The Global Political Economy of a Green Transition

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Abstract

From uneven development to unequal exposure to extreme weather events, the economic geography of climate change implies substantial heterogeneity regarding countries’ preferences for climate action. Yet, how this heterogeneity matters for sustaining high and effective levels of global climate action has not been analysed. This paper develops a novel geographical political economy model of climate action where countries’ choices are influenced by the evolution of greenhouse gas emissions, total participation in climate action and by the dispersion of economic geographical factors. Our results highlight that uneven geographic development can be a barrier to sustained high levels of global action.
1 Introduction

Since the late 1980s, countries across the world have taken steady but insufficient action to stop climate change. Current mitigation pathways put the world on a turbulent path to exceed 1.5°C by the 2030s (IPCC, 2022), despite increasing numbers of countries signing up to international climate agreements and implementing national laws to reduce emissions (Tenreyro and de Silva, 2021). However, not all countries face the same costs and preferences for taking climate action as can be seen in Figure 1 - due in part to several factors related to the economic geography (EG) of climate change including uneven development and the spatial distribution of climate damages (Peri and Robert-Nicoud, 2021). This raises two crucial research questions. Firstly, how does uneven development and other economic geographical factors influence global climate action? Secondly, which conditions, if any, leads to sustained high levels of global action and an effective low carbon transition?

Figure 1: Signatures of key climate deals (% in each region)

(a) Notes: Data from Tenreyro and de Silva (2021). Region abbreviations: A=Asia; E=Europe; EE=Eastern Europe; MENA=North Africa & the Middle East; SSA=Sub-Saharan Africa; LA=Latin America & the Caribbean; O=Oceania; NA=Northern America

This paper tackles these questions by building a novel geographical political economy (GPE) model
that brings together economic geography and global political economy insights regarding countries’ climate actions. The foundations of our modelling framework rest upon the bounded rational, heterogeneous agents models along the lines of Lux (1995) and Brock and Hommes (1997) among others. We have used this modelling approach for two reasons. Firstly, it allows us to model the impact of economic geographical factors, such as uneven development, on the dynamics of global climate action. Secondly, taking inspiration from different paradigms within GPE (for example see Sheppard, 2001; Plummer and Sheppard, 2006; Martin and Sunley, 2007; Bergmann, Sheppard and Plummer, 2009; and MacKinnon et al., 2009, among others), we use a framework which allows us to model out of equilibrium dynamics and the emergence of complex phenomena, both of which are relevant to understand the evolution of climate action.

In our model, each country’s preference for taking action depends on two types of factors: global (which are the same across countries) and idiosyncratic economic geographical (which vary from country to country). The global factors are: (i) the growth rate of the stock of greenhouse gas (GHG) emissions, which acts as a proxy for future expected damages due to climate change and (ii) the actions of other countries, which lowers the potential costs of action for each country and increases peer pressure effects. Drawing on relevant empirical literatures, we focus on four key economic geographical factors: uneven development influencing the relative costs of mitigation; unequal vulnerability to climate change, which in turn is an outcome of the geographical distribution of climate damages and uneven development; the concentration of extractive fossil fuel industries, which reflect the geographical distribution of fossil fuel deposits; and (the type of) political institutions.

We develop our model in two steps. First, we empirically analyse the distribution of the economic geographical factors across countries. Based on our analysis we note that there is not a simple North-South divide regarding the influence of the different factors on countries’ choices for climate action, but rather, if we take all four economic geographical factors together we observe a unimodal and relatively symmetric distribution. Moreover, we note that the relative importance of the economic geographical

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1 We leave the precise definition of climate action undefined in the paper for generality. However, two definitions of discrete climate action that are shown to effectively reduce emissions are (Tenreyro and de Silva, 2021): (i) whether countries have national laws adopting carbon taxes and/or Emission Trading Schemes; (ii) whether countries sign a major international climate agreement with quantified targets.

2 For simplicity we refer to these “idiosyncratic economic geographical factors” as “economic geographical”.

factors may change across countries over time. Second, based on these insights, we develop an evolutionary model where countries’ choices for action are influenced by (i), (ii) and also a measure of the variation of the economic geographical factors, which we call economic geographical dispersion or simply just dispersion. Not taking into account the economic geographical factors (or equivalently assuming zero dispersion) would correspond to the case where all countries are assumed to be identical. The global factors (i) and (ii) are endogenous in our framework and we focus on how dispersion and the relative importance between (i) and (ii) influence the dynamics of global climate action.

As is often the case in works within the broader family of complexity economics, our model gives rise to various qualitative behaviours characterising the dynamic cooperation for climate action across countries, including some complex dynamics. The different cases depend mainly on the dispersion of economic geographical factors and also on the effect of peer pressure and the initial conditions. Most crucially, to have sustained high levels of global climate action, there must be a low dispersion and a relatively high peer pressure effect. Without these two necessary conditions, climate action may increase in the short run but decline over the medium run. As dispersion captures the overall variation of the four economic geographical factors to which uneven development plays a key role, our results highlight the possible limitations to sustained global action posed by uneven geographical development.

Why does the dispersion of economic geographical factors influence global climate action? When all countries are similar (i.e. low dispersion), they take more or less the same action as their decisions are driven by global common factors, namely the growth rate of emissions and the participation of other countries. If peer pressure effects are strong enough, all countries will decide to take costly action and avoid free riding on other countries’ actions. On the other hand, if dispersion is high and/or peer pressure effects are weak, this will increase the influence of the free riding effect, leading to a decline in global action. We interpret this second case as a forewarning that the observed increase in climate action over the last decades does not necessarily imply that high levels of cooperation will be reached and sustained over time.

This paper is structured as follows. Section 2 discusses the contribution of this work in relation to relevant literatures. Section 3 discusses the empirical literature related to the economic geographical factors which influence preferences for taking climate action. We also use relevant data as proxies for these factors which allows us to map each of these, an index capturing their average and how this changes
over time. Section 4 introduces the model and section 5 presents the analytical results and simulations. Finally, section 6 provides a concluding discussion with directions for future research.

2 Contribution to relevant literatures

The paper contributes to and brings together different literatures. Our model builds on the economic geography of climate change literature (Conte et al. 2021; Castells-Quitana et al. 2021; Bosetti et al., 2021, Grimm, 2021) by developing an evolutionary model, the outcomes of which depend primarily on the variation of different economic geographical factors influencing global climate action decisions. By doing this we also connect EG works with relevant works within (global) political economy which analyse the influences of decisions regarding climate action and have close EG aspects (for example see Bättig and Bernauer, 2009; Scheidel et al., 2020; Tubi et al., 2012; Victor et al., 2022). Given our focus on global climate action, our work also contributes to the literature modelling participation in international environmental agreements (Calvo and Rubio, 2012; Hoel, 1997; van der Ploeg and de Zeeuw, 1992; Long, 2012) by highlighting the importance of economic geography aspects regarding making decisions for climate action. One key contribution to this literature is that through our work, we show that economic geography matters for the global political economy of a low carbon transition. Not taking into account global economic geographical variations of conditions which affect choices can lead to very different results.

