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Conglomerate Mergers with Bundling in the Unit Square

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Abstract

We study conglomerate mergers between providers of independent services with overlapping customers. In a unit square model, a generalist firm with a content advantage merges with a premium provider. The merger creates neither demand complementarities nor cost synergies, but bundling reshapes competition. When standalone services remain available, mixed bundling can make the merger privately profitable while strengthening price competition and raising consumer welfare for intermediate levels of content advantage. With a larger advantage, bundling can harm consumers by softening competition. Consumer harm can also arise under pure bundling commitments, including through an ecosystem mechanism based on double marginalization between non-merging providers. These conclusions remain relevant when the content advantage is endogenized through investments by the generalist providers.

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1 Introduction

Conglomerate mergers between providers of distinct services have attracted increasing antitrust scrutiny in digital markets. Transactions such as Microsoft's acquisition of Activision Blizzard with its premium games, Amazon's acquisition of MGM Studios with its premium video content and other acquisitions by big tech companies of specialist services have been widely debated. These mergers combine a leading generalist platform and a premium add-on with largely independent demand, and often lack clear technological complementarities. Nevertheless, regulators have expressed concerns about "ecosystem effects" affecting the same set of customers, leverage of market power from generalist services to premium services, and the potential softening of competition through bundling.[2]

From a traditional industrial organization perspective, such transactions are puzzling. When services are neither strong substitutes nor technological complements, and when cost synergies are limited, standard theory predicts that a merger should be competitively neutral, even if one of the merging firms has a large advantage in its market. Why, then, can such mergers be privately profitable? And under what conditions do they harm or benefit consumers?

This work provides a rationale for conglomerate mergers between providers of independent services with overlapping customer bases, beyond traditional explanations based on risk diversification, economies of scope and managerial incentives (Mueller, 1969; Montgomery, 1994). In our model consumers' preferences over generalist and premium services are independently and uniformly distributed on the unit square in the tradition of Matutes and Regibeau (1988, 1992), Salinger (1995), Nalebuff (2004), Gans and King (2006), Peitz (2008), Armstrong and Vickers (2010), Gayer and Shy (2016), Vincent (2024) and others. We consider a merger between a generalist firm with a content advantage and a premium provider. By construction, the merger generates neither demand complementarities nor cost synergies. Therefore, under separate pricing, it is neutral. However, once bundling is allowed, the merger fundamentally reshapes competitive interactions.

The underlying mechanism is simple. In the absence of bundling, firms compete only within their respective product dimensions. Bundling introduces cross-market competition between the merged entity and the rival generalist service. When the content advantage of the merging firm is moderate, substitutability is high and the main effect of bundling is to intensify price competition: the

[2] See Shapiro (2025) for a recent perspective on conglomerate mergers and the *Merger Guidelines* issued in 2023 by the U.S. Department of Justice and the Federal Trade Commission.

merged entity discounts the bundle, rivals respond aggressively, and consumer welfare increases.[3] When the advantage is large, the main effect of bundling is to increase effective differentiation between the bundle and the rival generalist service, so that consumers become less responsive to relative prices, allowing the merged entity to monetize its superior content through a higher bundle price: this starts softening competition, and can lead to consumer harm.[4]

We start by developing a baseline model in which two generalist firms compete *à la* Hotelling, while a premium provider is monopolistic. In this environment, a merger with pure bundling is profitable only when the merging firm's content advantage is sufficiently large. However, when the full set of pricing tools is available, the equilibrium strategy is mixed bundling: the merged entity offers a discounted bundle together with both standalone services when the content advantage is small, the bundle with only the premium service standalone when the content advantage is higher, and without the premium service when the quality advantage increases further. The equilibrium merger is always privately profitable and increases consumer surplus whenever the content advantage is not too large. Only when the content advantage becomes sufficiently large does bundling soften competition and reduce consumer welfare.

We then extend the analysis to competition between differentiated premium services. In particular, we consider a double Hotelling model with competition between two generalist services and between two premium providers. Such a framework has been analyzed by Vincent (2024) for symmetric generalist services without differences in content value, but provides novel insights when there is a content advantage for the generalist service that merges with a premium provider, which is the most relevant case for antitrust analysis.[5] Under pure bundling, when premium services are highly differentiated and have substantial market power, the merger strengthens competition and is both profitable and beneficial for consumers for intermediate levels of content advantage of the merging firm. However, when the full set of pricing tools is available, the equilibrium strategy is again some form of mixed bundling: in particular, for a large con-

[3] Competitive effects of bundling are well known since the work of Matutes and Regibeau (1988, 1992), though Zhou (2017) has recently shown that they may be reversed with more than two bundling firms. Related competitive effects emerge also in mergers of complements or vertically related products without bundling (see Lu, Moresi and Salop, 2007; Das Varma and De Stefano, 2020; De Stefano and Salinger, 2025).

