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The Global Political Economy of a Green Transition

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Abstract

Countries respond differently to climate change and while this resulting behavioral heterogeneity is empirically observed, its relationship to the evolution of global climate action has not been analysed. This paper fills this gap by developing a novel integrated assessment international political economy model (IPE-IAM). Our model shows the possibility of a number of outcomes, where high levels of sustained global action is only one possibility. We show that the model fits the observed increase in global climate action from 1989 to today, however, given estimated parameters, the expected trajectory to 2050 is that global climate action will not continue to rise and net zero will not be reached. In order to achieve a high level of sustained global action, a low degree of heterogeneity regarding countries' preferences for action and a strong peer pressure effect is required.

Keywords: Climate action, Evolutionary dynamics, Discrete choices, Integrated assessment, International political economy
JEL codes: C62, E71, F5, Q54, Q58

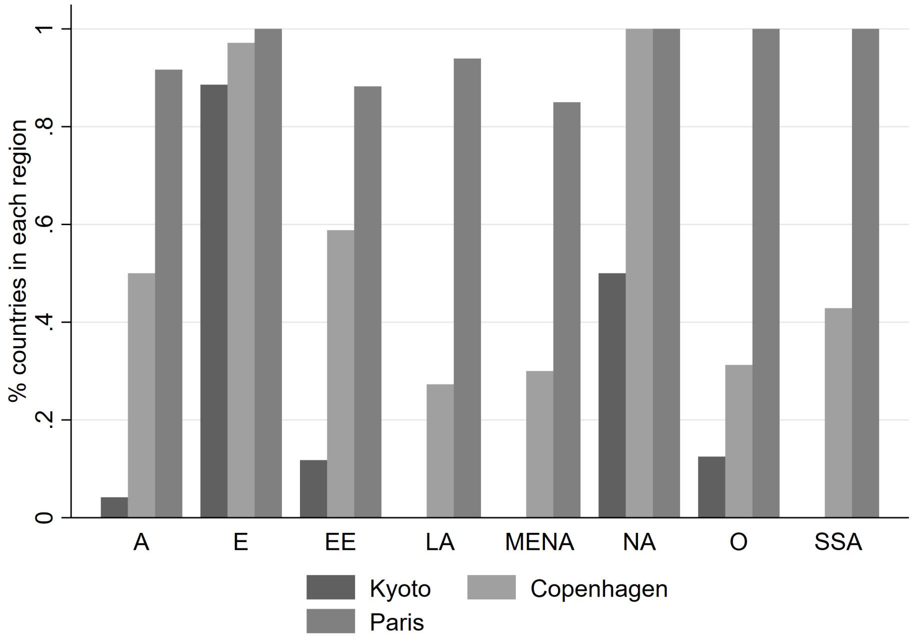
1 Introduction

Since the late 1980s, countries across the world have taken steady but insufficient action to stop climate change. Current mitigation pathways put the world on a turbulent path to exceed 1.5°C by the 2030s (IPCC, 2022), despite increasing numbers of countries signing up to international climate agreements and implementing national laws to reduce emissions (de Silva and Tenreyro, 2021). This is partly because countries disagree on the appropriate level of action. As Figure (1) illustrates, countries' preferences for climate action differ across both space and time (Lazkano et al., 2016; Li and Rus, 2019). These differences arise from several factors, including the unequal distribution of climate damages and the disparity in economic resources among countries (Peri and Robert-Nicoud, 2021; Yohe and Schlesinger, 2002). Taking into account this variation in preferences is relevant, especially in a dynamic context as countries' actions are known to have a positive impact on other countries' choices (Sauquet, 2014; Fankhauser et al., 2016; Carattini et al., 2023).

While an extensive literature has studied various aspects of climate action including mitigation and adaptation policies (for example see Li and Rus, 2019), common property resource use (for example see Sethi and Somanathan, 1996; Noailly et al., 2007; Osés-Eraso and Viladrich-Grau, 2007) and environmental agreements (for example see Battaglini and Harstad, 2016; Harstad, 2016; 2023; Wagner, 2016; Günther and Hellmann, 2017; de Silva and Tenreyro, 2021; Bellelli et al., 2023), the dynamic impact of heterogeneity in countries' preferences for action has not been considered. The present paper aims to fill this gap by answering two related questions: (i) what is the role of the variation of preferences in the global political economy of climate action; and (ii) which are the necessary conditions for sustained high levels of global action? We tackle these questions by developing a novel, evolutionary, international political economy, integrated assessment model (IPE-IAM). Our modelling framework is rooted in the discrete choice tradition (Manski and McFadden, 1981; McFadden, 1974, 1978, 2001; Train, 2009) which allows for incorporating relevant behavioural insights such as bounded rationality (Simon, 1957, 1979) and the use of heuristics (Tversky and Kahneman, 1974; Kahneman, 2003) which may vary across countries and time.

We assume heterogeneous countries, who choose in each period to take action or not. The choices are influenced by both global and idiosyncratic factors. The global factors are: (i) global emissions and (ii) the actions of other countries which have been shown to have a positive effect on overall action

Figure 1: Signatures of key climate deals (% in each region)



(a) Notes: Data from de Silva and Tenreyro (2021). Region abbreviations: A=Asia; E=Europe; EE=Eastern Europe; MENA=North Africa & the Middle East; SSA=Sub-Saharan Africa; LA=Latin America & the Caribbean; O=Oceania; NA=Northern America

(Fankhauser et al., 2016). Drawing on relevant empirical research, we identify four key idiosyncratic factors, which are heterogeneous across countries. These are related to: economic resources; vulnerability to climate change; the concentration of extractive fossil fuel industries; and (the type of) political institutions. Using data which capture these factors, we develop an index which confirms that the distribution of these factors is such that the choice of perturbed best response (logit) evolutionary dynamics is appropriate. We refer to the variation of these idiosyncratic factors as the *degree of heterogeneity*. Given that damages have an unequal global distribution depending on geographical and economic factors, climate damages are expected to increase the degree of heterogeneity over time (Kaufmann et al., 2017; Tol, 2018) making climate action more difficult especially for more vulnerable countries.

This logit modelling approach provides an analytically tractable framework which allows us to base the model on empirical observations and analyse the effects of heterogeneity on countries decisions. Our model gives rise to different potential outcomes, including low levels of action, increasing levels followed by a decline and sustained high levels of action. The outcome which actually dominates depends on the relative importance between peer pressure effects and the degree of heterogeneity. For sustained high levels of global climate action, strong peer pressure effects should be accompanied by a low degree of heterogeneity. To determine which type of dynamics best describe the current path, we empirically estimate our model through a logit panel analysis of the determinants of climate action. As a proxy for climate action we use data from the Climate Change Laws of the World Database (2020) on the frequency of mitigation laws from 1950 to 2019.

We simulate our model between 1989 (the first full operational year of the IPCC) and 2050 (a key date for countries' net zero targets). Given estimated parameter values and initial conditions, we first show that our simulated model fits the observed dynamics of global GHG emissions and global action over the last three decades from 1989 to today. Given the success of this validation exercise, we then plot the expected outcome from today to 2050. Interestingly we find that global climate action does not continue to rise but instead converges to a steady state that implies rising net emissions in the long run. In other words, the expected outcome is that countries will fail to achieve net zero. To get better insights, we present simulations where we change two key "policy" parameters: (i) the peer pressure effect and (ii) the degree of heterogeneity. Our main finding is that it is possible to reach a steady state where all countries take action and global emissions continually fall to net zero, but only if the peer pressure effect is high and the degree of heterogeneity is low. The intuition of the model is as follows. While peer pressure among countries positively influences others to take action, free-riding may still emerge as more measures to reduce emissions are taken. When there is a high degree of heterogeneity, countries which are taking action due to peer pressure but have relative strong biases against action will likely abstain as net emissions start to reduce. Since the effects of global action on emissions take time, a reduction in participation has a delayed effect on increasing emissions, but an immediate impact on peer pressure. The unsynchronised timing of these effects means that the decreased peer pressure from an in crease in abstention will lead more countries to abstain, further reducing peer pressure. Consequently, before net emissions rise enough to counteract this trend, peer pressure might shift to incentivise abstention instead. Without these two necessary conditions, climate action may increase in the short run but stop rising over the medium run. Moreover as climate damages vary across countries, they increase the level of heterogeneity, potentially making it more difficult to achieve high levels of global climate action. Timing is crucial for sustained global efforts, as the rising heterogeneity caused by increasing climate damages over time means that future levels of peer pressure will need to be higher than those required today to maintain strong collective action.

The paper contributes to and brings together different literatures. Given the structure of our model, our work contributes to the broader environmental economics literature which integrates natural processes related to climate change and environmental damages with social and economic variables. While the key focus of this broad family of Integrated Assessment Models (IAMs) has been to assess the environmental economic feedback effects of various fiscal and financial policies (for example see Nordhaus, 1992, 2014; Stern, 2013; Dafermos et al., 2017, 2018; Lamperti et al. 2018 among others), to our knowledge there are no works which integrate international political economic processes related to climate action with processes related to the carbon cycle and climate damages. Our work contributes to this literature by bringing in the analysis, empirically relevant global political economy aspects. Countries' preferences for action may change over time, due to both different parties being in power over time and political and economic reasons which vary geographically and are beyond the policy makers' control. We show that due to interactions, green transition dynamics might be different in the short run and the long run. Related to this, we do not assume optimal choices under a full information assumption as is the case for example in DICE (Nordhaus, 1992). Our model assumes choices are made through a discrete choice evolutionary framework which allows to directly link our theoretical model with empirical works.

Given our focus on climate action, our model creates links between IAMs and the boarder literature on climate action which has mainly focused on International Environmental Agreements (IEAs)[1]. More specifically, our model is empirically driven which is one of the exceptions compared to related works (Bellelli et al., 2023). In this way we aim to connect empirical findings related to the determinants of action (Bättig and Bernauer, 2009; Scheidel et al., 2020; Tubi et al., 2012; Victor et al., 2022) with dynamic models of environmental agreements (for example see van der Ploeg and de Zeeuw, 1992; Hoel, 1997; Long, 2012; among others and Calvo and Rubio, 2013 for a review) which are shown to be causally linked to climate action (de Silva and Tenreyro, 2021). Due to the evolutionary dynamics of our model, the closest paper within the dynamic IEAs literature is the one by Breton et al. (2010) who analyse

[1] For a recent review on IEAs see Bellelli et al. (2023)

stability of IEAs under the possibility of punishing non-signatories. Our model differs from Breton et al. (2010) in two fundamental ways. First, in our paper, action refers to the reduction of emissions rather than to signing an agreement. Second, our model focuses on the degree of heterogeneity which depends on damages (which in turn depend on the carbon cycle and a damage function).