The economic geographical factors discussed in the next section, are related to different literatures within GPE. First and foremost uneven development (Harvey, 1982; Smith, 1984) which has been one of the key influences of GPE (Jones, 2008; Sheppard 2011, 2013) is related directly with the first two of the economic geographical factors and indirectly with the other two as discussed in the next section. In our analysis, uneven development implies both unequal relative costs for action and also unequal vulnerability regarding climate damages, which are also unevenly distributed globally. Uneven development is also indirectly related with the other two factors, namely fossil fuel rents and institutions. Within GPE, the link between natural resources and uneven development was recently discussed in Arboleda (2020, 2022), while Peck and Theodore’s (2007) ‘variagated capitalism’ provides a framework which brings together uneven development and institutions. Furthermore the political economy of fossil
From a methodological viewpoint our approach creates novel links between complexity economics, evolutionary economic geography and GPE, which call for formal disequilibrium models able to generate complex dynamics. Our modelling framework shares similar methodological assumptions with the evolutionary, bounded rational, heterogeneous agents models literature starting from Lux (1995) Brock and Hommes (1997, 1998). These types of models have been traditionally part of growing literatures within behavioural finance (Chiarella et al., 2006; Dieci and Westerhoff, 2016, among others), behavioural macroeconomics (De Graauwe, 2012; Flaschel et al., 2018; Hommes and Lustenhouwer, 2019; among others) and have recently been used to study problems related to political polarisation (Di Guilmi and Galanis, 2021; Di Guilmi et al., 2022), physical distancing decisions (Galanis et al., 2021; Di Guilmi et al., 2022) and a low carbon transition (Cahen-Fourot et al., 2023; Campiglio et al., 2023). However, to the best of our knowledge, this paper is the first within this literature to study the global political economy of climate action.

An interesting link between this literature and works within (mathematical) geographical political economy is the use of a logit formulation to capture agent’s decision making processes. This logit assumption is not only the standard assumption in the models above but it has also been used in different contexts in works such as Sheppard and Barnes (1990), Sheppard et al. (1992) and Bergmann (2012). We differ from the above (economic) approaches however by not assuming the logit formulation on the basis of agents’ informational limitations but rather on the basis of heterogeneous preferences driven by economic geographical factors and to the best of our knowledge, this approach has not been taken in similar models. Focusing on heterogeneous preferences rather than information, or in the easiness to switch between choices as in Brock and Hommes (1997, 1998), has an interesting implication. Based on our approach, one can look at empirical works to derive the distribution that leads to the logit framework rather than assume it as given. In this way it is possible to get insights regarding the factors which influence the key logit parameters (in our case dispersion) which lead to different model outcomes. Given these insights it is also possible to further develop models in this tradition by endogenising the logit parameters.

Our evolutionary approach overlaps with influential works within evolutionary economic geogra-

\[3\] For this extension see discussion in Conclusion.
phy and GPE by combining heterogeneous interacting agents with possibly complex, out-of-equilibrium dynamics. For example Sheppard (2001) discusses the use of non-linear dynamic models as an alternative to the standard (neoclassical) equilibrium models while Bergmann et al. (2009) and Sheppard (2011) argue that approaches focusing on potentially complex out-of-equilibrium dynamics can be the appropriate mathematical modelling tools for critical geography and geographical political economy respectively. The interdependence between agents’ actions and emerging complex dynamics, as is the case in our model, are also discussed in Plummer and Sheppard (2006) in the context of alternative modelling approaches that can be used in economic geography and are identified as possible components of evolutionary economic geography (Boschma and Martin, 2007; Martin and Sunley, 2007).

3 The economic geography of climate action

Each country’s willingness to take climate action is affected by a combination of economic geographical factors. We focus here on the four main factors influencing the spatial distribution of preferences for taking action. At this point we should mention two issues related to our analysis. First, while there may be other channels which influence climate action, we focus on the main ones discussed in relevant literatures in closely related fields such as environmental economics and global political economy. Second, while each of these four factors is presented in isolation from the others in respective research, all factors have strong EG foundations.

The first factor regards the unequal distribution of economic resources across countries which is an outcome of uneven geographical development (Harvey, 1982; Smith, 1984; Jones, 2008; Sheppard 2013). It is well documented that the lack of resources in less developed parts is associated with lower levels of action (Bättig and Bernauer, 2009; Dolšak, 2009; Fankhauser et al., 2016). Bättig and Bernauer (2009) find that GDP per capita determines a country’s commitment to UN-based climate mitigation processes, while Fankhauser et al. (2016) find that richer countries are more likely to adopt a greater number of domestic climate laws. This is quite intuitive as lower economic resources imply higher relative costs, which can lead to higher economic and political constraints. Figure 2 shows the spatial distribution of current GDP per capita, as a proxy of uneven development and relative costs for action.

The second and related factor regards each country’s vulnerability to climate change damages (Ricke
Countries which are more vulnerable to climate change have a greater incentive to mitigate, and therefore take action (Torstad et al., 2020; Tubi et al., 2012). Vulnerability reflects the actual damages from extreme weather events (Pollard et al., 2008), in addition to the infrastructural capacity and resilience to respond to these damages. It therefore combines the geographical distribution of damages and extreme weather events with the unequal ability of countries to adapt, which itself is outcome of uneven development. Recent works on the EG of climate change discuss some of these effects with respect to population, fertility, migration, urbanisation and conflict (Conte et al. 2021; Castells-Quitana et al, 2021; Bosetti et al., 2021, Grimm, 2021).

Figure 3 presents the Notre Dame-Global Vulnerability Index, composed of 36 indicators capturing vulnerability to climate change damages across six sectors: food, water, health, ecosystem services, human habitat and infrastructure. Looking at both GDP per capita and vulnerability together, we can see the familiar uneven geographical development of the world market, split between two clear groups, the most vulnerable countries - Sub-Saharan Africa, South Asia, Small Islands and Central and South America - and the rest of the world.

Thirdly, the global distribution of fossil fuels is also known to influence climate action by concentrating extractive industries in certain countries (Brulle, 2018; Colgan et al., 2020; Dolphin et al., 2020; Lamb and Minx, 2020; Victor et al., 2022). These countries will be less willing to take climate action,
given that such mitigation unambiguously relocates economic value away from their economies (Bridge, 2008; IPCC, 2022). The centrality of fossil fuels in determining political economic relationships has been highlighted in several strands of the EG literature, from Huber’s analysis of oil’s centrality for Fordist wage relations in the USA (2013) to Arboleda’s (2020, 2022) work on combined and uneven extractivism.