[4] Bundling as a device to soften competition has been analyzed elsewhere (for instance by Carbajo, de Meza and Seidmann, 1990, Chen, 1997, Jeon and Menicucci, 2006, Amelio and Jullien, 2012, or Chen and Rey, 2025).

[5] For a general analysis of how bundling affects competition between two asymmetric multiproduct firms see Hurkens, Jeon and Menicucci (2019).

tent advantage the merger becomes profitable and involves the provision of the discounted bundle and the standalone generalist service at an increased price, without standalone premium service. Such a merger increases consumer welfare as long as the content advantage of the merging firm is not too high.

We finally consider competition between two perfectly substitutable premium services, showing that the conglomerate merger is neutral under mixed bundling, but harms consumers under a credible commitment to pure bundling. In such a case, pure bundling allows the non-merging premium service to gain market power and complement the non-merging generalist service: this generates a double marginalization that softens competition with the bundle of the merged entity, increasing prices and reducing consumer welfare. Such a mechanism resonates well with the ecosystem theory of harm of Chen and Rey (2025), which is also based on a softening of competition between the merged entity and the rivals.

The main policy implication emerging from our investigation is that conglomerate mergers are less likely to harm consumers when standalone services remain available and the content advantage is moderate, while concerns become more serious when bundling leads to the discontinuation of standalone services available before the merger (which, in practice, may require prohibitively high standalone prices). These conclusions remain relevant when the advantage is endogenized through investments in content creation by the generalist providers. This extension shows that bundling affects not only static price competition but also dynamic incentives to invest: different post-merger pricing regimes can generate different endogenous content advantage, which in turn reinforces or mitigates the static effects of the merger on profitability and consumer welfare.

Our work is related to the literature on conglomerate mergers in multi-firm environments (Choi, 2008; Vincent, 2024; Chen and Rey, 2025). The analysis of Choi (2008) is based on perfect complementarity between two services, each one provided by two competing firms, with differentiation between the four possible symmetric combinations of services. The model evaluates the impact of a merger between two providers of different services. A merger with separate pricing is profitable and strengthens competition because the merging firms internalize demand complementarities on their services and the rival firms react by reducing their prices as well, which creates benefits for consumers. However, a merger with unrestricted mixed bundling can harm consumers when there is high substitutability between all the combinations of services. We depart from this framework by eliminating any demand complementarity, so that the merger with separate pricing is entirely neutral. Nevertheless, the overlapping

set of customers of generalist and premium services allows the merger to become profitable under a restricted form of mixed bundling, which can either strengthen or soften competition depending on asymmetries between firms.

The closest work to our analysis of the double Hotelling model is the one by Vincent (2024), which, however, does not allow for differences in content value across services. In the Vincent model the merging firms have an incentive to adopt an unrestricted form of mixed bundling providing both standalone products, which strengthens competition but reduces profits compared to the pre-merger outcome. We depart from this framework allowing for a content advantage of the generalist provider engaged in the merger. This provides a rationale for the conglomerate merger, which becomes profitable for a large enough advantage, and is implemented in equilibrium through a restricted form of mixed bundling with the provision of the bundle and only one standalone service. In such a case, the merger can either strengthen competition when the content advantage is limited, or soften it when it is large.

A recent important work by Chen and Rey (2025) has developed a related theory of conglomerate mergers where bundling creates consumption synergies or one-stop shopping services for buyers and there is no content advantage for the merging firms. In that case bundling can arise either through a precommitment or as an equilibrium strategy (by setting prohibitively high standalone prices). This allows the merged entity to charge a premium for the bundle, but enhances product differentiation, softening competition with rival standalone offers, which can generate consumer harm. Contrary to that work, we abstract from consumption synergies and one-stop shopping benefits associated with bundling or other merger efficiencies, and we analyze the impact of the merger in function of the content advantage of the merging firm.

We finally relate to the recent literature on mergers and innovation, most of which is focused on mergers of firms producing substitute services (see Jullien and Lefouili, 2018, and Lefouili and Madio, 2026). A few works have analyzed the issue within mergers of firms producing complement services and investing in cost reductions with effects on the supply side (see Choi, 2008, and Etro, 2019). We focus on mergers of firms producing independent services and investing in content value with effects on the demand side.

The remainder of the paper is organized as follows. Section 2 introduces the baseline model with a monopolistic premium provider. Section 3 extends the framework to differentiated premium services. Section 4 analyzes pure bundling commitments under highly competitive premium services developing an ecosystem theory of harm for conglomerate mergers. Section 5 concludes.

2 Merger with a monopolistic premium service

We consider heterogeneous consumers characterized by types uniformly distributed on the unit square $(x, y) \sim U[0, 1]^2$. The type x represents the cost of distance from the generalist service of firm A and $1 - x$ is the one from the rival B. The type y represents the cost of distance from the premium service of firm C.[6]

The content values of the generalist services are respectively v_A and v_B, assumed large enough to ensure full market coverage, with an advantage $\Delta = v_A - v_B$ for the leading generalist service and $|\Delta| < 3$. In the static analysis we assume that the content advantage is non-negative to focus on mergers by a dominant firm. The maximum willingness to pay for the premium service is fixed at $u > 0$. Accordingly, there is no demand complementarity between generalist and premium services.

