envisaging an initial wage, plus a fixed bonus conditional on the productivity level being the one associated with the higher z (i.e., conditional on the worker being poor rather than rich).

For the rich, the SL equilibrium associated with $b = \overline{b}$ satisfies stability conditions analogous to (7) and (8). These determine the initial wage schedule

$$w_R\left(z, s_R\left(z|\overline{b}\right), x\right) = \int_{\widetilde{z}}^{z} \pi_z\left(t, s_R\left(z|\overline{b}\right), \widetilde{x}\right) dt + \int_{\widetilde{x}}^{x} \pi_x\left(z, s_R\left(z|\overline{b}\right), t\right) dt + \underline{w}_{SL} \tag{13}$$

where $\underline{w}_{SL} = \underline{w}_{WR} = w_0 + c\left(\widetilde{x}, \widetilde{z}\right)$, $\widetilde{x} > x_0$ is the FB value of education bought by rich and poor agents with ability $\widetilde{z}$. Analogous stability conditions determine the wage schedule of the poor,

[10] In the absence of this refinement, there may exist other SL partially separating equilibria where some of the rich of ability $z < \widetilde{z}$ go to university while some of the poor with $z > \widetilde{z}$ go straight to the labour market. There also exists a pooling equilibrium where agents of ability $z \geq \widetilde{z}$ choose $\widetilde{x}$, and firms hold to their priors.

$$w_P\left(z, s_P\left(z|\bar{b}\right), x\right) = \int\limits_{\widetilde{z}}^{z} \pi_z\left(t, s_P\left(z|\bar{b}\right), \widetilde{x}\right) dt + \int\limits_{\widetilde{x}}^{x} \pi_x\left(z, s_P\left(z|\bar{b}\right), t\right) dt + \underline{w}_{SL}. \tag{14}$$

The difference between the wage rates of a poor and a rich for any given (s, x) such that $s_P\left(z'|\bar{b}\right) = s_R\left(z|\bar{b}\right)$,

$$w_P\left(z', s_P\left(z|\bar{b}\right), x\right) - w_R\left(z, s_R\left(z|\bar{b}\right), x\right),\ z' > z,$$

is the productivity bonus that will be granted to the poor. In words, an employer hiring a worker with education x in a job of quality s must promise to pay this worker the bonus thus calculated if the productivity turns out to be $\pi\left(z', s_P\left(z|\bar{b}\right), x\right)$ rather than $\pi\left(z, s_R\left(z|\bar{b}\right), x\right)$, otherwise the worker will be offered a higher bonus by a firm with the same or a slightly lower s.

The functions that allocate jobs to agents, $s_R(.)$ for the rich and $s_P(.)$ for the poor, both increasing in z, are derived in Appendix C together with the equilibrium strategies of the two categories, $x_R(z|\bar{b})$ and $x_P(z|\bar{b})$. The matching condition is now

$$F(x_i) = \alpha F_R(x_i) + (1-\alpha)F_P(x_i) = H(s_i), \tag{15}$$

where $F_R(x)$ is the distribution of x induced by $x_R(z)$, and $F_P(x)$ that induced by $x_P(z)$, and i represents the rank of education level and firm quality.

All agents of ability $\widetilde{z}$ invest $\widetilde{x}$ as in WR. Above that ability level, investment behaviour depends on whether the agent is rich or poor. For the rich, the SL equilibrium satisfies an incentive-compatibility condition analogous to (10), namely that it must be unprofitable for a rich agent of ability z to choose the level of x appropriate for a rich agent of ability $z' \neq z$. Their investment strategy will then satisfy

$$x_R'(z|\bar{b}) = \frac{\pi_z\left(z, s_R\left(z|\bar{b}\right), x\right)}{c_x\left(x, z\right) - \pi_x\left(z, s_R\left(z|\bar{b}\right), x\right)}, \tag{16}$$

so that

$$x_R(z|\bar{b}) = \int\limits_{\widetilde{z}}^{z} x_R'(z|\bar{b}) dz + \widetilde{x}. \tag{17}$$