Figure 4 presents data on the sum of coal, oil and gas rents as a share of GDP from the World Bank. The MENA region is unique in having a large proportion of countries with fossil fuel rents above 20 percent of GDP. Moreover, several Sub-Saharan countries such as Angola and Gambon also have high oil rents as a proportion of GDP. We would therefore expect these countries, other factors being held equal, to be less willing to take action to mitigate against climate change. However, this blunt measure of fossil fuel rents may mask a more complicated geographical exposure to fossil fuel assets. Semeniuk et al. (2022) find that lost profits in the oil and gas sector due to expected climate mitigation policy are distributed via a vast global equity network of 1.8 million companies to the ultimate owners of these assets. The authors find that most of this market risk falls on private investors, overwhelmingly in OECD countries, including substantial exposure through pension funds and financial markets.

Fourthly, political institutions are likely to impact preferences for taking action. Countries with political institutions geared towards long term policy making are shown to be more willing to take
climate action (Davidson et al., 2021; Finnegan, 2022; Fredriksson and Neumayer, 2016; Genovese and Tvinneim, 2019; Keohane, 2001). Fredriksson and Neumayer (2016) argue that controlling corruption promotes climate mitigation policy adoption, due in part to the fact that effective climate mitigation policies require imposing short-term costs on voters for benefits that will arrive in the future, uncertainty about whether future benefits will materialise, and overcoming opposition from cost-bearing organised groups (Finnegan, 2022). The distribution of institutions are themselves outcomes of geographical factors, as outlined by Peck and Theodore’s (2007) concept of "variegated capitalism", with its focus on interpreting relatively concrete institutional conjunctures located within unevenly developed global economic systems.

Figure 5 presents the index on the control of corruption from the World Bank’s Worldwide Governance Indicators (WGI), where higher values indicates more control of corruption. The clusters of relatively strong political institutions in North America, Europe and the southern tip of Africa and South America has already been well documented in previous EG studies (Bosker and Garretsen, 2009).

Given that these four factors do not always cluster together, we lastly consider the spatial distribution of all four factors collectively by taking a simple unweighted sum of each factor.\[\text{Figure 6}\] presents \[\text{We take the z transformation of each factor to make them comparable. With the z transformation, we convert our variables/distributions to a set of z values with mean equal to 0 and a standard deviation equal to 1. In this way, all factors are comparable.}\]
two maps of this index. The upper panel shows the index for 2019 - the latest year with available data. Countries in darker red prefer to take climate action to those with lighter colours. The bottom panel shows the change in the index between 2005 and 2019. Preferences for action increased in red countries, declined in cream countries and stayed the same in orange ones.

Figure 6 points towards two findings that serve as foundations for modelling the geographical heterogeneity related to the global political economy of climate action. As the upper panel of Figure 6 shows several countries in the global south have both high (e.g. Niger) and low (e.g. Angola) preferences for taking action. This suggests that preferences are not binomially distributed between the global north and the global south. In fact, as can be seen in Figure 7 which plots the distribution of the index for a single year (2019), preferences seem to follow a relatively symmetric unimodal distribution, which may be reasonably proxied by a logistic distribution. Moreover, as the lower panel of Figure 6 shows, for most countries, the index changes over time, implying that for the same global factors (such the growth of the stock of GHG emissions) the economic geographical factors across countries which may change over time and can be represented as a shock. Both of these findings serve as the basis for using a logit

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5In the appendix we show that the preferences for action index is correlated with actual measures of climate action: (i) whether countries signed binding commitments at 2008 Copenhagen accords; (ii) whether countries have a carbon tax and/or ETS law (Tenreyro and de Silva, 2021).

6Niger has a relatively higher preference for taking action as it is particularly vulnerable to climate change, while Angola has a large fossil fuel sector.
Figure 6: Preferences for climate action index

(a) Notes: The preferences for action index is constructed from the unweighted sums of four variables: (i) GDP per capita in current USD (World Bank); (ii) vulnerability Index (Notre Dame-Global); (iii) sum of oil rents, natural gas rents and coal rents as percentage of GDP (World Bank); (iv) control of corruption index (Worldwide Governance Indicators from World Bank). The variables are z-transformed to make them comparable. Strong preferences for taking climate action are represented by higher values on the index. The upper panel takes the index value in 2019. The bottom panel demonstrates the change in the index between 2005 and 2019 where red indicates increased preferences, orange indicates no change, and cream indicates a decline in preferences. If the index changes in absolute terms by less than 0.1 we consider it not to have changed (i.e. orange).
framework for our discrete choice model discussed in the next section.\footnote{As shown in Galanis et al. (2022), the normal distribution can be well approximated by a logistic one and furthermore for dynamic discrete choice models the results using a logit framework are robust even under a probit assumption. Given the analytical tractability offered by the logit modelling framework, we choose this approach for our analysis in the next section.}

4 Model

4.1 Setup

Consider that the world economy is composed by $2N$ countries, each of which faces a binary decision: take a costly climate action ($C$) to reduce the growth rate of the stock of greenhouse gas emissions or abstain ($A$). Let $U_{jt}^j$ with $j = \{C, A\}$ be the country’s utility from each of the choices at $t$. Country
$i \in \{1, \ldots, 2N\}$ chooses $C$ in period $t$ if

$$V^c_i = U^C_i - U^A_i > 0.$$

(1)

Following the discrete choice literature (for example see McFadden, 1974; 1978; 2001 and Train, 2009), we assume that $V^c_i$ has two types of components: a ‘global’ one which is common across countries and is given by a column vector $v_t$ whose elements refer to specific global variables and an ‘idiosyncratic’ one, $\epsilon^c_i$, which varies across countries due to the economic geographical factors discussed in the previous section. Based on this we can express the preferences of country $i$ at time $t$ by

$$V^c_i = \beta v_t + \epsilon^c_i,$$

(2)

where $\beta$ is a row vector, with each of its elements capturing the importance of each of the elements in $v_t$. As justified in section 3, we assume a logit framework, where $\epsilon^c_i$ follows a logistic distribution with mean 0 and variance equal to $\frac{3s^2}{\pi^2}$, where $s$ is the scale parameter. We define $\gamma = \frac{1}{s}$ as the dispersion of the economic geographical factors influencing countries’ preferences for taking action. High (low) values of $\gamma$ correspond to a low (high) dispersion.

Given that $\epsilon$ incorporates different factors discussed in the previous section (uneven development, unequal vulnerability, fossil fuel rents, institutions), its dispersion captured by gamma depends on the combined variation of these factors across space. For example, more uneven development, implies in general higher dispersion of the economic geographical factions, hence lower $\gamma$.