From a methodological viewpoint our work has links with evolutionary game theoretic models which have studied common-pool resource problems (for example see Sethi and Somanathan, 1996; Noailly et al., 2007; Osés-Eraso and Viladrich-Grau, 2007). A key difference with these works is that instead of assuming the usual (in evolutionary game theory) replicator dynamics to govern the evolution of the agents' behaviour, here the evolution is modelled through a logit (perturbed best response) framework which is derived from empirical behavioural microfoundations. This modelling approach has its roots in the behavioural dynamic discrete choice literature starting from the works of Lux (1995) and Brock and Hommes (1997) who studied the effects of heterogeneous behaviours in financial markets. This heterogeneous interacting agents behavioural framework has been applied to different fields[2] including environmental economics focusing on the effects of heterogeneity in investment decisions (Cahen-Fourot et al., 2023), expectations (Campiglio et al., 2024) and attitudes towards green policies (Dávila-Fernández and Sordi, 2020; Dunz et al. 2021; Sordi and Dávila-Fernández, 2023). A key difference between our model and the previous ones, is that while all allow for heterogeneous agents, we focus on the effect of the degree of heterogeneity on climate action.

The structure of the rest of the paper is as follows. Section 2, introduces the setup of our model and the empirical observations upon which we base our analysis regarding the heterogeneity of preferences for climate action. Section 3 consists of two parts. The first part provides some analytical results of a baseline version of our model where the degree of heteorogeneity is exogenous. Then we present simulations of different cases of heterogeneity with the full IPE-IAM model. In Section 4 we estimate the equations of the model and discuss the robustness of our results under different specifications. The final section concludes.

[2]For example: behavioural finance (Lux, 1995; Brock and Hommes, 1997, 1998; Chiarella and He, 2002; Chiarella et al., 2006; Westerhoff and Dieci, 2006; Anufriev abd Tuinstra, 2013; Dieci and Westerhoff, 2016, among others), behavioural macroeconomics (De Grauwe, 2011, 2012; Flaschel et al., 2018; Hommes et al., 2018, 2019; Hommes and Lustenhouwer, 2019; Assenza et al., 2021, among others), voting (Di Guilmi and Galanis, 2021; Di Guilmi et al., 2022), physical distancing decisions (Galanis et al., 2021; Di Guilmi et al., 2022, Flaschel et al., 2022)

2 Model

Setup

Consider that the world economy is composed by $2N$ countries, each of which faces a binary decision at each point in time: take a costly climate action (C) to reduce the level of greenhouse gas emissions with the goal of reaching zero net emissions (E_t), or abstain (A). Let π_t^{ji} be the payoff of country $i = \{1, \ldots, 2N\}$ choosing strategy $j = \{C, A\}$ at t. The evolution of global action depends on the *fitness* measure captured by the relative payoffs of strategy C compared to strategy A, $\pi_t^i = \pi_t^{Ci} - \pi_t^{Ai}$, such that country i chooses C in period t if $\pi_t^i > 0$, chooses A if $\pi_t^i < 0$ and chooses randomly if $\pi_t^i = 0$.

Let n_t^C be the number of countries who take an action at time t and n_t^A the number of ones who don't, with $n_t^C + n_t^A = 2N$. Also let x_t be the relative share of countries which take climate action at t, such that

$$x_t = \frac{n_t^C - n_t^A}{2N}. \tag{1}$$

This implies that $x_t \in [-1, 1]$, for all t with $x_t > 0$ when $n_t^C > n_t^A$. As shown in de Silva and Tenreyro (2022), actions have an impact on the growth rate of countries' emissions. However it is not clear whether abstracting from country size, as we do here, implies that the relative share of participation has a statistically significant impact on the growth rate of global net emissions $\hat{E}_t$. Following this we assume and estimate[3] that the evolution of global emissions is given by:

$$\hat{E}_{t+1} = -\alpha x_t, \tag{2}$$

with $1 >> \alpha > 0$.

Preferences for action

Relative preferences of country i at t are assumed to depend on three types of factors. First, countries are taking action with the goal of reaching zero net emissions, such that as long as $E_t > 0$, the effect

[3]The estimation results can be found in column 2 of table (1) in the appendix. We also test the relation with a constant, $\hat{E}_{t+1} = \alpha_0 - \alpha x_t$. We find that α_0 is not significant and α is significant at significance level 5% level. These estimation results can be found in column 1 of table (1). Given that the constant is insignificant, we build the model using equation (2) without a constant.

on action is positive ($\frac{\partial \pi_t^i}{\partial E_t} > 0$). Second, following the empirical insights of Fankhauser et al. (2016), other countries' actions have a positive impact on the action of country i, which implies a coordination incentive. The influence of other countries' action can be approximated by x_t, such that the majority action has a positive effect on a single country taking action ($\frac{\partial \pi_t^i}{\partial x_t} > 0$). Third, there are 'idiosyncratic' factors influencing countries' decisions.[4] Based on these we can express preferences for action in the following linear form:

$$\pi_t^i = \beta_x x_t + \beta_e E_t - \epsilon_t^i, \tag{3}$$

where $\beta_x > 0$ and $\beta_e > 0$ capture the relative importance of x_t and E_t on actions and ϵ_t^i is distributed across counties.[5] Note that while we follow Fankhauser et al. (2016) who find a lack of empirical justification of *free rider effects*, equation (3) does allow for the possibility of 'free riding behaviour' when $\beta_e E_t - \epsilon_t^i < -\beta_x x_t$.

In the discrete choice literature (for example see McFadden, 1974; 1978; 2001 and Train, 2009), the standard (or baseline) assumption is that ϵ_t^i follows a logistic distribution, which gives rise to a logit structure. In order to see if this assumption is reasonable in our framework we develop an index which is a simple average[6] of four factors which are known in relevant literatures to have an impact on countries taking action. These are economic resources (Bättig and Bernauer, 2009; Fankhauser et al., 2016); vulnerability to climate damages (Tørstad et al., 2020; Tubi et al., 2012; Ricke et al., 2018); fossil fuel rents (Brulle, 2018; Colgan et al., 2021; Dolphin et al., 2020; Lamb and Minx, 2020; Victor et al., 2022) and democratic and long term political institutions (Davidson et al., 2021; Finnegan, 2022; Fredriksson and Neumayer, 2016; Genovese and Tvinnereim, 2019; Keohane, 2001).[7]

Figure (2) presents two maps of this index. The upper panel shows the index for 2019 - the latest year with available data. Countries in darker red prefer to take climate action to those with lighter

[4] These idiosyncratic factors can also capture variation of preferences with respect to E_t and x_t. For a discussion on taste variation see Train (2009) and Galanis et al. (2022).

[5] Allowing the idiosyncratic factors to influence the preferences in a negative way is without loss of generality, while it allows for a more intuitive discussion of the results.

[6] We take the z transformation of each factor to make them comparable. With the z transformation, we convert our variables/distributions to a set of z values with mean equal to 0 and a standard deviation equal to 1.

[7] One aspect of this relates to corruption. Fredriksson and Neumayer (2016) argue that controlling corruption promotes climate action, as effective climate mitigation policies requires overcoming opposition from cost-bearing organised groups (Finnegan, 2022).

colours. The bottom panel shows the change in the index between 2005 and 2019. Preferences for action increased in red countries, declined in cream countries and stayed the same in orange ones.

Figure (3) plots the distribution of the index for a single year (2019) and shows that preferences seem to follow a relatively symmetric unimodal distribution, which may be reasonably proxied by a logistic distribution.[8] Moreover, as the lower panel of Figure (2) shows, for most countries, the index changes over time, implying that for the same global factors the idiosyncratic factors change over time. In addition to this, there are likely to be a set of other non observable idiosyncratic shocks that influence a country's willingness to take action. For example, countries which had previously signed up to climate commitments may suddenly change direction due to political shocks, such as the USA dropping out of the Paris agreement in 2016, or the UK shifting its approach to net zero commitments in 2023. This provides empirical justification for assuming that preferences have an idiosyncratic time variant component. Both of these findings allow us to reasonably assume that ϵ_t^i follows a logistic distribution.

Let μ be mean of the distribution of ϵ_t^i and $\frac{s_t^2\pi^2}{3}$ its variance, where s_t is the value of the scale parameter of the logistic distribution at time t. We define $\gamma_t = \frac{1}{s_t}$ as the degree of heterogeneity of idiosyncratic influencing countries' preferences for taking action. High (low) values of γ_t correspond to a low (high) dispersion.[9] Given that ϵ_t^i incorporates different factors, its dispersion captured by γ_t depends on the combined variation of these factors across countries. As climate damages vary across countries and are expected to increase over time (Hsiang et al., 2022; Calel et al., 2020; Callahan and Mankin, 2022), countries' preferences for taking action will also become more heterogeneous, i.e. increasing the degree of heterogeneity, hence reducing γ_t. Formally this can be expressed through the following linear form:

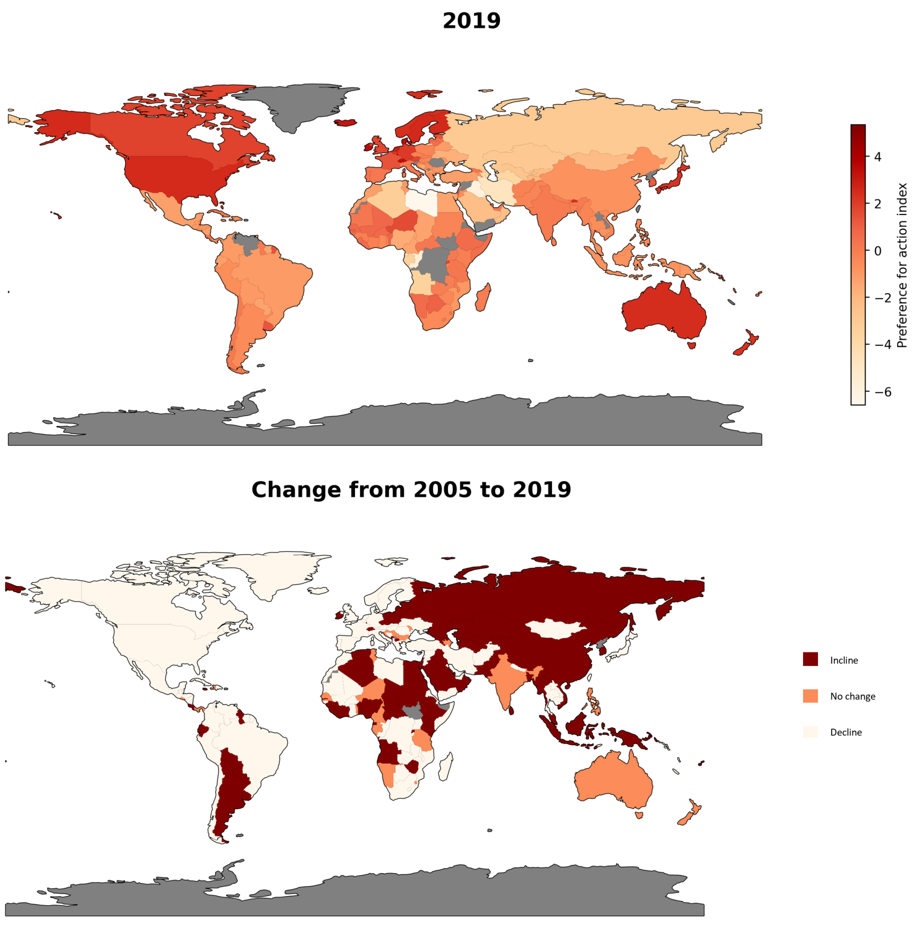
$$\gamma_{t+1} = \gamma - \delta\Omega_t, \tag{4}$$

where $\Omega_t \in [0, 1]$ captures the global level of damages and $\gamma > \delta > 0$, such that γ corresponds to the degree of heterogeneity excluding the impact of damages and δ is the marginal effect of damages on the degree of heterogeneity. Note that a high degree of heterogeneity also implies high importance

[8] Later we discuss alternative specifications.

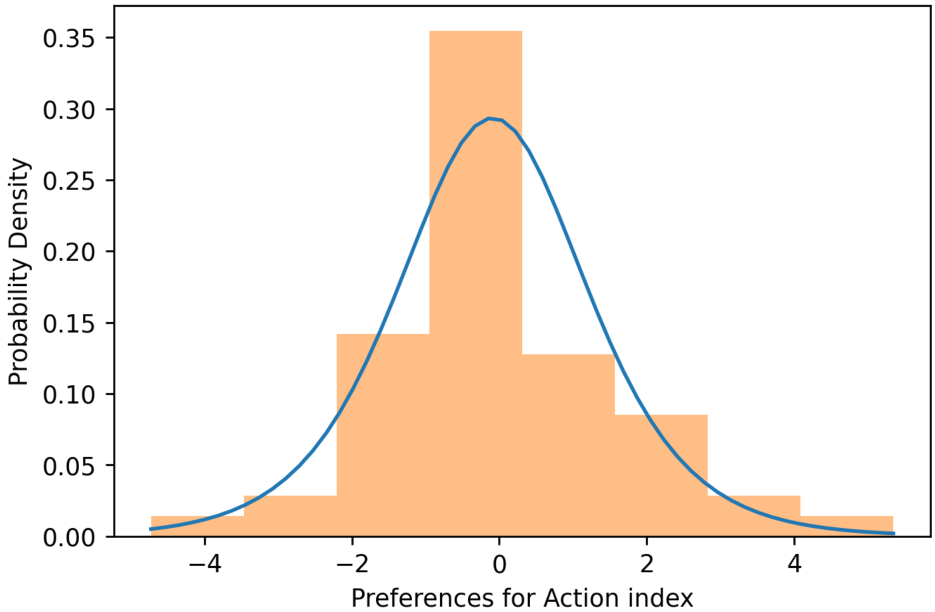
[9] The parameter γ_t is also known as *intensity of choice* in the relevant literature. For example see Brock and Hommes (1997).

Figure 2: Preferences for climate action index



(a) Notes: The preferences for action index is constructed from the unweighted sums of four variables: (i) GDP per capita in current USD (World Bank); (ii) vulnerability Index (Notre Dame-Global); (iii) sum of oil rents, natural gas rents and coal rents as percentage of GDP (World Bank); (iv) control of corruption index (Worldwide Governance Indicators from World Bank). The variables are z-transformed to make them comparable. Strong preferences for taking climate action are represented by higher values on the index. The upper panel takes the index value in 2019. The bottom panel demonstrates the change in the index between 2005 and 2019 where red indicates increased preferences, orange indicates no change, and cream indicates a decline in preferences. If the index changes in absolute terms by less then 0.1 we consider it not to have changed (i.e. orange).

Figure 3: Logistic PDF and Histogram of Preference for Action Index



(a) Notes: The line plots the fitted logistic probability density function (PDF) to the data. The bar chart represents the histogram with 8 bins.

of the idiosyncratic factors relative to the global ones. This is due to the fact that a high dispersion of idiosyncratic factors, means that there will always be a considerable number of countries that will almost always want to take action and also a considerable number of countries that almost never want to take action, regardless of the global factors.

Evolutionary dynamics

Given the logit structure underlining the choice mechanism, assuming that the number of countries $2N$ is sufficiently large, we can express the probability that a country chooses C at time t for given x_t and E_t as[10]

$$P(C|\pi_t) = \frac{e^{\gamma_t \pi_t}}{1 + e^{\gamma_t \pi_t}}, \tag{5}$$

where

$$\pi_t = \beta_x x_t + \beta_e E_t - \mu, \tag{6}$$

[10] For details of the derivation, see chapter 1 in Train (2009).

Then, the probability of no action is

$$P(A|\pi_t) = 1 - P(C|\pi_t) = \frac{1}{1 + e^{\gamma_t \pi_t}}, \tag{7}$$

High values of γ_t (low degree of heterogeneity) means v_t is relatively more important in determining the choices of the countries, while low values of γ_t (high degree of heterogeneity) means that v_t plays less of a role and the idiosyncratic factors become relatively more important. For example for $\gamma_t \to 0$ ($s_t \to \infty$), $P(C|v_t) = P(A|v_t) = \frac{1}{2}$, meaning that countries make their choices mainly due to the idiosyncratic factors.

Based on this, the evolution of x_t is given by

$$\Delta x_{t+1} = (1 - x_t)\frac{e^{\gamma_t \pi_t}}{1 + e^{\gamma \pi_t}} - (1 + x_t)\frac{1}{1 + e^{\gamma_t \pi_t}} = \frac{e^{\gamma_t \pi_t} - 1}{1 + e^{\gamma_t \pi_t}} - x_t, \tag{8}$$

which is the logit/perturbed best response dynamics used in evolutionary game theory (Sandholm, 2010). While this revision protocol is less commonly used compared to the replicator one, given its derivation from behavioural microfoundations, it allows for explicitly studying the effects of the degree of heterogeneity.

3 Results

3.1 Baseline model

Before we analyse the complete IPE- IAM which allows for the feedback effects between countries' climate action, GHG emissions and climate damages, we focus on the baseline scenario with no damages such that $\gamma_t = \gamma$. This allows us to get some analytical insights regarding the equilibria, where $\Delta x_t = \Delta E_t = 0$. We provide results regarding both the existence and the local asymptotic stability of the equlibria for different parameter values. Given the non-linearities, we complement the analytical results with simulations. The equilibria of our model are also known as *evolutionary equilibria* (Friedman, 1991, 1998) as they describe an equilibrium of an evolutionary process. However, in order not to confuse the notion of asymptotic stability, which implies that the outcome is Nash Equilibrium with the static notion of evolutionary stability which is not always implied by asymptotic stability (see Sandholm, 2010), we will refer to the evolutionary equilibria as steady states.

Assuming that $\Omega_t = 0$, means that the evolution of countries' participation in climate action is given by

$$x_{t+1} = \frac{e^{\gamma \pi_t} - 1}{1 + e^{\gamma \pi_t}} \tag{9}$$

Hence the economy in the baseline version of the model can be described by a system of two difference equations: (2) and (9).

Proposition 1. *Consider the economy described by* (2) *and* (9) *.*

(i) $(E, x) = (\frac{\mu}{\beta_e}, 0)$ *is a steady state for all parameter values.*

(ii) There exists a steady state $(E, x) = (0, x')$ *for all parameter values, with* $x' \in (0, 1]$ *for* $\mu < 0$ *and* $x' \in [-1, 0]$ *for* $\mu > 0$.

The first steady state corresponds to an outcome where net emissions are stable at a positive or negative level and the total number of countries are equally split between the ones taking action and the ones abstaining. Whether emissions are positive or negative at this steady state depends on the sign of μ. A positive (negative) μ implies that the mean of idiosyncratic factors is such that there is a bias against (in favour of) action, which in turn leads to positive (negative) net emissions. The second part of the Proposition shows two different possible outcomes with zero net emissions which depend on the average influence of idiosyncratic factors. As we see below, the degree of heterogeneity will determine the (local) stability of these steady states and also the existence of others.

Proposition 2. *Let* $\bar{\gamma} = \frac{2\left(\beta_x + 2\alpha\mu - \sqrt{(\beta_x + 2\alpha\mu)^2 - \beta_x^2}\right)}{\beta_x^2}$, *then* $(E, x) = (\frac{\mu}{\beta_e}, 0)$ *is*

(i) locally asymptotically stable for $\mu > 0$ *and* $\gamma < \frac{2}{\beta_x + \alpha\mu}$ *and a spiral node for* $\gamma > \bar{\gamma}$.

(ii) unstable otherwise.