We assume that there are no marginal or fixed costs of production. Therefore, there is also no cost complementarity between generalist and premium services. All firms set prices simultaneously in a Bertrand style. Our objective is to evaluate the impact of a merger between the generalist provider A and the premium provider C.[7]

2.1 Pre-merger analysis

In our basic setup the premium provider is a monopolist. When the three firms are independent in the pre-merger environment, they set prices p_i for $i = A, B, C$ for their standalone services, and consumers select one of the two generalist services with values $v_A - x - p_A$ and $v_B - (1 - x) - p_B$, and decide whether to purchase the premium service as an add-on with value $u - y - p_C$.

Under Hotelling competition between generalist services, consumers purchase the service of firm A when their type x is below the cutoff:

$$\hat{x} \equiv \frac{1 + \Delta + p_B - p_A}{2}$$

[6] Notice that the unit square representation is without loss of generality since the dimensions of a rectangular support can be absorbed into transport cost parameters, but the assumption of unitary transport costs in both dimensions provides a symmetric benchmark. Asymmetries in transport costs would affect boundaries across demand configurations, but not the economic mechanisms of this section.

[7] Zero costs are without loss of generality under common constant marginal costs within each service category, since prices can be expressed net of costs. We abstract from cost asymmetries to isolate the effects of bundling and content advantage. We abstract from merger-specific cost efficiencies to verify whether and when such conglomerate mergers can be both profitable and beneficial for consumers even without efficiency justifications.

and the one of firm B otherwise. Moreover, they purchase the premium service C when $y < u - p_C$ and nothing otherwise. Accordingly, the unit square is divided into the four regions of Fig. 1a, where consumers purchase either a generalist service alone, A or B, or a combination of generalist and premium services, AC or BC. This yields the following profit functions:

$$\pi_A = p_A \left(\frac{1 + \Delta + p_B - p_A}{2} \right), \pi_B = p_B \left(\frac{1 - \Delta - p_B + p_A}{2} \right), \ \pi_C = p_C \left(u - p_C \right)$$

The Bertrand equilibrium prices can be easily derived as:

$$p_A^{pre} = 1 + \frac{\Delta}{3}, \quad p_B^{pre} = 1 - \frac{\Delta}{3}, \ p_C^{pre} = \frac{u}{2}$$

as long as $u < 2$ to ensure that only some consumers purchase the premium service. The advantage of firm A increases its price and reduces the price of the rival B, but does not affect the price of firm C due to demand independence. Given these prices, the equilibrium threshold becomes $\hat{x} = \frac{1}{2} + \frac{\Delta}{6}$, and differs from the efficient threshold, which would allocate purchases ignoring prices, requiring $\hat{x}^e = \frac{1}{2} + \frac{\Delta}{2}$ for $|\Delta| < 1$. Therefore, the equilibrium provides too little consumption of both the generalist service with a content advantage and of the premium service. The pre-merger profits are:

$$\pi_A^{pre}(\Delta) = \frac{1}{2} \left(1 + \frac{\Delta}{3} \right)^2, \quad \pi_B^{pre}(\Delta) = \frac{1}{2} \left(1 - \frac{\Delta}{3} \right)^2, \ \pi_C^{pre} = \frac{u^2}{4} \qquad (1)$$

and total consumer surplus can be computed as:

$$CS^{pre}(\Delta) = v + \frac{\Delta}{2} + \frac{\Delta^2}{36} - \frac{u^2}{8} - \frac{5}{4}$$

where we set $v = v_B = v_A - \Delta$. In the rest of this section we set $u = 1$ for tractability, but the basic mechanisms described below extend to other values, although thresholds change.[8]

We now compare the pre-merger equilibrium with the post-merger one. The merger would be neutral under separate pricing, because there are no demand complementarities or cost synergies.[9] However, there is always an incentive to offer a bundle (see McAfee, McMillan and Whinston, 1989), therefore we now

[8] Our parameter restrictions allow us to express all results in function of the content advantage of the merging firm as the unique parameter, which is also the fundamental one to discriminate between different effects of the merger.

[9] Demand complementarities generate a merger that is profitable and beneficial for consumers under separate prices in Choi (2008). Cost synergies emerge when there is a positive cost per customer c, paid only once on the joint provision of generalist and premium services by the merged entity. In such a case, the merger is already profitable with sep-

examine the case of a merger with a commitment to pure bundling and then the case of a merger with mixed bundling that emerges naturally when the merged entity has a full set of pricing tools available.