Based on the logit structure we can express the probability that country $i$ chooses $C$ at time $t$ for given $v_t$ as

$$P(C|v_t) = \frac{e^{\gamma \beta v_t}}{1 + e^{\gamma \beta v_t}},$$

(3)

which also means that the probability of no action will be

$$P(A|v_t) = 1 - P(C|v_t) = \frac{1}{1 + e^{\gamma \beta v_t}},$$

(4)

The parameter $\gamma$ is also known as intensity of choice in the relevant literature. For example see Brock and Hommes (1997).
Note that high dispersion also implies, high importance of the economic geographical factors relative to the global ones. This is due to the fact that a high dispersion of economic geographical factors, means that there will always be a considerable number of countries that will always want to take action and also a considerable number of countries that never want to take action, regardless of the global factors. Put differently, high values of $\gamma$ (low dispersion) means $v_t$ is relatively more important in determining the choices of the countries, while low values of $\gamma$ (high dispersion) means that $v_t$ plays less of a role and the economic geographical factors become relatively more important. For example for $\gamma \to 0$ ($s \to \infty$), $P(C|v_t) = P(A|v_t) = \frac{1}{2}$, meaning that countries make their choices mainly due to the economic geographical factors.

Let $n_t^C$ be the number of countries who take an action at time $t$ and $n_t^A$ the number of ones who don’t, with $n_t^C + n_t^A = 2N$. Also let $x_t$ be the relative share of countries which take climate action at $t$, such that
\begin{equation}
  x_t = \frac{n_t^C - n_t^A}{2N},
\end{equation}
This implies that $x_t \in [-1,1]$, for all $t$ with $x_t > 0$ when $n_t^C > n_t^A$. We can express the shares as functions of $x_t$:
\begin{equation}
  n_t^C = (1 + x_t)N
\end{equation}
and
\begin{equation}
  n_t^A = (1 - x_t)N.
\end{equation}
From (3) to (7), we can express the evolution of $x_t$ as
\begin{equation}
  \Delta x_{t+1} = (1 - x_t) \frac{e^{\gamma \beta_{v_t}}}{1 + e^{\gamma \beta_{v_t}}} - (1 + x_t) \frac{1}{1 + e^{\gamma \beta_{v_t}}} = \frac{e^{\gamma \beta_{v_t}} - 1}{1 + e^{\gamma \beta_{v_t}}} - x_t,
\end{equation}
which can also be expressed as
\begin{equation}
  x_{t+1} = \frac{e^{\gamma \beta_{v_t}} - 1}{1 + e^{\gamma \beta_{v_t}}}.
\end{equation}

We consider two types of global factors corresponding to the elements of $v_t$. First, the growth rate of GHG emissions ($\dot{E}_{t+1}$) which provides a signal for future damages, given that the current stock of GHG ($E_0$) is already high. More specifically we assume that for $\dot{E}_{t+1} > 0$, GHG growth has a positive
effect on a country choosing to take action ($\partial \beta v_t / \partial \hat{E}_{t+1} > 0$), while for $\hat{E}_{t+1} = 0$, the choice will depend on other factors. Similarly if $\hat{E}_{t+1} < 0$ then the danger of future damages diminishes, leading to countries becoming on average less inclined to take action.

The second global factor is relative country participation. If the majority of countries participate this creates an incentive for participation, such that $\partial \beta v_t / \partial x_t > 0$. This is due to both the fact that possible future costs for reducing GHG will decrease if more countries participate in the present alongside peer pressure effects that have also been empirically observed (Fankhauser et al., 2016).

We can express these two effects of the global factors in the following way:

$$\beta v_t = \beta_x x_t + \beta_e \hat{E}_{t+1}, \tag{10}$$

where $\beta_x > 0$ and $\beta_e > 0$ capture the importance of the two factors.

The growth rate of the stock of GHG, $\hat{E}_{t+1}$, is assumed to be given by

$$\hat{E}_{t+1} = \alpha_0 - \alpha_1 r_t, \tag{11}$$

where $r_t \in [0, 1]$ captures the reduction rate at time $t$, with $\alpha_0 > 0$ representing an ‘autonomous’ growth rate of emissions (when no reducing action is assumed) and $\alpha_1 > \alpha_0$ capturing the relative effect of $r_t$ on $\hat{E}_{t+1}$. Equation $\text{(11)}$ shows that when $r_t$ is fixed, the growth rate of GHG is constant. This corresponds to the observed growth rate of emissions which has been on average equal to 0.38%.

Furthermore, we assume that the overall level of participation (captured by $x_t$) defines the reduction rate of emissions in the following way

$$\hat{r}_{t+1} = x_t (1 - r_t), \tag{12}$$

such that $\partial \hat{r}_{t+1} / \partial x_t > 0$ and also the effects of $x_t$ on $r_t$ are diminishing as $r_t$ moves towards 0 or 1, hence ensuring that $r_t \in [0, 1]$.

Using $\text{(11)}, \text{(10)}$ can be expressed in terms of $r_t$ as

$$\beta v_t = \beta_x x_t + \beta_e (\alpha_0 - \alpha_1 r_t). \tag{13}$$
The evolution of climate action globally is described by \( \text{[12]}, \text{[8]}, \text{given [13]} \). As we discuss below, the dispersion of economic geographical factors and the relative importance between the global variables influencing countries’ decisions, affect both the number of possible steady states and their stability properties.

4.2 Results

We start from the more general results which hold independently of the model’s parameter values and we move to the different possibilities regarding the importance of the different effects.

**Lemma 1.** The following steady states exist for any values for the parameters:

(i) \( (x_t, r_t) = (x^+, 0), \text{ with } x^+ > 0 \)

(ii) \( (x_t, r_t) = (x^-, 1), \text{ with } x^- < 0 \)

(iii) \( (x_t, r_t) = (0, \frac{a_0}{a_1}) \)

This shows the existence of three possible steady states: one where the majority of the countries take action and the growth rate of emissions is zero, one where the minority take action and the reduction rate of emissions is one (the maximum in this framework), and one where the countries are equally split between taking action or not and the reduction rate is such that the stock of GHG is stable over time. The steady state that will prevail depends not only on the parameter values which capture specific assumptions regarding behavioural characteristics and the dispersion of economic geographical factors across countries, but also on the stock of GHG which corresponds to the model’s initial conditions. The next Propositions characterise these steady states.

**Proposition 1.** For all parameter values, both \( (x_t, r_t) = (x^+, 0) \) and \( (x_t, r_t) = (x^-, 1) \) are unstable.