Proposition 2, shows that when the average idiosyncratic preferences are in favour of action ($\mu < 0$) then $(E, x) = (\frac{\mu}{\beta_e}, 0)$ cannot be stable. Intuitively, this occurs because the steady-state emission level is negative (since $\mu < 0$), triggering a free-rider effect even with the smallest shock. Countries with strong biases against taking action will be the first to exhibit this behavior. This, in turn, will influence other countries' reluctance to act, with this effect becoming increasingly pronounced over time—at least until emissions begin to rise again.

A positive μ leads to stability of $(E, x) = (\frac{\mu}{\beta_e}, 0)$ when the degree of heterogeneity is relatively high (low γ). For given levels of peer pressure (β_x) and impact of action on emissions (α), γ has to be low (less than $\bar{\gamma}$) otherwise positive net emissions would lead to higher incentives for action. If the degree of heterogeneity is not sufficiently high ($\gamma > \bar{\gamma}$), then cyclical dynamics will emerge around $(E, x) = (\frac{\mu}{\beta_e}, 0)$. The latter dynamics suggest that what we currently observe in global climate mitigation (i.e. increasing levels of climate action) might not necessarily lead towards high sustained levels but may be simply the upward trend of a cyclical variation.

The intuition of the cyclical dynamics is as follows. When there is significant heterogeneity among countries, those strongly opposed to taking action are likely to abstain once net emissions move towards approaching net zero levels. Since the impact of global efforts to reduce emissions takes time, a drop in participation will quickly weaken peer pressure, rather than immediately limiting emissions reductions. This weakened peer pressure will lead to more countries abstaining, hence further reducing peer pressure. As a result, before net emissions increase enough to reverse this trend, the majority of countries can shift towards abstention. As the effect of lowering participation on emissions becomes more apparent, these dynamics will slowly change with more and more countries shifting towards action.

This potential oscillatory convergence offers some interesting insights worth emphasising. When the degree of heterogeneity satisfies $\frac{2}{\beta_x + \alpha\mu} > \gamma > \bar{\gamma}$, the level of action may rise significantly above the steady state value of $x_t = 0$ before eventually decreasing. The extent to which participation exceeds the steady state level before declining will depend on the specific parameter values, which reflect the relative strength of the various factors influencing action. This suggests that the observed increase in climate action participation since the late 1980s could represent a short term transitory dynamic, with potential declines in action levels still to come.

Proposition 3. *There exists a* $\gamma^* = \gamma^*(\beta_x, \mu)$ *such that for* $\mu < \beta_x$ *and* $\gamma > \gamma^*$*, two more steady states exist:* $(E, x) = (0, x^1)$ *and* $(E, x) = (0, x^2)$ *, with* $x^1, x^2 \in (0, 1)$ *for* $\mu > 0$ *and* $x^1, x^2 \in (-1, 0)$ *for* $\mu < 0$*, while for* $\gamma = \gamma^*$*,* $x^1 = x^2$*.*

Proposition 3 shows the possibility of more steady states where the level of participation is positive (negative) when the average bias μ is also positive (negative). These steady states exist when the peer pressure effect is stronger than average negative biases for climate action and the degree of heterogeneity is sufficiently low. Note that when the average bias towards action is positive ($\mu < 0$), then the peer pressure condition $\mu < \beta_x$ always holds. For $\mu < 0$ ($\mu > 0$), the steady state level of participation to climate action is such that the minority (majority) participates, however due to the non linear nature of the model we are not able to have information about the local stability.

We next estimate the parameters of the model to get insights regarding it's medium-run dynamics under different specifications.

3.2 Simulations

3.2.1 Baseline estimated version

First we estimate the baseline version assuming a fixed degree of heterogeneity. More specifically, without loss of generality we assume $\gamma = 1$. For α, we estimate equation (2) using a simple ARDL time series regression at the global level. $\hat{E}_{t+1}$ is the yearly growth rate in global annual net Greenhouse gas emissions in CO2 equivalents, measured using data from Our World In Data. As this dataset includes land-use emissions (which can be negative), technically $\hat{E}_{t+1}$ (our dependent variable) captures the change in the *net* GHG emissions at the global level. x_t is given by the number of countries that pass at least one climate mitigation law in a given year minus the number of countries that do not pass any laws in that year, divided by the total number of countries. Data on climate mitigation laws is from the Climate Laws of the World database.

We estimate equation (2) for the period from 1950 to 2019 using robust standard errors. The time period stops in 2019 in order to not include the sharp drop in emissions due to the COVID-19 shutdown and starts in 1950 to allow for enough observations (70) to estimate the ARDL equation. The results are presented in column 2 of table (1) in the appendix. The estimated parameter α is -0.022 and this is significant at the 1% significant level.

To estimate β_x, β_e, μ, we estimate equation (3) using panel data at the country level. We use a dummy variable that equals 1 if a country adopts a mitigation law in a given year and 0 if it does not adopt a law, again from the Climate Laws of the World database. This panel regression is estimated using a logit random effects panel estimator, in order to estimate a value for the constant (μ, the average bias). The regressions are estimated on data from 1980 to 2019. The results are presented in table (2). The estimated $\beta_x = 1.098$, $\beta_e = .181, \mu = 9.77$ and they are significant at the 10%, 1% and 1% level respectively.

We first present a validation exercise, which compares the model simulation of the two endogenous variables (E_t and x_t) with their actual observed values. We set the initial conditions to be their observed values in 1989. This is the year the IPCC became operationalised and therefore could be considered as a good starting point for global climate negotiations. Net emissions in 1989 were $E_0 = 36.8$ in billion metric tons of CO2 equivalent emissions and the relative share of countries taking action was $x_0 = -0.97$. Figure (4) shows the trajectory of the simulation (given initial conditions and estimated baseline parameters) and the observed values from 1989 to 2019. The simulated model aligns closely with the observed data, effectively capturing the dynamics of emissions and global climate action over the past three decades.

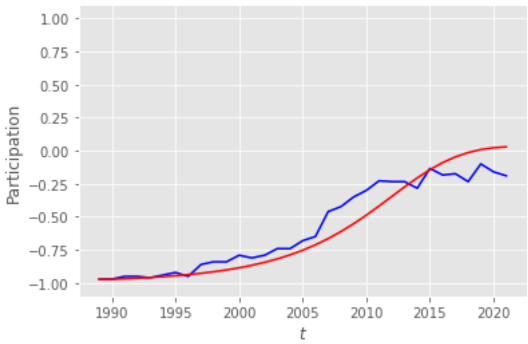
Next, figure (5) simulates the baseline model over the coming decades (until 2050). The model expects participation to convergence to a low level of participation ($x^* = 0$), corresponding to positive net emissions ($E^* \approx 54.4$). This outcome is interesting and relevant from a policy perspective as it shows that the increase in climate participation observed over the last three decades is not expected to continue over the next three decades. Participation will plateau and net zero emissions will not be reached. This result can be seen in Proposition 2: local asymptotic stability of $(E, x) = (\frac{\mu}{\beta_e}, 0)$ requires $\mu > 0$ and $\gamma < \frac{1}{\beta_x + \alpha\mu}$ which for the estimated parameter values becomes $1 < \frac{2}{1.098+0.02*9.77} = \frac{2}{1.2934}$ showing that the steady state is locally stable. Also note that $\bar{\gamma} = \frac{2\left[1.4908-\sqrt{(1.4908)^2-(1.098)^2}\right]}{(1.098)^2} = \frac{2(1.4908-\sqrt{2.2224-1.205604})}{1.205604} \approx 0.8 < \gamma = 1$, which implies that the steady state is also a spiral node.[11]

We now explore what happens to the dynamics of our endogenous variables when we adjust two key parameters: β_x and γ. As Propositions 2 and 3 show β_x and γ define to a great extent the dynamics of the model. Moreover, policy makers have some control over these parameters as they can either exerting more peer pressure on countries (β_x) and/or reducing global inequalities (γ).

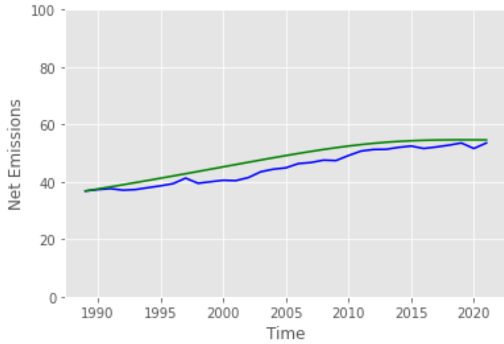
Figure (6) illustrates the impact of increasing the peer pressure effect (β_x). We consider three

[11] While the steady state is a spiral node for the estimated parameter values, this is not directly obvious from figure (5). To add clarity we present phase plots for different values of γ and β_x in the appendix.

Figure 4: Actual and Simulated x_t and E_t



(a) red line: Baseline Simulated Model, blue line: Actual Values



(b) green line: Baseline Simulated Model, blue line: Actual Values

Parameters for simulation: $E_0 = 36.8$, $x_0 = -0.97$, $\gamma = 1$, $\beta_x = 1.1$, $\beta_e = 0.18$, $\alpha = 0.02$, $\mu = 9.77$

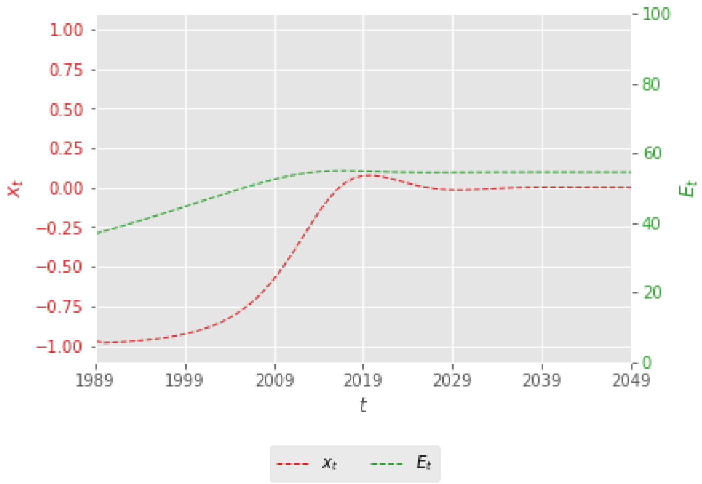
scenarios: the baseline ($\beta_x = 1.1$), blue line ($\beta_x = 3$) and ($\beta_x = 15$).[12] Increasing the peer pressure from the baseline to $\beta_x = 3$ (but not enough so that β_x is greater than μ), we observe clear oscillations. There is a shift from high levels of participation to high levels of abstention among countries. Concurrently, there is a pronounced decline in net emissions followed by a surge, without the attainment of a long-term equilibrium.