2.2 Merger with pure bundling

Let us consider a commitment to a *pure bundling* strategy by the merged entity. Consumers have to choose between a bundle AC with price P and value $v_A + 1 - x - y - P$, and a generalist service B without premium add-on with value $v_B - (1-x) - p_B$.[10] Bundling introduces direct competition between the merged entity and the rival, and the unit square is divided in two regions, with the bundle purchased by types with $y < \Delta + 2 - P + p_B - 2x$. The effect on competition depends on the magnitude of the content advantage Δ.

When the difference between generalist services is small, namely for $\Delta < 1$, the relevant region of the unit square where the bundle is purchased is a trapezoid in the lower left hand side of the unit square. By computing its area, we obtain the following profit functions:

$$\pi_M^{PB} = P\left[\frac{3 + 2(\Delta - P + p_B)}{4}\right] \text{ and } \pi_B^{PB} = p_B\left[\frac{1 - 2(\Delta - P + p_B)}{4}\right] \quad (2)$$

which are strictly concave in the own price. The best response functions of the two firms exhibit strategic complementarity in prices. In Bertrand equilibrium the price of the bundle is:

$$P^{PB} = \frac{7 + 2\Delta}{6}$$

which is always below the sum of pre-merger prices $p_A^{pre} + p_C^{pre} = \frac{9+2\Delta}{6}$, and the price of the rival firm is:

$$p_B^{PB} = \frac{5 - 2\Delta}{6}$$

which is also below the pre-merger level p_B^{pre} throughout the relevant range.[11] The strengthening of competition is not sufficient to infer the impact on con-

arate prices and always beneficial for consumers, because the merged entity internalizes cost complementarities and strengthens competition. For instance, with $\Delta = 0$, we have separate prices $p_A^{SP} = \frac{12+8c+c^2-c^3}{12-c^2} < p_A^{pre} = 1 + c$ for the merging generalist service, $p_B^{SP} = \frac{12+10c-c^3}{12-c^2} < p_B^{pre} = 1 + c$ for the rival, and $p_C^{SP} = \frac{6+3c-c^2}{12-c^2} < p_C^{pre} = \frac{1+c}{2}$ for the premium service. The merger is also profitable under bundling for a large enough c, without qualitative impact on the following results.

[10] As usual we assume that components of a bundle cannot be discarded or combined with a rival standalone service.

[11] Checking the adjacent demand regions using the complete piecewise demand, neither firm obtains a higher profit when the rival adopts the candidate equilibrium price. At the equilibrium prices it can be verified that the corner type $(x, y) = (1, 0)$ stops purchasing the

sumers because they lose some of the options available before the merger, but the post-merger consumer surplus can be computed as:

$$CS^{PB} = v + \frac{19\Delta}{36} + \frac{\Delta^2}{36} - \frac{35}{36}$$

which is always above the pre-merger level.[12] The merger with pure bundling leads to the following profits:

$$\pi_M^{PB}(\Delta) = \frac{1}{2}\left(\frac{7+2\Delta}{6}\right)^2 \quad \text{and } \pi_B^{PB}(\Delta) = \frac{1}{2}\left(\frac{5-2\Delta}{6}\right)^2 \tag{3}$$

Since it can be verified that $\pi_M^{PB} < \pi_A^{pre} + \pi_C^{pre}$, the merger with pure bundling is not profitable in this range of small content advantage. For instance, the case with $\Delta = 0$ provides $p_A^{pre} = p_B^{pre} = 1$, $p_C^{pre} = 0.5$ and $\pi_A^{pre} + \pi_C^{pre} = 0.75$ with $CS^{pre} = v - 1.13$ before the merger, and $P^{PB} = 1.17 < 1.5$, $p_B^{PB} = 0.86 < 1$ and $\pi_M^{PB} = 0.68$ with $CS^{PB} = v - 0.98$ after the merger. It is already well understood that a monopolist can adopt pure bundling of goods with heterogenous and uncorrelated valuations to expand demand and create benefits for consumers (see for instance Nalebuff, 2004, and Gayer and Shy, 2016),[13] but in our duopolistic setup pure bundling has the additional effect of strengthening competition (Matutes and Regibeau, 1988), which here dissipates all the profits of the bundling firm.

When $\Delta \in [1, 3)$, the unit square is divided as in Fig. 1b into a triangle in the upper right-hand corner representing the demand for the non-merging firm, and a residual pentagon representing the demand for the bundle. Since the height of the triangle is $1 + P - \Delta - p_B$ and the base is half of that, the profits of the two firms can be derived as:

$$\pi_M^{PB} = P\left[1 - \frac{(1+P-\Delta-p_B)^2}{4}\right] \quad \text{and} \quad \pi_B^{PB} = p_B\frac{(1+P-\Delta-p_B)^2}{4} \tag{4}$$

which are again strictly concave in the own prices. The first order conditions can be solved for the Bertrand equilibrium with prices:

$$P^{PB} = \frac{5\Delta - 5 + 3\sqrt{17 - 2\Delta + \Delta^2}}{8}$$

bundle when $\Delta = 1$, which defines the limit of this regime. While our focus is on a positive advantage for the merging firm, the equilibrium regime applies for $\Delta \in [-2, 1]$ and similar qualitative implications emerge also for $\Delta \in (-3, -2)$.