We do not have an incentive-compatibility condition for the participating poor, because these agents borrow all that the government is willing to lend them, and their choice of x is thus determined by

$$c\left(x, z\right) = b,. \tag{18}$$

so that

$$x_P'\left(z|b\right) = -\frac{c_z}{c_x} \tag{19}$$

and

$$x_P\left(z|\overline{b}\right) = \int_{\widetilde{z}}^{z} x'_P(z)dz + \widetilde{x}. \tag{20}$$

Recall that a poor of ability $\overline{z}$ will buy the same amount of x, $\overline{x}$ as an equally talented rich,

$$x_R(\overline{z}|\overline{b}) = x_P\left(\overline{z}|\overline{b}\right).$$

Note also that poor agents are not be liquidity constrained if their z is equal to $\widetilde{z}$, but will be if it is even only slightly larger because, in view of (16), $x'_R(z|\overline{b})$ tends to infinity in a neighborhood of $\widetilde{z}$, and the level of x chosen by the rich thus increases very rapidly as z rises.

The equilibrium is illustrated in Figure 2. The two dashed curves represent the investment strategies of poor and rich agents for $b = \overline{b}$. Given that, for any given $\widetilde{z} < z < \overline{z}$, a poor agent will invest less x than a rich one, the $x_R(z)$ curve lies above the $x_P(z)$ curve everywhere except at the two extremes, $(\widetilde{z}, x(\widetilde{z}))$ and $(\overline{z}, x(\overline{z}))$. Using an argument analogous to that used in Appendix B for the case of wealth redistribution, it can be shown that the LF curve lies above the $x_R(z|\overline{b})$ curve, while the WR curve lies below the $x_R(z|\overline{b})$ and intersects the $x_P(z|b)$ curve. We can see from the diagram that the rich overinvest in SL less than in LF, but still more than in WR. The poor of ability $z > \widetilde{z}$ underinvest with respect to WR up to a certain point. Beyond that point, however, the poor will overinvest.

At intermediate values of b, $\overline{b} < b < b_0$, the poor will not be liquidity constrained not only for $z = \widetilde{z}$ and $z = \overline{z}$, but also for a range of other values of z. Let $\underline{z}(b)$ denote the lowest value of z for which the poor are not liquidity constrained given b. The poor with $z \geq \underline{z}(b)$ will then buy the same amount of x as the rich of the same ability, but those with $\widetilde{z} < z < \underline{z}(b)$ will buy less. Therefore, there will again be two investment functions, one for those who are not liquidity constrained (which now include the poor with $z \geq \underline{z}(b)$ as well as all the rich) and one for those who are (the poor with $z < \underline{z}(b)$). For $z = \underline{z}(b)$, the two functions have the same value. For $\widetilde{z} < z < \underline{z}(b)$, the amount invested by those who are not liquidity constrained (i.e., in this case, by the rich) is higher than that invested by those who are.

Let $\underline{x}(b)$ denote the amount of education bought by agents of ability $\underline{z}(b)$. For $x < \underline{x}(b)$, jobs with the same s will again be assigned at random to agents with the same x but different z. The wage schedule is

$$w_U\left(z, s_U\left(z|b\right), x\right) = \int_{\widetilde{z}}^{z} \pi_z\left(t, s_U\left(z|b\right), \widetilde{x}\right) dt + \int_{\widetilde{x}}^{x} \pi_x\left(z, s_U\left(z|b\right), t\right) dt + \underline{w}_{SL}$$

for those (rich or poor) who are not liquidity constrained, and

$$w_P\left(z, s_P\left(z|b\right), x\right) = \int_{\widetilde{z}}^{\underline{z}(b)} \pi_z\left(t, s_P\left(z|b\right), \widetilde{x}\right) dt + \int_{\widetilde{x}}^{\underline{x}(b)} \pi_x\left(z, s_P\left(z|b\right), t\right) dt + \underline{w}_{SL}$$

for those who are.