Increasing the peer pressure effect further to $\beta_x = 15$ (such that it is greater than μ) leads to all countries taking action and therefore a sustained decline in emissions (the black line). This is the only case where there is a sustained high level of action and the potential to reach net zero.

What this shows is that a very strong peer pressure effect is needed in order to get sustained participation. This is because there is a "fight" between peer pressure on the one hand and free riding on the other. In the high sustained participation case, the high peer pressure effect offsets the free-riding effect. Looking at equation (9) can help gain more insights regarding this balance. An increase in peer pressure β_x (with $\beta_x > \mu$) effectively offsets the (negative) impact of average idiosyncratic biases against action ($\mu > 0$) influencing countries' decisions. In the case with oscillations, the peer pressure is not enough to offset free riding. When emissions start to decline, due to an increase in climate action, countries will start to free ride, as there is not a strong enough peer pressure effect, leading to a turning point in participation.

[12]We use these scenarios of $\beta_x = 3$ and $\beta_x = 15$ to simulate two cases where β_x is smaller and larger than μ as discussed in proposition 3. See the Phase plots for different values of β_x in the Appendix.

Figure 5: Participation and Emissions with estimated parameters and exogenous γ



Parameters: $E_0 = 36.8$, $x_0 = -0.97$, $\gamma = 1$, $\beta_x = 1.1$, $\beta_e = 0.18$, $\alpha = 0.02$, $\mu = 9.77$

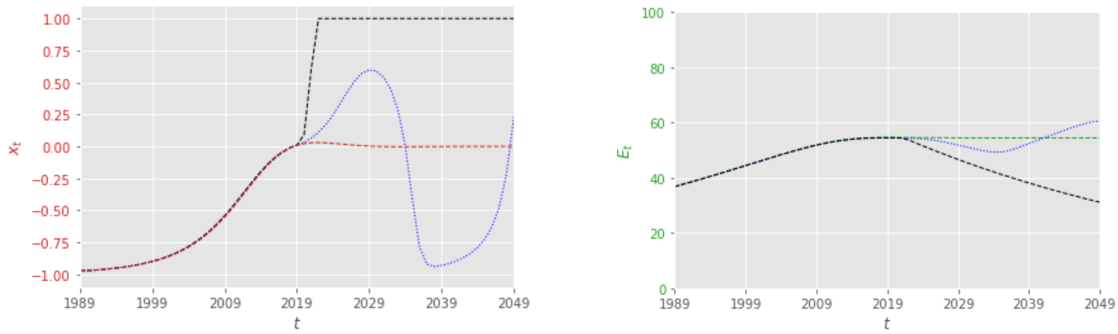
Lastly, Figure (7) shows how the simulation changes when we decrease the degree of heterogeneity (i.e. increasing γ), keeping the other baseline parameters and initial conditions constant. We consider three scenarios: the baseline ($\gamma = 1$), blue line ($\gamma = 3$) and the black line ($\gamma = 15$). If we increase gamma (reduce inequality) from the baseline to $\gamma = 3$, we also observe oscillations in participation - and by extension emissions. Reducing inequality further to $\gamma = 15$ increases the amplitude of these oscillations but does not lead to a situation where participation is sustained at high levels. If countries are more equal, their behaviour is similar. They move together in the same direction. Either they all participate together or, due to declining emissions and the free riding effect, they all decide not participate.[13]

Does the degree of heterogeneity not therefore matter in achieving sustained global climate action, and therefore net zero? To answer this question, figure (8) presents a bifurcation diagram which shows the stability (and instability) of x_t for different values for the degree of heterogeneity (bifurcation parameter). We set $\beta_x = 15$ as this was the only case where sustained participation is possible, as discussed above in figure (6).

Consistent with the analytical results and our simulations, the bifurcation shows that relatively high heterogeneity (low γ) for a given level of peer pressure leads to stability of the steady state with $x_t = 0$, as indicated by the straight line observed in the beginning of the diagram. Increasing γ leads initially to

[13]Technically there is less dispersion in the logistic distribution, so all countries are close to the mean.

Figure 6: Evolution of x_t and E_t for different values of β_x



(a) red line: Baseline Model

(b) green line: Baseline Model

Note: after 2019 $\beta_x = 3$ (Blue line) and $\beta_x = 15$ (Black line)
Parameters: $E_0 = 36.8$, $x_0 = -0.97$, $\beta_e = 0.18$, $\gamma = 1$, $\alpha = 0.02$, $\delta = 0$, $\mu = 9.77$

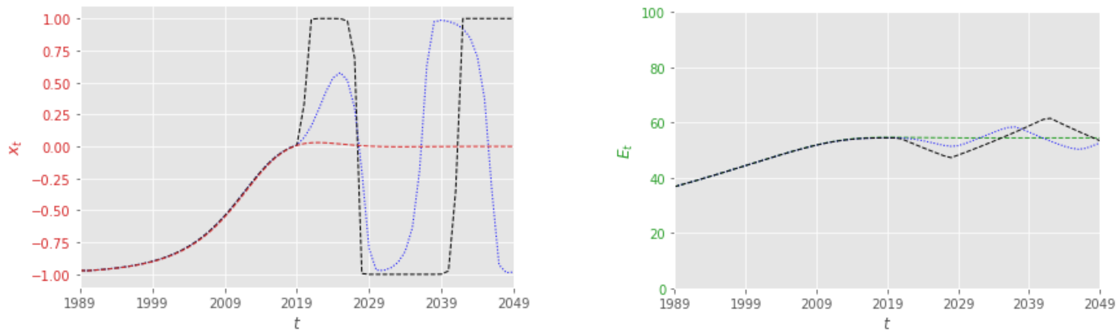
cyclical dynamics corresponding to the light red area in the diagram. The more pronounced red areas on the top and bottom of the diagram for values of γ around 2 to a bit higher than 8, show that the cyclical dynamics can be such that for example a short period of high participation which might appear as stable can be followed by a sharp decrease. Finally we observe stability of the high participation steady state for high values of γ.

This shows that heterogeneity among countries must be low (i.e. γ greater than around 0.75) in order to achieve the aim of net zero emissions. In other words, we need both a strong peer pressure effect and a low degree of heterogeneity in order to achieve sustained global climate action.

3.2.2 IPE-IAM

Up to now we treated the degree of heterogeneity, captured by γ, as exogenous. This has allowed us to gain intuition regarding the effects of heterogeneity on participation to climate action and net zero. However, as both damages and the capacity to deal with these varies greatly across countries, it is likely that more global emissions will increase the degree of heterogeneity (i.e. a reduction in γ). These effects could imply that an endogenous degree of heterogeneity may make it harder to reach high levels of climate action. However, given the nonlinear dynamics observed above and a further introduction of nonlinearities related to the carbon cycle and climate damages, it is far from obvious whether new possibilities might emerge.

Figure 7: Evolution of x_t and E_t for different values of γ



(a) red line: Baseline Model

(b) green line: Baseline Model

Note: after 2019 $\gamma = 3$ (Blue line) and $\gamma = 15$ (Black line)
Parameters: $E_0 = 36.8$, $x_0 = -0.97$, $\beta_x = 1.1$, $\beta_e = 0.18$, $\alpha = 0.02$, $\delta = 0$, $\mu = 9.77$

In our IPE-IAM, the degree of heterogeneity evolves according to equation (4) which depends on climate damages (Ω_t). Regarding the specific functional form of damages we consider two cases which are widely used in related literature. The first is according to DICE2013 (Nordhaus, 2014)

$$\Omega_t = 1 - \frac{1}{1 + 0.0022(T_t^{AT})^2}, \tag{10}$$

and the second follows Weitzman (2012):

$$\Omega_t = 1 - \frac{1}{1 + (T_t^{AT}/20.46)^2 + (T_t^{AT}/6.081)^{6.754}}. \tag{11}$$

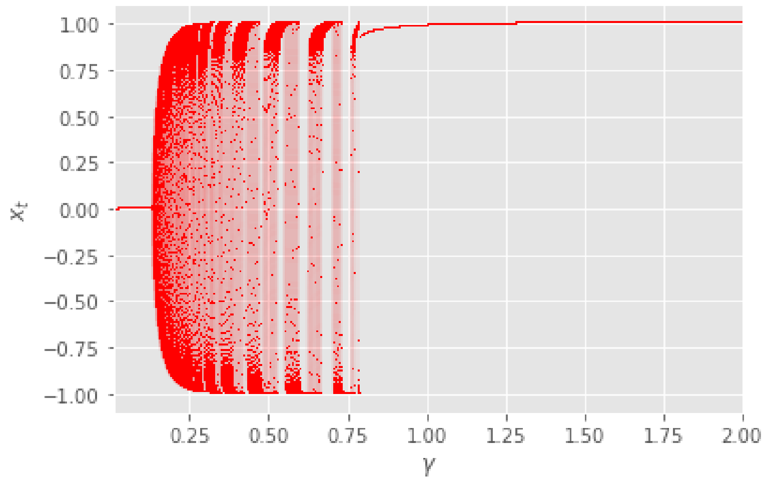
In both cases T_t^{AT} is the atmospheric temperature at t and is given by the *carbon cycle*[14].

In figure (9), we present the dynamics of climate action of the IPE-IAM for the two different damage functions above and we compare these dynamics with the baseline version for the estimated values showed above.

We note in figure (9)a that the dynamics for the estimated parameter values for the two versions of the IPE-IAM are almost indistinguishable from the dynamics of the baseline version. There are only

[14]For the equations and the values of the parameters used, please see Appendix.