[12] Similar competition effects emerge in the analysis of mergers of complements (see Lu, Moresi and Salop, 2007; Das Varma and De Stefano, 2020; Akgun *et al.*, 2020; Etro and Moresi, 2024; De Stefano and Salinger, 2025)

[13] See Gayer and Shy (2016) on positive marginal costs. See also Menicucci, Hurkens and Jeon (2015) on non-uniform distributions.

for the bundle, and:

$$p_B^{PB} = \frac{1 - \Delta + \sqrt{17 - 2\Delta + \Delta^2}}{8}$$

for the rival provider.[14] Both of these prices remain below the corresponding pre-merger levels for a low enough Δ, but an increase of the advantage of the merging firm generates more product differentiation pushing for a softening of competition, and for a large enough advantage these prices overcome pre-merger levels.

This implies that the equilibrium profit of the merged entity $\pi_M^{PB}(\Delta)$ increases rapidly with its advantage and reaches joint pre-merger profits when $\pi_M^{PB}(\tilde{\Delta}) = \pi_A^{pre}(\tilde{\Delta}) + \pi_C^{pre}$ which can be solved for $\tilde{\Delta} \approx 1.07$. This allows the merger to become profitable for any $\Delta \in (1.07, 3)$.

Integrating utility over the different regions at the equilibrium prices, we obtain that for a low enough content advantage the merger increases consumer welfare $CS^{PB}(\Delta)$ because the impact of price reductions compensates for the loss of variety, but when the content advantage increases, the gradual softening of competition reduces the benefits for consumers until $CS^{PB}(\bar{\Delta}) = CS^{pre}(\bar{\Delta})$ for $\bar{\Delta} \approx 1.65$. This leaves a range of intermediate levels of content advantage $\Delta \in (1.07, 1.65)$ where the merger with pure bundling is both profitable and beneficial for consumers through the expansion of consumption for the high content generalist service and the premium service. Only when the advantage of the generalist firm is large enough, do softening of competition and loss of variety start harming consumers.

2.3 Merger with mixed bundling

Let us move to the analysis of the merger under *mixed bundling*. In general, we allow the merged entity to offer the bundle at price P, together with standalone services at prices p_A and p_C, while the rival generalist provider sets the price p_B. Consumers choose between the bundle AC, the standalone generalist services A and B, and the mix & match combination BC whose value is $v_B + 1 - (1 - x) - y - p_B - p_C$.

Under unrestricted mixed bundling, consumers purchase all the four options available according to the demand regions in Fig. 1c. The analysis of this general case is provided in the Appendix, where we show that the associated

[14] Also in this case neither firm obtains a higher profit in adjacent demand regions when the rival adopts the candidate equilibrium price. Notice that at $\Delta = 1$ there is continuity in the profit functions and in the equilibrium prices $P^{PB} = 1.5$ and $p_B^{PB} = 0.5$.

equilibrium is compatible with the sale of the standalone generalist service of the merged entity only for a small content advantage, namely for $\Delta < 1/9 \approx 0.11$. When the content advantage increases in this range, the merged entity reduces the gap between the price of the bundle P and the price of its standalone general service $p_A < P$ to attract more buyers to its valuable bundle. When $\Delta > 1/9$, the equilibrium does not involve $p_A < P$ and we obtain a regime of restricted mixed bundling where the merged firm is effectively providing only the bundle and the standalone premium service (in practice through a prohibitively high price of the standalone generalist service). In what follows we focus our analysis on this case of restricted mixed bundling.