The first steady state \( (x^+, 0) \) captures a situation where the majority is taking action while the reduction rate is zero. It is quite intuitive to note that this would be an unstable steady state as the emissions would be growing, pushing even more countries to take action. Similarly the second steady state \( (x^-, 1) \) is unstable as a reduction rate equal to one leads to a reduction of emissions which will lead to even less countries to take action.

\(^9\)For proofs of Lemmas and Propositions see Appendix
Proposition 2. There exists a $\gamma^* = \frac{2\beta_x \alpha_0 (1 - \frac{a_0}{a_1})}{\beta_e}$, such that the steady state $(x_t, r_t) = (0, \frac{a_0}{a_1})$ is

(i) locally stable for $\gamma < \gamma^*$

(ii) unstable for $\gamma > \gamma^*$

Proposition 2 states that the stability of $(x_t, r_t) = (0, \frac{a_0}{a_1})$ depends on the dispersion parameter $\gamma$ such that if the dispersion of the economic geographical factors is relatively high, then the steady state is stable. A low dispersion implies a relatively high importance of economic geographical factors compared to global ones and in this case countries are split between taking action or not.

Figure 8 shows $\gamma^*$ on a two-dimensional plane (contour plot). Given $\beta_e$, for a higher $\beta_x$, the steady state becomes unstable for a relatively higher dispersion (low $\gamma^*$). This indicates a possible complementarity between the relative importance of two types factors: the influence of how other countries act and the relative influence of the economic geographical factors. However the relationship between these two types of factors goes beyond leading to instability and can lead to further qualitative differences as the next Proposition shows.

Figure 8: contour plot of $\gamma^*$ for different values of $\beta_x$ and $\beta_e$

Proposition 3. Let $\beta_x > \beta_e \alpha_0$, then for values of $\gamma$ sufficiently high there exist steady states with $r_t = 0$ and $x_t < 0$ and $r_t = 1$ and $x_t > 0$.  

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Proposition 3 shows that more steady states can exist under two conditions. First, the importance of relative participation should be high compared to the effects of GHG emission growth. Intuitively, this means that if the effect of participation is high (which implies low relative costs and/or high peer pressure), then even if GHG emissions are declining, the incentives for not taking climate action are low. Second, the dispersion of economic geographical factors should be relatively low. This necessary condition captures the fact that for the previous effect to be able to take place, the relative importance of the economic geographical factors should be low, which is true only when the dispersion is low.

The following figures provide a graphical representation of the dynamics around the steady states discussed in Propositions 2 and 3. Figure 9 shows the evolution of \( x_t \) and \( E_t \) for different values of \( \gamma \) when \( \beta_x \gtrless \beta_e \alpha_0 \). In the four quadrants, starting from the same initial values \( (x_0 = -0.75, E_0 = 40, r = 0.5) \), we show how different values of \( \gamma, \beta_x \), and \( \beta_e \) for given \( \alpha_0 \), correspond to different trajectories regarding climate action and, consequently, the evolution of emissions, \( \hat{E}_t \).

In the upper left quadrant, the dispersion of economic geographical factors is very high (\( \gamma \) is low) while at the same time, the effect of peer pressure is relatively less important than the effect of GHG emissions in influencing countries decisions to take action (\( \beta_x < \beta_e \alpha_0 \)). The evolution of climate action converges in an oscillatory manner towards the steady state where countries are equally split between taking action and not (\( x_\infty = 0, r_\infty = \frac{\alpha_0}{\alpha_1} \)). After some periods of fluctuations, the stock of emissions converges to a value close to the initial one.

A decrease in dispersion (an increase in \( \gamma \)) leads to qualitatively different behaviour among countries (figure 9b). Here, we observe fluctuations which do not converge to a steady state. Instead, periods with a high level of global action are followed by periods in which countries prefer to abstain from intervention. After an initial increase, \( E \) will fluctuate and then slowly decline; leading to no significant change.

In figure 9c and 9d we consider the case in which the effect of peer pressure is now relatively more important than emissions in influencing action (\( \beta_x > \beta_e \alpha_0 \)). In figure 9c, the high dispersion also leads to some kind of a cyclical behaviour, although with more complexity and a slower reduction in action. However, even in this case we do not observe any significant reduction in GHG emissions. There is a long-run decreasing trend, however after 60 years the stock of emission will still be at the initial level.
Differences with case b) are driven by a combination of effects: on the one hand a higher peer effect or, conversely, the lower free riding reaction; on the other hand higher dispersion. Last but not least, the reduction of dispersion, when $\beta x > \beta_0 \alpha_0$, leads to the emergence of a new steady state (Proposition 3) with $r_t = 1$ and $x_t > 0$. In this case, $E$ continues to grow before starting to monotonically decline.

Our results highlight the fact that if the dispersion of economic geographical factors is very high (for example due to greater uneven development), it will be practically impossible for some countries to take climate action, meaning that the maximum overall peer pressure will not be sufficient to counteract the negative effect that a reduction in GHG can have on climate action. As GHG emissions decline, more countries will stop taking action, further weakening the peer pressure effect. As there is a delay between action and its effects on the growth (or reduction) rate of GHG emissions, the reduction in
global climate action will not lead directly to a change in the sign of \( \dot{E}_t \), hence the process where there is a negative \( \dot{E}_t \) and a continuous reduction of \( x_t \) continues and changes only after \( \dot{E}_t \) becomes positive.

The overall stability for each of the two cases for different values of \( \gamma \) can be portrayed by the bifurcation diagrams showed in figure [10]. In both cases the evolution of climate action loses stability moving to a cyclical behaviour when \( \gamma \) obtains higher values. The first bifurcation diagram where \( \beta_x \) is high, shows that the new steady states become stable for high values of \( \gamma \) while the second exhibits non trivial dynamics. The complementarity relationship between \( \gamma \) and \( \beta_x \) for given \( \beta_e \) and \( \alpha_0 \) is clearly shown.

Figure 10: Bifurcation Diagram of \( x_t \) for \( \gamma \in (0, 5) \)

In summary, the bifurcations show three possible cases, the first two of which exist for all possible values for \( \beta_x \), \( \beta_e \) and \( \alpha_0 \). The first case is when there is high dispersion corresponding to low levels of global action. The second case is when there is a lower dispersion but this is not sustained over time, even though there is the possibility of an increase of global action against climate change. In order to have the third and best case - i.e. high sustained levels of global action - there are two necessary conditions: a low dispersion and a high relative importance of the peer effect in the decision making process. The bifurcation diagrams also show hints of complex dynamics with the possibility of chaos. We next turn to phase plots to get a better insight regarding the possibility of chaos.