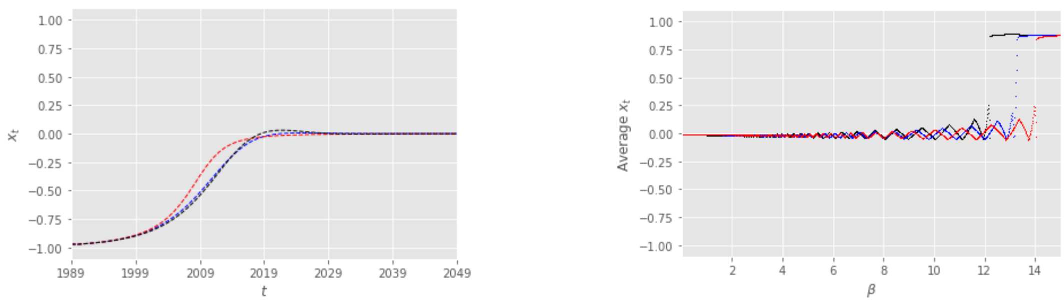
Figure 8: Bifurcation Diagram of x_t for γ, with $\beta_x > \mu$



Parameters: $E_0 = 36.8$, $x_0 = -0.97$, $\beta_x = 15$, $\beta_e = 0.18$, $\alpha = 0.02$, $\delta = 0$, $\mu = 9.77$

small differences in reaching the long-run steady state. This shows that the intuition of the baseline model also holds here, as such the baseline version is a good approximation of the IPE-IAM.

Figure 9: Climate action and damages



(a) $\beta_x = 1.1$ and $\gamma = 1$

(b) for different values of β_x and $\gamma = 2$

Black Line: Baseline Model ($\delta = 0$); Blue line: DICE ($\delta = 0.99$); Red line: Weitzman ($\delta = 0.99$). Parameters: $E_0 = 36.8$, $x_0 = -0.97$, $\beta_e = 0.18$, $\alpha = 0.02$, $\mu = 9.77$.

We next compare the IPE-IAM with the baseline model regarding the feasibility reaching the high level global action steady state. Figure (9)b depicts the average participation for varying values of β_x with damages using the DICE function (blue line) and the Weitzman specification (red line) and without damages influencing the degree of heterogeneity (black line).[15] Just like above, convergence to the high levels of action steady state requires higher peer pressure when the effects of damages are taken into account. However, what is interesting is that introducing damages requires a higher peer pressure effect to reach thie high level of action steady state. Without damages, $\beta_x = 12$ is sufficient, while with the DICE and Weitzman damage function $\beta_x = 13$ and $\beta_x = 14$ is required respectively. In other words, introducing more damages makes it harder to reach the steady state (i.e. there must be more peer pressure).

On the one hand this result is surprising as one might expect damages to incentivise countries to take more action in order to avoid future costs. In fact the opposite seems to be occurring. This is because damages influence the degree of heterogeneity. By increasing the degree of heterogeneity (i.e. reducing γ), damages increase the number of countries who are likely to not take action in any case. To compensate against this, a higher peer pressure effect is required to achieve high sustained levels of global action.

3.3 IPE-IAM with 200 countries

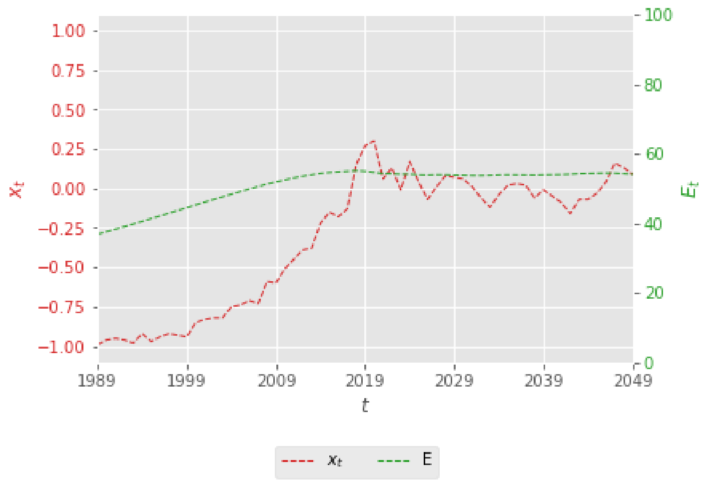
In order to have analytical results in the baseline version of the IPE-IAM, we assumed a *large* population of countries which was implicitly approximated by a continuum. For robustness, we now consider a setup where the world economy is composed by $N = 200$ countries.[16] In this case, for each country i, the idiosyncratic impact on action, captured by ϵ_t^i is drawn by a logistic distribution with the same assumptions about its scale parameter as outlined above. Figure (10) shows the global action and emission simulated (single run) dynamics for 200 countries, using the estimated parameter values, without taking into account the effects of damages on heterogeneity.

Note that assuming a discrete number of countries does not qualitatively alter the results shown in Figure (5). The only noticeable difference is that the dynamics become less smooth. This is because, with discrete countries, the idiosyncratic characteristics can no longer be approximated as before and instead manifest as shocks at each time step, as outlined in Section 2. In order to further check whether

[15]Note this is similar to the bifurcation diagram above, but showing the average value of x_t rather than all values of x_t to make it comprehensible.

[16]We assume that those countries are of the same "size" in both economic and political terms. We leave the case of heterogeneous size for further analysis.

Figure 10: Evolution of Participation and Net Emissions with 200 countries and constant γ_t



Parameters: $N = 200$, $\beta_x = 1.1$, $\beta_e = 0.18$, $\alpha = 0.02$, $\mu = 9.77$, $E_0 = 36.8$, $\gamma = 1$

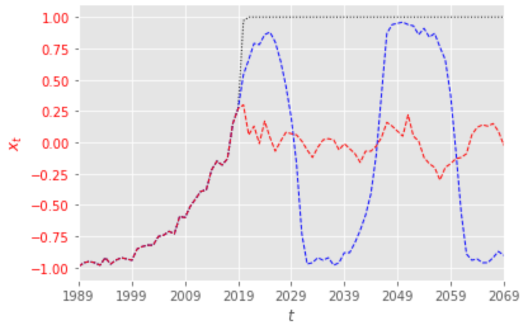
the discrete number of countries has a qualitative impact on the results regarding different outcomes, in figure (11), we re-run the model for different values of β_x.

Also in this case, the behaviour is qualitative similar to what we find with a large number of countries (see figure (6)). In order to prompt the majority of countries to implement measures to combat climate change and achieve a significant reduction in net emissions, it is essential that β_x is large enough. If this is not the case, following an initial surge in participation, the substantial reduction in net emissions gives rise to the free-riding effect. This consequently leads to a consistent reduction in the number of acting countries and prevents the zero-net-emissions goal from being achieved.

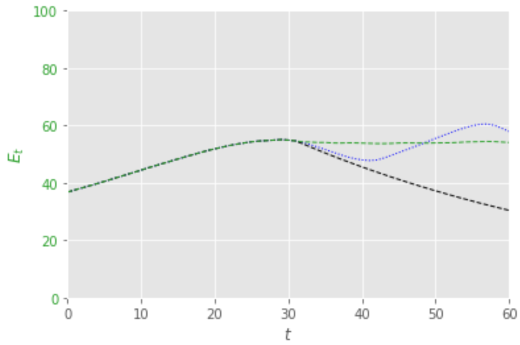
Finally, Figure (12) presents the time series of relative participation x_t for 200 countries. The chart compares three different model versions: the baseline model, represented by the black line; the IPE-IAM using the DICE damage function, shown by the blue line; and the IPE-IAM employing the Weitzman damage function, depicted by the red line.

When comparing this to Figure (9)a, it becomes evident that the results remain robust when instead of a continuum, 200 countries are assumed. As expected, when we have a discrete number of countries, the differences between the baseline version and the versions with endogenous degree of heterogeneity are less obvious. The graphs collectively demonstrate that the core insights derived from the model hold

Figure 11: Evolution of Participation and Net Emissions with constant γ_t and different β_x



(a) red line: Baseline Model



(b) green line: Baseline Model

Note: after 2019 $\beta_x = 3$ (Blue line) and $\beta_x = 15$ (Black line).
Parameters: $E_0 = 36.8$, $\beta_e = 0.18$, $\alpha = 0.02$, $\mu = 9.77$, $\gamma_t = 1$

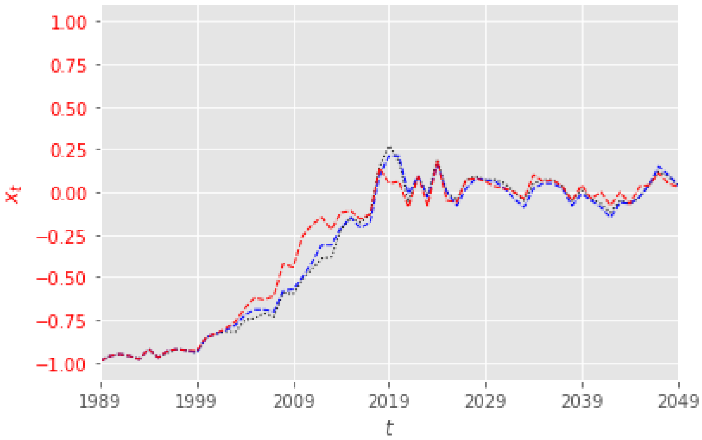
true, despite reducing the number of countries from a large, unspecified value to 200. This suggests that the model's conclusions are not significantly affected by the number of countries involved, maintaining consistency in its predictions.

4 Conclusion

Over the past decades an increasing number of countries have taken climate action, from implementing domestic mitigation laws to signing quantifiable binding targets at international climate agreements. Despite these collective efforts, they are not sufficient to prevent global temperatures from rising. As we face an increasingly warmer world, there is no assurance that countries will continue to pursue more ambitious climate actions, especially given the growing complexity of geopolitical and economic crises. These crises can divert attention and resources away from climate commitments, as governments are likely to prioritise immediate political stability and economic recovery over long-term environmental goals. To effectively mitigate against climate change, we need a better understanding of the conditions under which most countries will adopt sustained levels of global action.