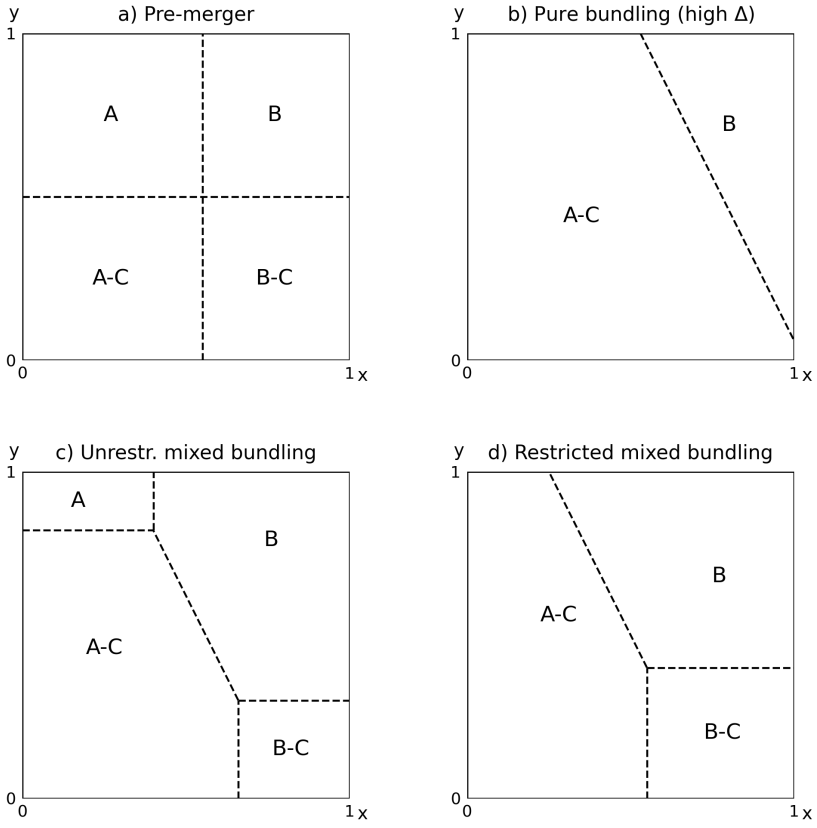


Fig. 1: Demands in the baseline model

When the merged entity provides the bundle and the standalone premium service, the unit square is divided in three regions as in Fig. 1d. To derive the

demand of the alternative options, notice that all consumers with $y < 1 - p_C$ purchase the premium service, and their relevant cutoff for the choice of the bundle over the mix & match combo is:

$$\hat{x}^* = \frac{1 + \Delta + p_B + p_C - P}{2}$$

For the other consumers, the relevant cutoff for the choice of the bundle over the standalone generalist service of the rival becomes:

$$\hat{x}(y) = \frac{2 + \Delta + p_B - P - y}{2}$$

This allows us to compute the demand for the bundle as:

$$\begin{aligned} D_{AC} &= (1 - p_C)\hat{x}^* + \int_{1-p_C}^{1} \hat{x}(y)dy \\ &= \frac{1 + \Delta + p_B + p_C - P}{2} - \frac{p_C^2}{4}, \end{aligned}$$

and the demand for the mix & match combination as:

$$\begin{aligned} D_{BC} &= (1 - p_C)(1 - \hat{x}^*) \\ &= \frac{(1 - p_C)(1 - \Delta - p_B - p_C + P)}{2}, \end{aligned}$$

with the residual demand in the unit square for the standalone service B.

Profits become:

$$\pi_M^{MB} = PD_{AC} + p_C D_{BC} \quad \text{and} \quad \pi_B^{MB} = p_B \left(1 - D_{AC}\right) \tag{5}$$

For a given price $p_C^{MB} = p$ of the premium service, the first order conditions for prices P of the merged entity and p_B of the rival provide:[15]

$$P^{MB} = \frac{6 + 2\Delta + 6p - 5p^2}{6} \quad \text{and} \quad p_B^{MB} = \frac{6 - 2\Delta - p^2}{6}$$

where the equilibrium price of the premium service solves the cubic equation $12 + 2(\Delta - 9)p = p^2(9 - 13p)$, which requires a small enough advantage to have an interior solution. We have verified that the merger firm's Hessian is negative definite at the equilibrium prices for any $\Delta < 1.12$, the profit of the rival firm is strictly concave in its price, and there are no profitable deviations from the candidate equilibrium prices.

[15] The first order condition for the bundle price is $P = \frac{1}{2}\left(1 + \Delta + p_B + 2p - \frac{3}{2}p^2\right)$ and the one for the price of the non-merging firm is $p_B = \frac{1}{2}\left(1 - \Delta + P - p + p^2\right)$. They show strategic complementarity between P and p_B and can be solved as in the text. The first order condition for the price of the premium service provides the remaining equation in the text.

The equilibrium can be computed as a function of content advantage. When this is low mixed bundling is in the unrestricted regime. For instance, when $\Delta = 0$ we can compute the equilibrium prices:

$$P^{MB} = 1.30 < 1.5,\ \ p_A^{MB} = 1.29 > 1,\ p_B^{MB} = 0.93 < 1 \text{ and } p_C^{MB} = 0.66 > 0.5$$

The merged entity attracts more customers toward the bundle by reducing its price and increasing the standalone prices, which increases the total price of the mix & match combination in spite of a lower price by the rival. This allows the merging firms to increase their profits to $\pi_M^{MB} = 0.78 > 0.75$. Nevertheless, the strengthening of competition with the rival increases consumer welfare to $CS^{MB} = v - 1.07 > CS^{pre}$, and increases also total welfare. Notice that there are well known cases where a multiproduct monopolist can adopt mixed bundling in the presence of consumers with heterogenous and uncorrelated valuations and still increase profits and consumer welfare (Nalebuff, 2004), but here we have shown that a merger with mixed bundling remains profitable also when mixed bundling strengthens competition with a rival firm.