Figure [11] shows phase plots for \( x_t \) and \( r_t \), using the same parameter values as in Figure [9]. This representation allows us to observe the co-evolution of our two endogenous variables used in the formal results above. Panels (b) and (c) correspond to parameter values for which the possibility of chaos
Figure 11: Evolution of $x_t$ and $r_t$ for different parameter values

(a) $\gamma = 0.9$ and $\beta_x < \beta_0 \alpha_0$

(b) $\gamma = 2$ and $\beta_x < \beta_0 \alpha_0$

(c) $\gamma = 0.9$ and $\beta_x > \beta_0 \alpha_0$

(d) $\gamma = 2$ and $\beta_x > \beta_0 \alpha_0$

Parameters: $\alpha_0 = 0.08$, $\alpha_1 = 0.1$

appears in the bifurcation diagrams. We note that while the cyclical behaviour is relatively complex, the dynamics are not chaotic ones, hence confirming the insights of Figure 9.

5 Conclusion

Over the past four decades, an increasing number of countries have taken climate action, from implementing domestic mitigation laws to signing quantifiable binding targets at international climate agreements. These collective efforts however have not done enough to keep temperatures from rising. As we enter a hotter world, it is not guaranteed that countries will continue to take more ambitious action. To effectively mitigate against climate change, we need a better understanding of the conditions...
under which most countries will adopt sustained levels of global action.

Economic geography, and in particular uneven geographical development, is deeply relevant for understanding the evolution of global climate action. Uneven development has left many countries without the resources they need to mitigate. This lack of economic resources combined with the unequal global distribution of climate damages, also means that many countries are particularly vulnerable to extreme weather events and other effects of climate change. More indirectly, uneven development has shaped and been influenced by countries’ political institutions and also their access to fossil fuels. Taking all these together, each country’s costs and preferences for taking climate action continues to be influenced by a set of specific economic geographical factors that are heterogeneous across space. At the heart of our analysis is exactly the extent of these spatial differences between countries - what we refer to as the dispersion of economic geographical factors.

Understanding global climate action requires combining economic geography and global political economy into a geographical political economy framework. We develop a novel evolutionary model and study its dynamics regarding global participation in climate action. Our modelling approach not only allows for incorporating heterogeneous economic geographical factors in a simple model, but also provides an example of a disequilibrium GPE model where the (often complex) dynamics are a key part of the analysis. In this way, we hope that our model provides an example of a formal GPE framework where both space and out-of-equilibrium behaviour matter.

Our key finding is that economic geography plays a crucial role and a high level of sustained global action is only possible if there is low dispersion. In other words, this implies that uneven geographical development can be a significant barrier in achieving lasting and effective global climate action. If the dispersion is not low (i.e. uneven development is not sufficiently curtailed or reversed), short run increases in climate action may be followed by a decline later, alongside other non trivial dynamics that make the evolution less predictable.

The purpose of this work has been to take a first step towards analysing the effects of economic geographical factors and their dispersion on climate action in an analytically tractable framework. Having done this, we necessarily abstracted from two elements which could provide interesting insights if included in a more detailed analysis. First, countries are also heterogeneous with respect to their size, hence their respective decisions have different weights in the overall process. Second, we have implicitly
assumed an underlining fully connected network structure, where climate action decisions of one country have the same effect across all other countries. However, countries are connected in different ways due to a number of geographical, economic and political reasons. Both of these issues define directions which can expand the existing paper through a more detailed empirical analysis and/or an agent based model.

Another direction is related to the empirical aspect of heterogeneity. While we have already discussed four economic geographical factors which influence countries’ decisions, we have not conducted a comprehensive analysis regarding further factors and their relative effects on taking climate action. Answering these types of questions is not only relevant for policy but also can create new insights regarding the relationship between (international) social justice and a low carbon transition. Lastly, we have assumed two global factors influencing countries’ behaviour where the expected damages are captured by the growth rate of emissions, given the current stock level. Our framework can be further expanded in this direction by including other factors and non-linearities related to climate damages.

In order to study the effects of economic geographical variation, we have necessarily treated the dispersion of economic geographical factors as an exogenous variable. However it may be reasonable to assume that in the long run this should be viewed as an endogenous variable. While it is far from obvious how this would change over time based on our current framework, including other factors and climate damages may also enable us to endogenise dispersion and this is another interesting direction for future work.
References


Appendix

Data sources

<table>
<thead>
<tr>
<th>Name</th>
<th>Source</th>
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</thead>
<tbody>
<tr>
<td>Vulnerability to climate damages</td>
<td>ND-GAIN Country Index</td>
</tr>
<tr>
<td>GDP per capita</td>
<td>World Bank in current USD</td>
</tr>
<tr>
<td>Fossil fuels rents/GDP</td>
<td>World Bank</td>
</tr>
<tr>
<td>Control of corruption index</td>
<td>Worldwide Governance Indicators</td>
</tr>
<tr>
<td>Has Carbon Tax and/or ETS</td>
<td>Climate Change Laws Database</td>
</tr>
<tr>
<td>Has quantified targets in 2008</td>
<td>Tenreyro and de Silva (2021)</td>
</tr>
</tbody>
</table>

Logit Regression: the relationship between preferences for action index and actual discrete measures of climate action

We quickly test whether the preferences for action index is broadly correlated with actual measures of climate action. To test this we analyse the relationship between the index and two discrete measures of taking climate action recently highlighted by Tenreyro and de Silva (2021): (i) whether countries have national laws adopting carbon taxes and/or emission trading schemes; (ii) whether countries signed the 2008 Copenhagen Accord with quantified targets. Table 2 presents the results of estimating two cross sectional logit regressions with each of these measures as dependent variables respectively. As can be seen, the odds ratio for both logit regressions are statistically significant, suggesting the index is correlated with both measures of taking climate action. Looking at column 1, a one unit increase in the preferences for action index is associated with a 1.487 increase in the odds of a country adopting carbon taxes and/or emission trading schemes. Looking at column 2, a one unit increase in the index is associated with a 1.779 increase in the odds of a country adopting quantified targets in the 2008

---

We use a logit regression as both dependent variables are binary. If a country has taken action, each measure equals 1, otherwise it is 0. The logit regression is estimated using Maximum Likelihood Estimation. Note this the logit regression is different to the logistic density function below which refers to the distribution of the index.
Table 2: Logit Regressions: The relationship between preferences for action index and actual measures of taking climate action

<table>
<thead>
<tr>
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<th>(1)</th>
<th>(2)</th>
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<tr>
<td>Preference index</td>
<td>1.487**</td>
<td>1.779***</td>
</tr>
<tr>
<td></td>
<td>(0.265)</td>
<td>(0.244)</td>
</tr>
<tr>
<td>Observations</td>
<td>168</td>
<td>176</td>
</tr>
<tr>
<td>Year</td>
<td>2019</td>
<td>2008</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
Column 1 and 2 have different measure of climate action i.e. dependent variables:
(1) Whether has Carbon Tax or ETS in 2019;
(2) Whether adopted quantified targets at Copenhagen.
Data: (1) Climate Change Laws Database
Data: (2) Tenreyro and de Silva (2021)
* \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)

Copenhagen Accord. In other words, the simple index is consistent with the two discrete measures of taking climate action highlighted in the literature.