The functions that allocate jobs to agents, $s_U(.)$ for those who are not liquidity constrained and $s_P(.)$ for those who are, are derived in Appendix C together with the equilibrium strategies of the two categories, $x_U(z|b)$, and $x_P(z|b)$. Up to $\underline{x}(b)$, $x_U(z|b)$ coincides with $x_R(z|b)$, and the matching condition is

$$F(x_i) = \alpha F_R(x_i) + (1-\alpha)F_P(x_i) = H(s_i),$$

where i represents the rank of education level and firm quality. Above $\underline{x}(b)$, the matching condition becomes

$$F(x_i) = \frac{G(z_i) - (1-\beta)}{\beta} = H(s_i),$$

and we have then the same job allocation as in WR. The educational investment of those who are not liquidity constrained is still governed by (17), but the s associated with each z is now different, because the poor can buy more x. The amount invested by an unconstrained agent of ability $z > \underline{z}(b)$. will then be

$$x_U(z|b) = \int_{\underline{z}(b).}^{\overline{z}} x_U'(z|b)dz + \underline{x}(b).$$

The investment strategy of the rationed poor, $x_P(z|b)$, with $\widetilde{z} < z < \underline{z}(b)$ will still satisfy (19) and (20) with different $s_P(z|b)$.

The equilibrium associated with $\overline{b} < b < b_0$ is illustrated in Figure 3. For the poor with $\widetilde{z} < z < \underline{z}(b)$, the investment strategy is represented by the $x_P(z|b)$ curve. Notice that the relatively less talented among these agents invest less, and the relatively more talented more, than in WR. The investment strategy of the rich talented enough to withstand the competition of the poor is represented by the $x_U(z|b)$ curve. The extremely talented poor, namely those with $z \geq \underline{z}(b)$ who, at this level of b, are not liquidity constrained in their investment decisions, behave like the extremely talented rich. The upper part of the $x_U(z|b)$ curve (less steep than the rest) represents, therefore, the investment strategies of both wealth categories. As b rises, the $x_U(z|b)$ curve gets closer to the WR curve, and the segment of the $x_U(z|b)$ curve common to rich and poor agents gets longer. For b sufficiently large ($b \geq b_0$), nobody will be rationed, and the $x_U(z|b)$ curve will coincide with the WR curve.

7 Conclusion

We have shown that, in an economy where graduate jobs are allocated by a matching tournament, and some of the potential participants cannot acquire the education required to do a graduate job not because they are not sufficiently able, but because they are poor (and cannot borrow against their expected future earnings), the government can raise efficiency and ex-ante equity by redistributing wealth. Alternatively, if redistribution is either not feasible or excessively costly, the government can still raise welfare by borrowing wholesale and lending to students. Both policies raise efficiency, because (a) they replace some of the less able rich with some of the more able poor in the performance of graduate jobs, and (b) bring everyone's educational investment closer to the efficient level. Interestingly, these policies reduce overinvestment relative to the laissez faire. They also raise equity, but perfect ex-ante equity can be achieved only by redistributing initial wealth. We have also shown that an education price subsidy financed by a wealth tax would raise efficiency less than purely redistributive taxes and subsidies, because it would encourage overinvestment. The policy would raise efficiency even less if, as is generally the case in practice, the price subsidy is financed at least in part by distortionary taxation. If it can, the government should then go for straight wealth redistribution.[11] If it cannot, either because wealth is imperfectly observable or for political reasons, but it can borrow on the wholesale money market, the government should go for student loans.[12]

The equilibrium with student loans has an interesting feature. Contrary to what we are used to see in tournament models, graduate jobs of the same quality are assigned to graduates with the same educational level, but different abilities. As poor agents cannot invest in their education more than the government is prepared to lend them, these agents will in fact enter the tournament with less education than rich participants of the same ability. Given that personal ability can be inferred ex post, because the productivity of the match, and all the factors other than personal ability that contribute to such productivity, are observable, competition among employers to secure the best workers will result in an ex-post wage improvement for the poor who invested in their education less than the equally talented rich carrying out a graduate job of the same quality. This constitutes another departure from the existing literature.