When $\Delta > 0$ we obtain similar results, with a merger that is profitable and beneficial for consumers, and when $\Delta > 1/9$ the merged entity stops providing A as a standalone service and provides it only within the bundle, and keeps providing the standalone premium service C. Increasing Δ the merged entity keeps reducing the price of the bundle (compared to pre-merger levels) to attract more customers, and monetizes also on the mix & match sales through a higher price of the standalone premium service, which diverts further customers to the bundle. However, when Δ is large enough, namely for $\Delta > 1.12$, the standalone premium service is no longer provided and the equilibrium boils down to *de facto* pure bundling (as analyzed in the previous sub-section), where the merger is profitable but beneficial for consumers only when $\Delta < 1.65$.[16]

Summing up, the equilibrium with mixed bundling (i.e. where the merged entity has access to the full set of pricing instruments) involves the bundle with both standalone services by the merged entity when its advantage is limited (as in Fig. 1c), the bundle with the premium standalone service when the advantage is intermediate (as in Fig. 1d), and without the premium standalone service when the advantage is large (as in Fig. 1b). The associated profits for the merged entity are weakly higher than under pure bundling ($\pi_M^{MB} \geqslant \pi_M^{PB}$)

[16] Only for an extremely high content advantage, namely for $\Delta \in (2.67, 3)$, the merged entity finds it profitable to reintroduce the standalone general service, but also under this form of restricted mixed bundling the merger remains harmful for consumers. We omit the analysis of this regime because it does not affect the parameter region where the merger is both profitable and beneficial for consumers.

and the merger is always privately profitable when mixed bundling is adopted:

$$\pi_M^{MB} > \pi_A^{pre} + \pi_C^{pre} \quad \text{for any } \Delta < 3$$

Direct computation of consumer surplus in the unrestricted and restricted mixed bundling regimes, together with the pure bundling result (once the equilibrium becomes *de facto* pure bundling), shows that the merger is both profitable and beneficial for consumers when:

$$\Delta < 1.65$$

We summarize as follows our main findings:

Proposition 1. *Consider a Hotelling duopoly between generalist firms and a premium monopoly with independent demand. A merger of the generalist provider with a content advantage and the premium monopoly is profitable under mixed bundling and is also beneficial for consumers when the content advantage is below a threshold.*

Given these findings, it is important to endogenize the content advantage to evaluate the dynamic effects of the merger in this context, which is our next step.

2.4 Endogenous content advantage

The content advantage can be endogenized by considering preliminary investments in content value. This allows us to examine the dynamic impact of the merger. Our main purpose is to show that there are conditions where the merger increases aggregate investment and the advantage of the merging firms, which in turn can augment their profitability and also the associated benefits for consumers.

Let us consider costly investments e_A and e_B by the generalist firms that raise the willingness to pay of their customers. The cost $C(e_i)$ is increasing, strictly convex and twice differentiable in the investment for each firm $i = A, B$. Given a common basic content $\bar{v}$, we have $v_A = \bar{v} + e_A$ and $v_B = \bar{v} + e_B$ with $\Delta = e_A - e_B$. We examine a two stage game. In the first stage, investments are selected simultaneously by the two generalist providers on the basis of their expected profits. In the second stage, the price game takes place as before under mixed bundling for a given content advantage. Of course, we look for a subgame perfect equilibrium.

In a pre-merger scenario, the two generalist providers expect the net profits:

$$\Pi_A^{pre} = \frac{1}{2}\left(1 + \frac{e_A - e_B}{3}\right)^2 - C(e_A) \quad \text{and} \quad \Pi_B^{pre} = \frac{1}{2}\left(1 - \frac{e_A - e_B}{3}\right)^2 - C(e_B) \tag{6}$$

where the first term represents the gross equilibrium profits in (1) from the static price game as a function of the investment differential, and the second term represents the cost of investment.

Each firm selects its investment to maximize profits, delivering the following best response functions:

$$\frac{1}{3}\left(1 + \frac{e_A - e_B}{3}\right) = C'(e_A) \quad \text{and} \quad \frac{1}{3}\left(1 - \frac{e_A - e_B}{3}\right) = C'(e_B)$$

The Nash equilibrium investments and the endogenous content advantage satisfy:

$$\frac{1}{3} = C'(e^{pre}) \quad \text{with } \Delta^{pre} = 0 \tag{7}$$

under the second order condition $C''(e^{pre}) > 1/9$. The investment game in content creation has the nature of a rent-seeking arm race, since both firms incur investment costs without obtaining a relative content advantage in equilibrium.