**Proof of Lemma 1**

From (12), \( \Delta r_{t+1} = 0 \) if

(i) \( r_t = 0 \). Note that \( \frac{e^{\gamma \beta_v t} - 1}{1 + e^{\gamma \beta_v t}} \in (-1, 1) \), with its sign depending on whether \( \beta_v t \) is positive or negative. When \( r_t = 0 \), \( \beta v_t = \beta x_t + \beta e(\alpha_0 - \alpha_1) \), with \( \beta v_t > 0 \), iff

\[
x_t > -\frac{\beta e(\alpha_0 - \alpha_1)}{\beta x}, \tag{14}
\]

in which case we also have \( e^{\beta v_t} > 1 \). Note that for \( x_t \to 1 \), \( \Delta x_{t+1} < 0 \). while for \( x_t = 0 \), \( \Delta x_{t+1} > 0 \). Hence, from continuity of \( \Delta x_{t+1} \), there exists an \( x^+ > 0 \), for which \( \Delta x_{t+1} = 0 \).

(ii) \( r_t = 1 \). For \( r_t = 1 \), \( \beta v_t = \beta x + \beta e(\alpha_0 - \alpha_1) \). Note that based on our assumptions \( \alpha_0 - \alpha_1 < 0 \), hence \( \beta v_t < 0 \) iff

\[
x_t < -\frac{\beta e(\alpha_0 - \alpha_1)}{\beta x}, \tag{15}
\]
Thus, for $x_t = 0$, $\Delta x_{t+1} < 0$, while for $x_t \to -1$, $\Delta x_{t+1} > 0$. Thus, from continuity of $\Delta x_{t+1}$, there exists an $x^- < 0$, for which $\Delta x_{t+1} = 0$.

(iii) $x_t = 0$. For $x_t = 0$, $\Delta x_{t+1} = 0$ if $\beta v_t = 0$, which holds for $r_t = \frac{a_0}{a_1}$.

\[ \Box \]

**Proof of Proposition 1**

The general form of the Jacobian is given by

\[
J(x_t, r_t) = \begin{bmatrix}
2\gamma \beta_x e^{\gamma \beta_x x_t} / (1 + e^{\gamma \beta_x x_t})^2 & -2\gamma \beta_x \alpha_1 e^{\gamma \beta_x x_t} / (1 + e^{\gamma \beta_x x_t})^2 \\
0 & 1 + x_t(1 - 2r_t)
\end{bmatrix}
\]

For local stability the following conditions should be satisfied

1. $1 + Tr(J) + Det(J) > 0$
2. $1 - Tr(J) + Det(J) > 0$
3. $Det(J) < 1$,

where $Tr(J)$ and $Det(J)$, correspond to the trace and determinant of the Jacobian respectively.

For $(x^+, 0)$,

\[
J(x^+, 0) = \begin{bmatrix}
2\gamma \beta_x e^{\gamma \beta_x x^+} / (1 + e^{\gamma \beta_x x^+})^2 & -2\gamma \beta_x \alpha_1 e^{\gamma \beta_x x^+} / (1 + e^{\gamma \beta_x x^+})^2 \\
0 & 1 + x^+
\end{bmatrix}
\]

with

\[
Tr(J) = 2\gamma \beta_x e^{\gamma \beta_x x^+} / (1 + e^{\gamma \beta_x x^+})^2 + 1 + x^+ > 0
\]

and

\[
Det(J) = (1 + x^+)[2\gamma \beta_x e^{\gamma \beta_x x^+} / (1 + e^{\gamma \beta_x x^+})^2] > 0
\]

Hence condition 1 always holds. For $Det(J) < 1$,

\[
x^+ < \frac{(1 + e^{\gamma \beta_x x^+})^2}{2\gamma \beta_x e^{\gamma \beta_x x^+}} - 1.
\]
For $1 - Tr(J) + Det(J) > 0$ the following should hold

$$-2\gamma_\beta e^{\gamma_\beta x^+}/(1 + e^{\gamma_\beta x^+})^2 - x^+ + (1 + x^+)[2\gamma_\beta e^{\gamma_\beta x^+}/(1 + e^{\gamma_\beta x^+})^2] > 0$$

which is true if

$$x^+2\gamma_\beta e^{\gamma_\beta x^+}/(1 + e^{\gamma_\beta x^+})^2 > x^+$$

or equivalently when

$$\frac{(1 + e^{\gamma_\beta x^+})^2}{2\gamma_\beta e^{\gamma_\beta x^+}} < 1.$$  \hspace{1cm} (17)

From (16) and (17), we have that $x^+ < 0$, but $x^+ > 0$. Hence conditions 2 and 3 cannot hold simultaneously, hence the steady state is unstable.

For $(x^-, 1)$,

$$J(x^-, 1) = \begin{bmatrix}
2\gamma_\beta e^{\gamma_\beta x^-}/(1 + e^{\gamma_\beta x^-})^2 & -2\gamma_\beta e^{\gamma_\beta x^-}/(1 + e^{\gamma_\beta x^-})^2 \\
0 & 1 - x^-
\end{bmatrix}$$

with

$$Tr(J) = 2\gamma_\beta e^{\gamma_\beta x^-}/(1 + e^{\gamma_\beta x^-})^2 + 1 - x^- > 0$$

and

$$Det(J) = (1 - x^-)[2\gamma_\beta e^{\gamma_\beta x^-}/(1 + e^{\gamma_\beta x^-})^2] > 0$$

Hence condition 1 always holds. For condition 3 to be satisfied, the following should hold

$$x^- > 1 - \frac{(1 + e^{\gamma_\beta x^-})^2}{2\gamma_\beta e^{\gamma_\beta x^-}}.$$  \hspace{1cm} (18)

For condition 2 to hold, the following should be true:

$$-2\gamma_\beta e^{\gamma_\beta x^-}/(1 + e^{\gamma_\beta x^-})^2 + x^- + [2\gamma_\beta e^{\gamma_\beta x^-}/(1 + e^{\gamma_\beta x^-})^2](1 - x^-) > 0$$

or

$$x^-2\gamma_\beta e^{\gamma_\beta x^-}/(1 + e^{\gamma_\beta x^-})^2 < x^-$$
which is equivalent to
\[
\frac{(1 + e^{\gamma \beta x^-})^2}{2\gamma \beta e^{\gamma \beta x^-}} < 1. \tag{19}
\]