In this paper, we developed an IPE-IAM to analyse the relative importance of different factors which are empirically known to influence countries' decisions regarding climate action. The key motivating

Figure 12: Evolution of Participation with 200 countries and endogenous γ_t



Black Line: Baseline Model ($\gamma_t = 1$); Blue line: DICE ($\delta = 0.99$); Red line: Weitzman ($\delta = 0.99$). Parameters: $N = 200$, $\beta_x = 1.1$, $\beta_e = 0.18$, $\alpha = 0.02$, $\mu = 9.77$, $E_0 = 36.8$

issue that we set to address here has been the fact that countries globally have heterogeneous climate action preferences due to the different idiosyncratic factors. While this heterogeneity is discussed in empirical works on climate action, its role in shaping overall global climate action has not yet been analysed until now. Our model shows that incorporating this missing aspect is crucial for understanding the different possibilities of global action and emissions dynamics.

The outcomes of the model depend on the relative importance of the various factors influencing decisions and to a great extent on the degree of heterogeneity across countries preferences. A key finding is that when the degree of heterogeneity is high, more countries with strong preferences against taking action are likely to abstain as net emissions near zero, compared to when heterogeneity is low. Since the effects of global efforts to reduce emissions take time to manifest, this decline in participation weakens peer pressure quickly, rather than immediately reducing emissions. This weakened peer pressure from countries with idiosyncratic preferences against action will result in more countries abstaining, further reducing the collective peer pressure. As a result, before net emissions increase enough to reverse this trend, peer pressure will instead shift toward further abstention from action.

Using available data to estimate the parameters in model's behavioural equations and set initial conditions, we note that the expected outcome is one where participation stops increasing and global net emissions continue to remain positive. In other words, globally we fail to make it to net zero emissions. While the model does a good job at reproducing the observed increase of participation in climate action seen over the last decades, action is expected to decline over the coming decades. To achieve high levels of global participation in emissions reduction efforts, two key conditions must be met: relatively low heterogeneity among countries and strong peer pressure. Our analysis demonstrates that these conditions are both necessary and sufficient to counteract the average negative idiosyncratic factors that act as barriers to climate action across countries. Low heterogeneity implies that countries have more aligned interests and face similar costs and constraints, making collective action through peer pressure more feasible.

The insights of our model have direct policy implications. Since the degree of heterogeneity in climate action preferences is shaped, in part, by global inequalities, our findings underscore the importance of addressing these disparities not only from a normative perspective but also from a practical standpoint. Inequalities can directly impact the ability of countries to participate in coordinated climate efforts and indirectly their willingness. Additionally, while increasing peer pressure is crucial to encourage stronger climate commitments, the timing of these efforts is equally important. Climate damages disproportionately affect countries, exacerbating existing inequalities, increasing heterogeneity over time, making it more challenging to achieve a high level of global action in the future.

Since the primary goal of this work was to develop an IPE-IAM focused on analysing how heterogeneity influences climate action dynamics, we have deliberately simplified certain aspects that would be crucial for exploring other research questions. For instance, we have implicitly assumed a fully connected network structure, where the climate action decisions of one country exert an equal influence on all other countries. In reality, however, countries are interconnected in diverse ways due to various geographical, economic, and political factors. Considering these complex global political and economic network structures, along with explicitly incorporating the varying sizes of countries—not only in terms of their emissions impact but also their relative economic and political power—offers promising avenues for future research. Such an approach would allow for a more nuanced understanding of how the interconnectedness and influence of different countries shape global climate action and policy dynamics.

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Appendix

Carbon Cycle

$$M_t^{AT} = Stock_t^E + \phi_{11} M_{t-1}^{AT} + \phi_{21} M_{t-1}^{UP}, \tag{12}$$

$$M_t^{UP} = \phi_{12} M_{t-1}^{AT} + \phi_{22} M_{t-1}^{UP} + \phi_{32} M_{t-1}^{LO}, \tag{13}$$

$$M_t^{LO} = \phi_{23} M_{t-1}^{AT} + \phi_{33} M_{t-1}^{LO}, \tag{14}$$

where M_t^{AT}, M_t^{UP} and M_t^{LO} correspond to the mass of carbon in reservoir for atmosphere, upper oceans and lower oceans respectively. And $Stock_t^E$ is the stock of emissions.

$$F_t = \eta\{log_2[M_t^{AT}/M_p^{AT}]\} + F_t^{EX}, \tag{15}$$

where F_t is the total radiative forcing and F_t^{EX} is the exogenous radiative forcing increasing at a rate f per unit of time, given by

$$F_t^{EX} = F_{t-1}^{EX} + f \tag{16}$$

$$T_t^{AT} = T_{t-1}^{AT} + \xi_1\{F_t - \xi_2 T_{t-1}^{AT} - \xi_3[T_{t-1}^{AT} - T_{t-1}^{LO}]\}, \tag{17}$$

where T_t^{LO} is the temperature of the lower oceans.

$$T_t^{LO} = T_{t-1}^{LO} + \xi_4[T_{t-1}^{AT} - T_{t-1}^{LO}] \tag{18}$$

To run the carbon cycle model, we use the following initial conditions and parameters:

$M_0^{AT} = 3120$, $M_0^{UP} = 5628.8$, $M_0^{LO} = 36706.7$, $F_0^{EX} = 0.28$, $T_0^{AT} = 1$, $T_0^{LO} = 0.0068$, $Stock_0^E = 1730$, $f = 0.005$, $\eta = 3.8$, $\xi_1 = 0.027$, $\xi_2 = \eta/3$, $\xi_3 = 0.018$, $\xi_4 = 0.005$, $\phi_1 1 = 0.9817$, $\phi_2 1 = 0.0080$, $\phi_1 2 = 0.0183$, $\phi_2 2 = 0.9915$, $\phi_2 3 = 0.0005$, $\phi_3 2 = 0.0001$, $\phi_3 3 = 0.9999$, $M_p^{AT} = 2156.2$.

Estimation results for parameter values

The dependent variable is the yearly growth rate of GHG emissions. The first column estimates the equation without a constant to demonstrate its insignificance, as discussed above.The second column estimates equation (2).

Table 1: Estimating α

	(1)	(2)
Relative Participation (t-1)	-0.014**	-0.022***
	(0.006)	(0.003)
Constant	0.008	
	(0.005)	
R^2	0.042	0.511
N	70	70

Notes: Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

To estimate β_x, β_e , μ, we estimate equation (3) using country level panel data. π_t^i is a dummy variable that equals 1 if a country adopts a mitigation law in a given year and 0 if it doesn't adopt a law using the Climate Change Laws Database . x_t is constructed using the method above and E_t is the global level of net GHG in billion metric tons of C02 equivalent using the Our World In Data dataset. This panel regression is estimated using a logit random effects panel estimator, given that the dependent variable is binary and the constant needs to be estimated (which a fixed effects estimator does not allow for). The regressions are estimated on data from 1980 to 2019. (**??**).

Data and summary statistics

Table (3) sets out an overview of all the data used in these estimations and the introductory empirical motivation for the paper. Table (4) presents the summary statistics for the regressions to estimate α. Table (5) presents the summary statistics for the other regression estimation.

The parameter values are:

- $\alpha = 0.02$

Table 2: Estimating β_x, β_e , ϵ_t^i

	(1)
Relative Participation (t)	1.104**
	(0.542)
Net Emissions (t)	0.177***
	(0.029)
Idiosyncratic Factors (t)	-9.777***
	(1.591)
N	7772

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Data sources

Name	Source
Vulnerability to climate damages	ND-GAIN Country Index
GDP per capita	World Bank in current USD
Fossil fuels rents/GDP	World Bank
Control of corruption index	Worldwide Governance Indicators
Relative number of mitigation laws x_t	Climate Change Laws Database
Signatures of key climate deals (% in each region)	Tenreyro and de Silva (2021)
Annual GHG emissions in CO2 equivalents GHG	Our World In Data

- $\beta_x = 1.1$
- $\beta_e = 0.18$
- $\mu = 9.77$

Table 4: Summary statistics for equation 2

	mean	p50	min	max	sd
Growth rate of Emissions (t)	.018688	.0179086	-.0436626	.0787522	.0188763
Relative Participation (t)	-.8220485	-.9899498	-1	-.1	.2868757
Observations	70				

Table 5: Summary statistics for equation 3

	mean	p50	min	max	sd
Payoffs	.1556557	0	0	1	.3625513
Relative Participation	-.6886887	-.81	-1	-.1	.3183948
Net Emissions	42.24234	40.54943	32.15487	53.5514	6.795554
Observations	7992				

Proofs

Proposition 1

Note that $\hat{E}_{t+1} = 0$ for $x_t = 0$ or $E_t = 0$.

(i) If $x_t = x_{t+1} = 0$, then $\frac{e^{\gamma(\beta_e - \mu)} - 1}{1 + e^{\gamma(\beta_e - \mu)}} = 0$ for $E = \frac{\mu}{\beta_e}$.

(ii) Note that $-1 < \frac{e^{\gamma(\beta_x x - \mu)} - 1}{1 + e^{\gamma(\beta_x x - \mu)}} < 1$ is increasing in $x \in [-1, 1]$. From continuity and Bolzano's theorem there exists a $x' \in (-1, 1)$: $\frac{e^{\gamma(\beta_x x' - \mu)} - 1}{1 + e^{\gamma(\beta_x x' - \mu)}} = x'$. Note that $\frac{e^{-\gamma\mu} - 1}{1 + e^{-\gamma m u}} > 0$ for $\mu < 0$ and $\frac{e^{-\gamma\mu} - 1}{1 + e^{-\gamma m u}} < 0$ for $\mu > 0$. This implies that $x' > 0$ for $\mu < 0$ and $x' < 0$ for $\mu > 0$.