Overinvestment occurs in our as in all models of the same type because the agent's native ability is not directly observable by the employer ex ante, and education is a signal of ability. In the presence of wealth inequalities and imperfect credit markets, however, education is a distorted signal.[13] All means of directly ascertaining a worker's native ability, from cognitive tests to the gathering of "soft information" as advocated by Gary-Bobo and Trannoy (2008), are thus beneficial, not only because they reduce unproductive signalling, but also because they tend to redress the distortions induced by it, and thus raise allocative efficiency. As the authors themselves point out, however, such means

[11] A similar conclusion is reached by Hoff and Lyon (1995) in a non-tournament setting.

[12] Our analysis does not account for uncertainty. Cigno and Luporini (2009) argue that, in the presence of uncertainty, student loans are dominated by a scholarship scheme financed by a graduate tax. In that article, graduate jobs are allocated by a conventional labour market, but the result would hold even if they were allocated by a matching tournament as in the present one.

[13] As emphasized in Hoff and Lyon (1995), lenders have in fact no way and no reason to distinguish between a borrower who is willing to offer collateral because his educational project has a high probability of success, and one who does so because the collateral staked is only a small fraction of his wealth.

do not reduce the underinvestment of the poor. Conversely, as we have just shown, relaxing the borrowing constraints of poor high-ability students improves the job allocation, and brings individual educational investments closer to their efficient levels.

8 Appendix A. Laissez faire vs. first best

We want to demonstrate that, for all $z \in [\widetilde{z}, \overline{z})$, $s_{LF}(z) > s_{FB}(z)$. This follows from the fact that in both FB and LF we have assortative matching with

$$s_{FB}(z) = H^{-1}\left(\frac{G(z) - (1-\beta)}{\beta}\right) < H^{-1}(G(z)) = s_{LF}(z),$$

because $\frac{G(z)-(1-\beta)}{\beta} < G(z)$ $\forall z \in [\widetilde{z}, \overline{z})$ and $H^{-1}(.)$ is monotonically increasing.

9 Appendix B. Wealth redistribution

We want to show that the WR curve lies above the FB curve everywhere except at the starting point, and everywhere below the LF curve, as shown in Figure 1. We know that, for $z < \widetilde{z}$, $x_{LF}(z) > x_{WR}(z) = x_{FB}(z) = 0$. We also know that $x_{WR}(\widetilde{z}) = x_{FB}(\widetilde{z})$. We must demonstrate that $x_{LF}(\widetilde{z}) > x_{WR}(\widetilde{z}) = x_{FB}(\widetilde{z})$, and that $x_{LF}(z) > x_{WR}(z) > x_{FB}(z)$ for $z \in (\widetilde{z}, \overline{z})$.

$x_{WR}(z) > x_{FB}(z)$ follows from the fact that $x_{WR}(z)$ is obtained integrating (11) from $\widetilde{x}$ (the proof is analogous to that of Proposition 3 in Hopkins, 2012). To prove that $x_{WR}(z) < x_{LF}(z)$, notice first of all that, in WR, graduates of ability $\widetilde{z}$ are matched with firms of quality $s = 0$, while in LF they are matched with firms of quality $s = \widetilde{s} > 0$. Then, $x_{WR}(\widetilde{z}) < x_{LF}(\widetilde{z})$ because $x_{WR}(\widetilde{z})$ is the efficient investment level for an agent of ability $\widetilde{z}$, i.e. the level obtained by maximizing $\pi(0, x, \widetilde{z}) - c(x, \widetilde{z})$, while $x_{LF}(\widetilde{z})$ is calculated by integrating (10) from x_0 and consequently higher than $argmax\,(\pi(\widetilde{s}, x, \widetilde{z}) - c(x, \widetilde{z}))$, which is in turn higher than $x_{WR}(\widetilde{z})$ because $\pi_{sz} > 0$ by assumption. Hence, at $z = \widetilde{z}$, the $x_{WR}(z)$ curve lies below the $x_{LF}(z)$ curve.