From (18) and (19), we get that for local stability we need for \(x^- > 0\), which is not true. \(\square\)

**Proof of Proposition 2**

\[
J \left( 0, \alpha_0 \over \alpha_1 \right) = \begin{bmatrix}
\gamma \beta_2 / 2 & -\gamma \beta_1 / 2 \\
\alpha_0 / \alpha_1 - (\alpha_0 / \alpha_1)^2 & 1
\end{bmatrix}
\]

with
\[
Tr(J) = \gamma \beta_2 / 2 + 1 > 1
\]

and
\[
Det(J) = \frac{\gamma \beta_2}{2} + \frac{\gamma \beta_1 (1 - \alpha_0 / \alpha_1)}{2} > 0
\]

Note that stability conditions 1 and 2 are satisfied. For \(Det(J) < 1\),
\[
\gamma < \frac{2 \beta_1 \alpha_0 (1 - \alpha_0 / \alpha_1)}{\beta_2} \tag{20}
\]

\(\square\)

**Proof of Proposition 3**

(i) For \(r_t = 0\), for the existence of steady states with \(x_t = x \in [-1, 0]\) it is sufficient to show that

(a) for \(x = 0\), \(\frac{e^{\gamma \beta x} - 1}{1 + e^{\gamma \beta x}} > 0\)

(b) for \(x = -1\), \(\frac{e^{\gamma \beta x} - 1}{1 + e^{\gamma \beta x}} > -1\)

(c) there exists a \(\hat{\gamma}\) such that for \(\gamma > \hat{\gamma}\), \(\frac{e^{\gamma \beta x} - 1}{1 + e^{\gamma \beta x}} < x\), for some \(x \in (-1, 0)\)

(a) follows directly from the assumption of \(\frac{\beta \alpha_0}{\beta x} < 1\).
(b) for $x = -1$, we need to show that

\[ e^{\gamma \beta v} - 1 > -1 - e^{\gamma \beta v} \]

or that

\[ 2e^{\gamma \beta v} > 0 \]

which is always true.

(c) We need to show that there exist $\gamma$, such that for some $x \in (-1, 0)$, \(\frac{e^{\gamma \beta v} - 1}{1 + e^{\gamma \beta v}} < x\) or equivalently,

\[ e^{\gamma \beta v} - 1 < x(1 + e^{\gamma \beta v}) \]

Note that the right hand side is negative. The left hand side is negative iff

\[ e^{\gamma \beta v} < 1, \]

which is true if and only if $\beta v < 0$, or

\[ \beta x + \beta e \alpha_0 < 0 \]

or,

\[ x < -\frac{\beta e \alpha_0}{\beta_x}, \quad (21) \]

where $\frac{\beta e \alpha_0}{\beta_x} \in (-1, 0)$ due to the assumption that $\beta_x > \beta e \alpha_0$. Hence assuming that (21) holds, we need to prove that there exist $\gamma$ for which

\[ e^{\gamma \beta v}(1 - x) < 1 + x, \]

or that

\[ e^{\gamma \beta v} < \frac{1 + x}{1 - x} \]
which is true for

\[ \gamma > \frac{\ln(1 + x) - \ln(1 + x)}{\beta_x x + \beta_e \alpha_0} \]

Hence for \( \beta_e \alpha_0 < \beta_x \) and \( \beta_x x + \beta_e \alpha_0 < 0 \), there exist \( \gamma \) for which \( r_t = 0 \) and \( x_t < 0 \).

(ii) \( r_t = 1 \), it is sufficient to show that

(a) for \( x = 0 \), \( \frac{e^{\gamma \beta_x} - 1}{1 + e^{\gamma \beta_x}} < 0 \)

(b) for \( x = 1 \), \( \frac{e^{\gamma \beta_x} - 1}{1 + e^{\gamma \beta_x}} < 1 \)

(c) there exists a \( \tilde{\gamma} \) such that for \( \gamma > \tilde{\gamma} \), \( \frac{e^{\gamma \beta_x} - 1}{1 + e^{\gamma \beta_x}} > x \), for some \( x \in (0, 1) \)

(a) For \( x = 0 \), \( \beta v = \beta_x (\alpha_0 - \alpha_1) < 0 \) which implies that \( \frac{e^{\gamma \beta_x} - 1}{1 + e^{\gamma \beta_x}} < 0 \).

(b) \( \frac{e^{\gamma \beta_x} - 1}{1 + e^{\gamma \beta_x}} < 1 \) is true for all \( x \in [-1, 1] \)

(c) Given that \( x > 0 \), for \( \frac{e^{\gamma \beta_x} - 1}{1 + e^{\gamma \beta_x}} > x \), for some \( x \in (0, 1) \) to hold, it is necessary that \( \beta v = \beta_x x + \beta_e (\alpha_0 - \alpha_1) > 0 \) or that

\[ x > -\frac{\beta_e (\alpha_0 - \alpha_1)}{\beta_x} > 0. \]

Then for these \( x \), \( \gamma \) should be such that

\[ e^{\gamma \beta_x} (1 - x) > 1 + x, \]

or,

\[ \gamma > \frac{\ln(1 + x) - \ln(1 + x)}{\beta_x x + \beta_e (\alpha_0 - \alpha_1)} \]

For a better intuition of the proofs, we provide a graphical representation of our arguments. In figure\[12\] we assume \( \beta_x < \beta_e \alpha_0 \) and plot \( \Delta x_{t+1} = 0 \) for \( r_t = 0 \) and \( r_t = 1 \) respectively for various values of \( \gamma \). Note that in this case the steady states presented in Lemma 1 are the only ones which exist for different values of \( \gamma \).

Next we assume \( \beta_x > \beta_e \alpha_0 \) and plot \( \Delta x_{t+1} = 0 \) for \( r_t = 0 \) and \( r_t = 1 \) respectively, again for different values for \( \gamma \) (figure\[13\]). In this case we see that for higher values of \( \gamma \), two more steady states may exist for \( r_t = 0 \) and the same is true for \( r_t = 0 \). Intuitively the ‘extreme’ steady states with \( x_t \) close to 1 with \( r_t = 1 \) and \( x_t \) close to \(-1\) with \( r_t = 0 \) should be stable.
Figure 12: steady states for $r_t = 0$ and $\beta_x < \beta_e \alpha_0$

Parameters: $\beta_x = 1.1$, $\beta_e = 50$, $\alpha_0 = 0.04$, $\alpha_0 = 0.1$

Figure 13: Looking for zeros high $\beta_x / \beta_e$

Parameters: $\beta_x = 3.1$, $\beta_e = 20$, $\alpha_0 = 0.04$, $\alpha_0 = 0.1$