Proposition 2

Writing the evolution of E_t as

$$E_{t+1} = E_t - E_t \alpha x_t$$

the general form of the Jacobian of (2) and (9) is given by

$$J(E_t, x_t) = \begin{bmatrix} 1 - \alpha x_t & -\alpha E_t \\ 2\gamma\beta_e e^{\gamma(\beta_x x_t + \beta_e E_t - \mu)}/(1 + e^{\gamma(\beta_x x_t + \beta_e E_t - \mu)})^2 & 2\gamma\beta_x e^{\gamma(\beta_x x_t + \beta_e E_t - \mu)}/(1 + e^{\gamma\beta(\beta_x x_t + \beta_e E_t - \mu)})^2 \end{bmatrix}$$

At $(E, x) = (\frac{\mu}{\beta_e}, 0)$, $e^{\gamma(\beta_x x_t + \beta_e E_t - \mu)} = e^0 = 1$ the Jacobian becomes

$$J(\frac{\mu}{\beta_e}, 0) = \begin{bmatrix} 1 & -\frac{\alpha\mu}{\beta_e} \\ \gamma\beta_e/2 & \gamma\beta_x/2 \end{bmatrix}$$

with

$$Tr(J) = 1 + \gamma\beta_x/2 > 0$$

and

$$Det(J) = \gamma\beta_x/2 + \alpha\mu\gamma/2$$

For stability $|Tr(J)| < |Det(J)| + 1 < 2$, or

1. $1 + Tr(J) + Det(J) > 0$
2. $1 - Tr(J) + Det(J) > 0$
3. $Det(J) < 1$,

As $Tr(J) > 0$, the first condition is redundant as if the second holds, so will the first. The second condition implies that $1 + Det(J) > Tr(J)$, or

$$1 + \gamma\beta_x/2 + \alpha\mu\gamma/2 > \gamma\beta_x/2 + 1$$

or

$$\alpha\mu\gamma/2 > 0$$

which is true only if $\mu > 0$. The third condition requires

$$\gamma\beta_x/2 + \alpha\mu\gamma/2 < 1$$

or

$$\gamma\beta_x + \alpha\mu\gamma < 2$$

or

$$\gamma(\beta_x + \alpha\mu) < 2$$

For $\mu > 0$, the stability condition is

$$\gamma < \frac{2}{\beta_x + \alpha\mu}$$

The steady state is spiral node if for $\gamma < \frac{2}{\beta_x+\alpha\mu}$, also $Tr^2 < 4Det$. The last is true when

$$(1 + \gamma\beta_x/2)^2 < 2\gamma\beta_x + 2\alpha\mu\gamma,$$

or

$$(1 - \gamma\beta_x/2)^2 < 2\alpha\mu\gamma,$$

or

$$1 - \gamma(\beta_x + 2\alpha\mu) + (\gamma\beta_x)^2/4 < 0$$

which implies that γ should be between the solutions of the above, which are given by

$$\gamma = \frac{2\left(\beta_x + 2\alpha\mu \pm \sqrt{(\beta_x + 2\alpha\mu)^2 - \beta_x^2}\right)}{\beta_x^2} \tag{19}$$

or

$$\gamma = \frac{2\left(\beta_x + 2\alpha\mu \pm \sqrt{4\alpha\mu(\beta_x + \alpha\mu)}\right)}{\beta_x^2}$$

We need to prove that given that $\gamma < \frac{2}{\beta_x+\alpha\mu}$ the following should also hold:

1.

$$\frac{2}{\beta_x + \alpha\mu} < \frac{2\left(\beta_x + 2\alpha\mu + 2\sqrt{\alpha\mu(\beta_x + \alpha\mu)}\right)}{\beta_x^2}$$

or

$$\frac{1}{\beta_x + \alpha\mu} < \frac{\beta_x + 2\alpha\mu + 2\sqrt{\alpha\mu(\beta_x + \alpha\mu)}}{\beta_x^2}$$

or

$$\beta_x^2 < (\beta_x + \alpha\mu)^2 + 2(\beta_x + \alpha\mu)\sqrt{\alpha\mu(\beta_x + \alpha\mu)} + \alpha\mu(\beta_x + \alpha\mu)$$

which is always true.

2.

$$\beta_x^2 > (\beta_x + \alpha\mu)^2 - 2(\beta_x + \alpha\mu)\sqrt{\alpha\mu(\beta_x + \alpha\mu)} + \alpha\mu(\beta_x + \alpha\mu)$$

or

$$3\beta_x\alpha\mu + 2(\alpha\mu)^2 - 2(\beta_x + \alpha\mu)\sqrt{\alpha\mu(\beta_x + \alpha\mu)} < 0$$

or

$$4(\beta_x + \alpha\mu)^2[\alpha\mu\beta_x + (\alpha\mu)^2] > [3\beta_x\alpha\mu + 2(\alpha\mu)^2]^2$$

or

$$4(\beta_x + \alpha\mu)^2 > 3\beta_x\alpha\mu + 2(\alpha\mu)^2$$

or

$$4\beta_x^2 + 8\beta_x\alpha\mu + 4(\alpha\mu)^2 > 3\beta_x\alpha\mu + 2(\alpha\mu)^2$$

or

$$4\beta_x^2 + 5\beta_x\alpha\mu + 2(\alpha\mu)^2 > 0$$

which is true.

□

Proposition 3

For $E = 0$, we consider two cases regarding μ:

First case: $\mu > 0$

In this case the midpoint ($x' = \frac{\mu}{\beta_x}$) of the sigmoid $F(x) = \frac{e^{\gamma(\beta_x x - \mu)} - 1}{1 + e^{\gamma(\beta_x x' - \mu)}}$ is positive which implies that if there exist three steady states, two of these will be for $x' > 0$. Consider two further cases:

- $\mu > \beta_x$. In this case $x = F(x)$ only for some $x' < 0$, hence only one steady state can exist.

- $\mu < \beta_x$. To prove that there exist two values of $x > 0$ for which $x = F(x)$, it is sufficient to prove that for some $x > 0$, $F(x) > x$, or

$$\frac{e^{\gamma(\beta_x x' - \mu)} - 1}{1 + e^{\gamma(\beta_x x' - \mu)}} > x. \tag{20}$$

For $x > 0$, after taking the natural logarithms in both sides and some rearrangement, (20) can be equivalently expressed as

$$\gamma\beta_x x > ln(x+1) - \ln(1-x) + \gamma\mu. \tag{21}$$

Note that the RHS of (21) is a strictly increasing and convex function of x, equal to $\gamma\mu$ for $x = 0$. This implies that there exists a value for $\gamma\beta_x$, call this c for which the LHS of (21) is tangent to the RHS of (21). Then for values higher than c, (21) holds. At $\gamma\beta_x = c$ the derivatives of the two sides of the inequality should be equal:

$$c = \frac{1}{1+x} + \frac{1}{1-x}, \tag{22}$$

which after some rearrangement and given that $x > 0$ gives the point at which the two lines are tangent:

$$x = \sqrt{1 - \frac{2}{c}}, \tag{23}$$

which is also the value of x for which

$$\gamma\beta_x x = ln(x+1) - \ln(1-x) + \gamma\mu. \tag{24}$$

Note that as $x > 0$ for (23) to be able to hold,

$$1 > \frac{2}{c}$$

which implies that γ should be such that

$$\gamma > \frac{2}{\beta_x}, \tag{25}$$

which in turn implies that for $\gamma < \frac{2}{\beta_x}$, $F(x)$ crosses x only for $x < 0$.

Substituting $x = \sqrt{1 - \frac{2}{\gamma\beta_x}}$ gives the value of γ given β_x and μ for which $F(x)$ is tangent to x for $x > 0$. Call this value of $\gamma(\beta_x, \mu)$, γ^*. Then γ^* is the solution of

$$\gamma\beta_x\sqrt{1 - \frac{2}{\gamma\beta_x}} = ln\left(1 + \sqrt{1 - \frac{2}{\gamma\beta_x}}\right) - \ln\left(1 - \sqrt{1 - \frac{2}{\gamma\beta_x}}\right) + \gamma\mu. \tag{26}$$

Then for $\gamma > \gamma^* > \frac{2}{\beta_x}$, there exist three values of x, (two positive and one negative) for which (**??**) holds.

Second case: $\mu < 0$

Here, the midpoint ($x' = \frac{\mu}{\beta_x}$) of the sigmoid $F(x) = \frac{e^{\gamma(\beta_x x - \mu)} - 1}{1 + e^{\gamma(\beta_x x' - \mu)}}$ is negative which implies that if there exist three steady states, two of these will be for $x' < 0$. Note that in this case μ is always smaller than $\beta_x < 0$. As was the case above, in order for three steady states to exist, there must be values of $x < 0$ for which

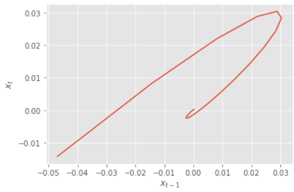
$$\frac{e^{\gamma(\beta_x x - \mu)} - 1}{1 + e^{\gamma(\beta_x x - \mu)}} < x. \tag{27}$$

which is equivalent to (20). Hence for three equilibria to exist we need $\mu < \beta_x$ and $\gamma > \gamma^*$.

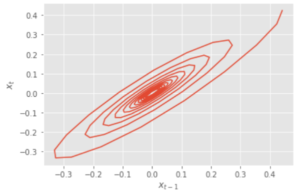
□

Phase Plots

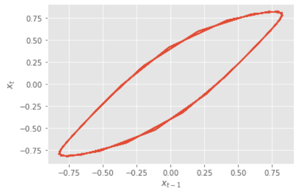
Figure 13: Phase Plot of x_t for different values of γ



(a) $\gamma = 1$



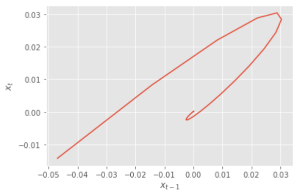
(b) $\gamma = 1.5$



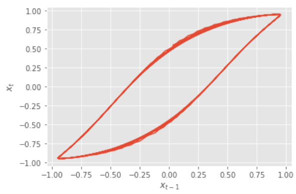
(c) $\gamma = 2$

Parameters: $E_0 = 36.8$, $x_0 = -0.97$, $\beta_x = 1.1$, $\beta_e = 0.18$, $\alpha = 0.02$, $\mu = 9.77$

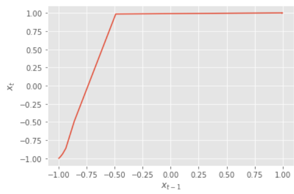
Figure 14: Phase Plot of x_t for different values of β_x



(a) $\beta_x = 1.1$



(b) $\beta_x = 3$



(c) $\beta_x = 15$

Parameters: $E_0 = 36.8$, $x_0 = -0.97$, $\gamma = 1$, $\beta_e = 0.18$, $\alpha = 0.02$, $\mu = 9.77$