Now, take any $z \in (\widetilde{z}, \overline{z})$. Considering that the slope of the $x_{WR}(z)$ curve is given by (11), while that of the $x_{LF}(z)$ curve is given by (10), the two curves cannot cross. Notice, first of all, that the numerator of (11) cannot be higher than the numerator of (10) because $\pi_{sz} > 0$ by assumption, $s_{LF}(z) > s_{WR}(z)$ for any $z \in (\widetilde{z}, \overline{z})$, and $\pi_{zx} = 0$. Moreover, the denominator of these expressions is increasing in x. Consequently, if there existed values of z such that $x_{WR}(z) > x_{LF}(z)$, the slope of the $x_{WR}(z)$ curve should be lower than that of the $x_{LF}(z)$ curve. Considering that $x_{WR}(\widetilde{z}) < x_{LF}(\widetilde{z})$, for the two curves to cross at a value $z' \in (\widetilde{z}, \overline{z})$, it would have to be true that $x_{WR}(z)$ is steeper than $x_{LF}(z)$ in some interval belonging to $(\widetilde{z}, z')$. But, for any $z = z' + \delta$, with δ arbitrarily small, the slope of $x_{WR}(z)$ should become lower than that of $x_{LF}(z)$, thus contradicting $x_{WR}(z) > x_{LF}(z)$. Neither can the two curves coincide from point $z' \in (\widetilde{z}, \overline{z})$ upwards. Given that $\pi_{sx} = 0$, this would imply that (11) and (10) have the same denominator. Then, the numerators should be the same too, but this is impossible because $\pi_{sz} > 0$ implies that, for any given x, the numerator of (11) is lower than the numerator of (10).

10 Appendix C. Student loans

Our LF and WR equilibria differ from the matching equilibrium in Hopkins (2012) in that they apply only to a segment of the population (to the rich in LF, to the more able agents, irrespective of wealth, in WR) rather than to the entire population. In the presence of student loans, the Hopkins approach can be applied to all the agents with $z \geq \widetilde{z}$ if the maximum loan available is large enough to allow each of these agents to participate in the tournament, $b \geq b_0$. In the case where $b < b_0$, an adapted version of the Hopkins approach can be applied to the two wealth categories. In the main text, we constructed two separate wage functions, (13) for the rich and (14) for the poor, ensuring stability of the bargaining solution as in Hopkins' Proposition 2 provided that the matching functions $s_P(z|b)$ for the poor and $s_R(z|b)$ for the rich are increasing in z. Where x is concerned, Hopkins' results now apply only to the rich, because the poor are liquidity constrained. In Step 6 and Step 7 below, we show that it is possible to construct two matching functions, $s_P(z|b)$ for the poor and $s_R(z|b)$ for the rich, such that there is assortative matching over z within wealth categories, and over x for the population as a whole. Using these functions, it is possible to replicate Hopkins' Proposition 3 regarding the existence of a matching equilibrium. Before proceeding with the construction of the $s_P(z|b)$ and $s_R(z|b)$ functions, however, we need to demonstrate that, if an SL equilibrium exists for $b < b_0$, it will be such that the least able rich participating in the tournament has the same ability level, and buys the same amount of education, as the least able of the participating poor. The same occurs for the most able agents, i.e. those of ability $z = \overline{z}$. This is demonstrated in steps 1 to 5 where we take it for granted that $x_R(z)$ are increasing functions $x_P(z)$. Such functions are then constructed in steps 6 and 7.

Step 1. The amount of x bought by a rich of ability z cannot be lower than the amount bought by a poor of the same ability, because the latter is liquidity constrained. Therefore the $x_R(z)$ curve cannot lie below the $x_P(z)$ curve. The minimum ability level at which an agent invests in education cannot be lower for the poor than for the rich.

Step 2. There cannot exist an equilibrium where some rich agents of ability $z \leq \overline{z}$ buy more x than the poor of ability $z = \overline{z}$. If such an equilibrium existed, there would in fact be a level of z, z^m, and a corresponding level of x, x^m, such that the rich of ability $z \geq z^m$ for whom it is optimal to buy $x \geq x^m$ separate themselves from the poor. That, however, cannot be an equilibrium because the employer hiring a graduate of education level x^m would be better-off hiring a worker of education level $x^m - \delta$ with δ arbitrarily small. By so doing, he would in fact have a positive probability of hiring a poor of ability $\overline{z} > z^m$. Therefore, if an equilibrium exists, all agents of ability $z = \overline{z}$.buy the same amount of education $\overline{x}$.

Step 3. There cannot exist an equilibrium where the smallest x bought by the rich, x^n, is larger than the the smallest x bought by the poor. Recalling Step 1, by an argument analogous to the one in Step 2, a firm hiring a graduate of education level x^n would in fact be better-off hiring a worker of education level $x^n - \delta$ because, if it did that, it would then hire a poor of ability level higher than the mean of agents choosing x^n.

Step 4. There cannot exist an equilibrium where the smallest x bought by the rich is smaller than the smallest x bought by the poor, x^q. Suppose that such an equilibrium exists. Let z^q be the ability level of the poor, and z^r that of the rich, buying x^q in this equilibrium. We know from Step 1 that $z^r < z^q$, and that there will thus be rich of ability even lower than z^r buying positive amounts of x. Consider a level of x, x'=$x^q - \delta$ with δ arbitrarily small. If it is profitable for a rich of ability $z' < z^r$ to buy x', it will be even more profitable to buy that amount for a poor of ability level z'' between z' and z^q, such that $c(x^q - \delta, z'') = b$. Hence, the equilibrium in question cannot exist.

Therefore, if an equilibrium exists for $b < b_0$, it will be such that the least able rich participating in the tournament will buy the same amount of education, as the least able of the participating poor.

Step 5. There cannot exist an equilibrium satisfying the intuitive criterion (Cho and Kreps, 1987) such that the lowest chosen level of x, say $\widehat{x}$, is chosen by rich and poor with different ability levels. If such an equilibrium existed, $\widehat{x}$ would in fact be chosen by rich of ability z' and poor of ability z'', $z'' > z'$. Then, $\underline{w}_{SL}$ would have to satisfy $\underline{w}_{SL} = w_0 + c\left(\widehat{x}, z'\right)$, where $c\left(\widehat{x}, z'\right) > c\left(\widehat{x}, z''\right)$. Since $c_{xz} < 0$, however, there is a level of x, $\widehat{x} - \delta$, such that $\underline{w}_{SL} - c(\widehat{x} - \delta, z' - \varepsilon) < w_0$ while $\underline{w}_{SL} - c(\widehat{x} - \delta, z'' - \varepsilon) > w_0$ for ε arbitrarily small. If the intuitive criterion is to be satisfied, firms cannot attribute a positive belief to $z = z' - \varepsilon$ if they observe $x = \widehat{x} - \delta$. But, if the belief $z = z'' - \varepsilon$ is attached to $x = \widehat{x} - \delta$, then firms of quality $s = 0$ could offer the wage $w = \underline{w}_{SL} - k = w_0 + c\left(\widehat{x} - \delta, z'' - \varepsilon\right)$ where $k > 0$. The offer would be acceptable by the poor of ability $z'' - \varepsilon$, and the firms of quality $s = 0$ would be better off. Hence, such an equilibrium cannot exist.

Therefore, if an equilibrium exists for $b < b_0$, it will be such that the least able rich participating in the tournament not only will buy the same amount of education but will also have the same ability level, as the least able of the participating poor. Moreover, all agents of ability $z = \overline{z}$ buy the same amount of education $\overline{x}$ independently of their being rich or poor. Given that the measure of graduate jobs is the same in SL as in FB and WR, the common minimum ability level will then be $\widetilde{z}$, and the common minimum investment level $\widetilde{x}$.

Step 6. There is a value of b, $\widetilde{b} \leq \overline{b} < b_0$, such that there exists an equilibrium where rich and poor agents of ability $\overline{z}$ buy the same level of education $\overline{x}$, while rich and poor agents of ability $\widetilde{z}$ buy the same level of education $\widetilde{x}$. For $\widetilde{z} < z < \overline{z}$, the rich buy more education than the poor.

Recall that there is a value of b, $\widetilde{b}$, which allows poor agents of ability $\widetilde{z}$ to buy the efficient amount of education $\widetilde{x}$. Consider a value of b such that $\widetilde{b} \leq b < b_0$. Denote by $x_P(z|b)$ the function that solves $c(x, z) = b$. Given our assumptions on $c(x,z)$, $x_P(z|b)$ is continuos, convex and strictly increasing in z, with derivative $x'_P(z|b) = -\frac{c_z}{c_x}$. Clearly, $x_P(z|b)$ and x'_P are increasing in b. Notice that the inverse functions $x_P^{-1}(x|b)$ is defined and is decreasing in b.

Let $\overline{x}_R$ denote the level of education chosen by a rich of ability $\overline{z}$, and define, over $[\widetilde{x}, \overline{x}_R]$, the distribution function

$$F(x) = \alpha F_R(x|b) + (1-\alpha)F_P(x|b) = \alpha G\left[x_R^{-1}(x|b)\right] + (1-\alpha)G\left[x_P^{-1}(x|b)\right] \quad (21)$$

where

$$x_R(z|b) = \int_{\widetilde{z}}^{z} x'_R(t)dt + \widetilde{x} \text{ with } x'_R(z) = \frac{\pi_z\left(z, s_R\left(z\right), x\right)}{c_x\left(x,z\right) - \pi_x\left(z, s_R\left(z\right), x\right)}, \quad (22)$$

$x_P(z|b)$ has already been defined, and $s_R(z)$ is still to be determined. Given our assumptions, the function $x_R(z|b)$ is concave if $s'_R(z) > 0$. Assortative matching implies

$$F\left(x_i\right) = \phi\left(\alpha G\left[x_R^{-1}(x_i|b)\right] + (1-\alpha)G\left[x_P^{-1}(x_i|b)\right]\right) = H\left(s_i\right). \quad (23)$$

so that an agent buying education level x_i is matched to a firm of the same rank s_i.

Given $G\left[x_P^{-1}(x|b)\right]$ and $H(s)$, (23) together with (22) determines $s_R(z)$ and implies $s'_R(z) > 0$. $s_P(z)$ then indirectly results from (23).

Equation (23) is not necessarily an equilibrium condition. For it to be an equilibrium, it must be the case that $\overline{x}_R = x_R(\overline{z}|b) = x_P(\overline{z}|b)$, where $x_P(\overline{z}|b)$ solves $c(x,\overline{z}) = b$. Were it the case that $\overline{x}_R > x_P(\overline{z}|b)$, there would not exist an equilibrium for this level of b (see Step 2). An equilibrium can however be obtained by raising b. Doing the latter would in fact raise $x_P(z|.)$ and thus reduce $x_R(z|.)$ because the all the rich other than those with ability $z = \widetilde{z}$ will be hired by less productive firms than they otherwise would. At the same time, the fact that $x'_R(z|b)$ tends to infinity in a neighborhood of $\widetilde{x}$ ensures that the $x_R(z|b)$ curve will still lie everywhere above the $x_P(z|b)$ curve. Given the convexity of $x_P(z|.)$ and concavity of $x_R(z|.)$, a point where $\overline{x}_R = x_R(\overline{z}|b) = x_P(\overline{z}|b)$ will then be reached by sufficiently raising b.

Step 7. For $b_0 > b > \overline{b}$, there exists an equilibrium where some of the poor are not liquidity constrained, and thus buy the same level of education as the rich of the same ability. If the $\overline{x}_R$ is less than $x_P(\overline{z}|b)$, poor agents of sufficiently high ability will not be liquidity constrained. The demonstration is analogous to that used in Step 6. The equilibrium has the following characteristics. Let $\underline{x}$ be the common value of x at the point where the $x_R(z|b)$ curve crosses the $x_P(z|b)$ curve. Let $\underline{z}(b)$ denote the level of z such that $x_P(z|b) = x_R(z|b) = \underline{x}$. Agents of ability $z < \underline{z}(b)$ buy $x_P(z|b)$ if they are poor, $x_R(z|b)$ if they are rich. In other words, in the notation used in the main text, $x_U(z|b) = x_R(z|b)$ for $z < \underline{z}(b)$. For this ability range, $s_P(z)$ and $s_R(z) = s_U(z)$ are derived using the same procedure that we used in Step 6. For $z > \underline{z}(b)$, there will be a unique $s_U(z|b)$ such that

$$G\left(z\right) = F\left(x\right) = H\left(s\right).$$

The associated investment function is $x_U(z|b) = \int\limits_{\underline{z}(b)}^{z} x'_U(t)dt + \underline{x}$, where $x'_U(z) = \frac{\pi_z(z, s_U(z|b), x)}{c_x(x,z) - \pi_x(z, s_U(z|b), x)}$ for rich and poor alike.

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Figure 1. Wealth Redistribution vs Laissez Faire and First Best

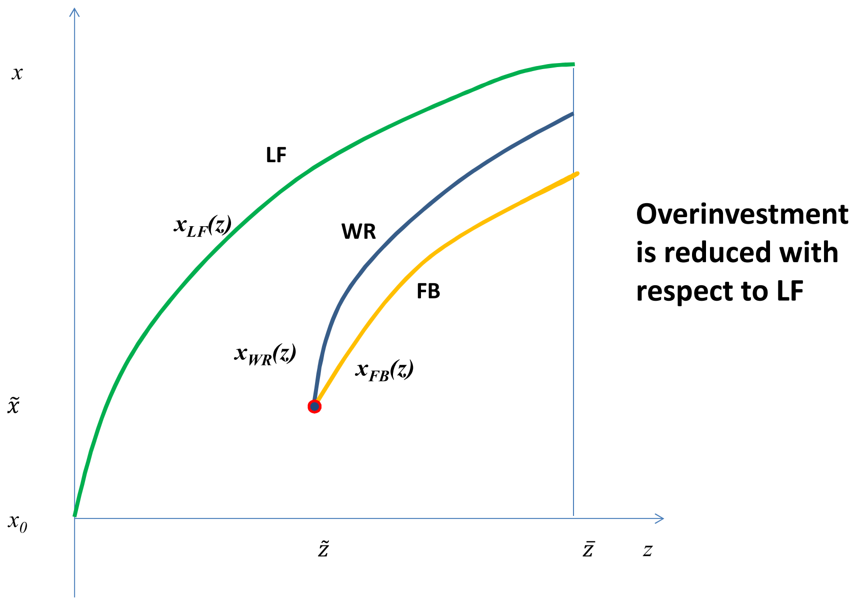

Figure 2: SL equilibrium with *b*=$\bar{b}$

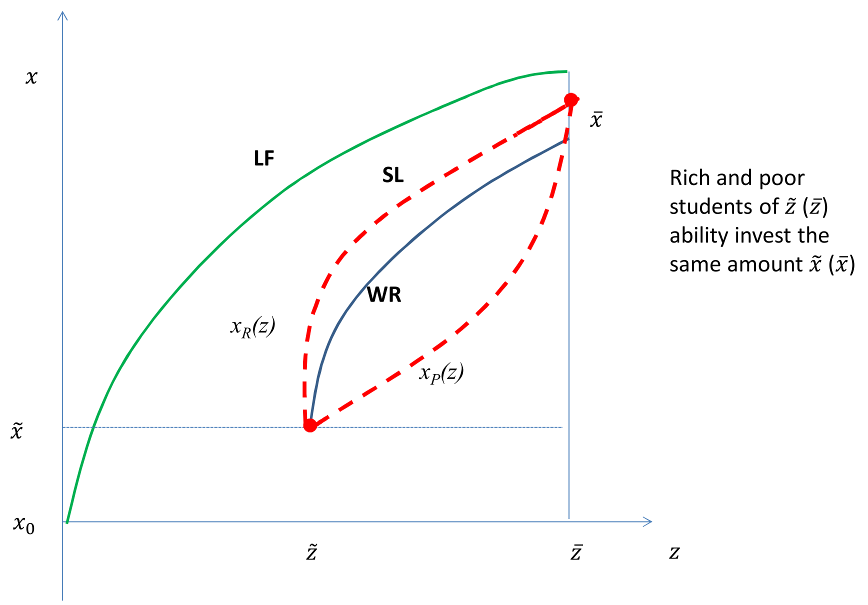

Figure 3: SL equilibrium with $b > \overline{b}$