To exemplify the analysis, let us consider a quadratic cost $C(e) = e^2/2$. This delivers the following best response functions:

$$e_A = \frac{3 - e_B}{8} \quad \text{and} \quad e_B = \frac{3 - e_A}{8}$$

which exhibit strategic substitutability. The Nash equilibrium investments and the endogenous content advantage can be derived as:

$$e^{pre} = \frac{1}{3} \quad \text{with } \Delta^{pre} = 0$$

which implies the joint net profits $\Pi_A^{pre} + \pi_C^{pre} = 0.69$ for the merging firms and consumer welfare $CS^{pre} = \bar{v} - 0.79$.

In the post-merger scenario with mixed bundling the expected profits are:

$$\Pi_M^{MB} = \pi_M(\Delta) - C(e_A), \quad \Pi_B^{MB} = \pi_B(\Delta) - C(e_B) \tag{8}$$

where the gross profit functions are derived from the static price game analyzed above and, assuming differentiability, the profit of the merging entity $\pi_M(\Delta)$ is increasing in Δ while the one of the rival $\pi_B(\Delta)$ is decreasing in Δ. The best response functions satisfy $e_A(\Delta) = \phi\left(\pi'_M(\Delta)\right)$ and $e_B(\Delta) = \phi\left(|\pi'_B(\Delta)|\right)$ for an increasing function $\phi(z) \equiv C'^{-1}(z)$, and the equilibrium content advantage is a fixed point of the following equation:

$$\Delta = e_A(\Delta) - e_B(\Delta) \tag{9}$$

Assuming that there is a solution, this implies a positive content advantage if the marginal profitability of investment is higher for the merged entity. For instance, numerical computations for our example with quadratic costs provide a unique Nash equilibrium with the following investments:

$$e_A^{MB} = 0.35 \quad \text{and} \quad e_B^{MB} = 0.33 \quad \text{with } \Delta^{MB} = 0.02$$

which is clearly consistent with the regime of unrestricted mixed bundling. We have verified the global investment incentives using the complete piecewise continuation equilibrium, including deviations generating $\Delta < 0$ or inducing a different bundling regime; neither firm has a profitable deviation from this equilibrium. The dynamic impact of the merger is a slight increase of aggregate investment to $e_A^{MB} + e_B^{MB} = 0.68 > 0.67$, due to an expansion of the investment of the merged entity. The merger is profitable ($\Pi_M^{MB} = 0.72 > 0.69$), and is also beneficial for consumers due to the aggregate increase in content value, which leads to $CS^{MB} = \bar{v} - 0.73 > CS^{pre} = \bar{v} - 0.79$.[17]

In conclusion, the merger can generate endogenous differences in content value and increase its profitability while remaining beneficial for consumers. It is important to remark that the nature of the post-merger equilibrium depends on the specification of the cost function. Within a differentiable regime, an interior investment equilibrium is characterized by (9), and a positive content advantage arises when the marginal profitability of investment is larger for the merged entity.

3 Merger with differentiated premium services

We now replace the monopolistic premium service with two differentiated premium providers to study the impact of competition in this market on the merger's effects. By extending the basic framework with competition between two generalist providers and two specialist providers in the unit square, we obtain a double Hotelling model *à la* Vincent (2024). However, we depart from that by allowing for a content advantage of the generalist firm that merges with a premium provider, which is the relevant case for antitrust analysis and gives novel insights.

The generalist services are the same as before, with the type x representing the distance from A and $1 - x$ representing the distance from B. The specialist

[17] Under a merger with commitment to pure bundling the equilibrium choices can be computed as $e_A^{PB} = 17/42$ and $e_B^{PB} = 11/42$ with $\Delta^{PB} = 1/7$, which preserves the aggregate investment concentrating most of it on the merging firm, but does not make the merger profitable ($\Pi_M^{PB} = 0.66 < 0.69$).

services are differentiated on the other dimension, with the type y representing the distance from the service of firm C, and $1-y$ now representing the distance from the new firm D. The maximum willingness to pay for each specialist service u is now assumed large enough to ensure full market coverage.

3.1 Pre-merger analysis

When the four firms are independent, they simultaneously set prices p_i for $i = A, B, C, D$ for their standalone services, and consumers select one of the two generalist services and one of the two specialist services. The unit square is divided into four regions in which consumers purchase AC, BC, AD or BD as shown in Fig. 2a. The equilibrium prices and profits of the generalist providers are the same as before, and the specialist providers compete *à la* Hotelling in a similar way. This leads to the following prices:

$$p_A^{pre} = 1 + \frac{\Delta}{3}, \quad p_B^{pre} = 1 - \frac{\Delta}{3}, \quad p_C^{pre} = p_D^{pre} = 1$$

and profits:

$$\pi_A^{pre}(\Delta) = \frac{1}{2}\left(1 + \frac{\Delta}{3}\right)^2, \quad \pi_B^{pre}(\Delta) = \frac{1}{2}\left(1 - \frac{\Delta}{3}\right)^2, \quad \pi_C^{pre} = \pi_D^{pre} = \frac{1}{2} \quad (10)$$

Total consumer surplus becomes:

$$CS^{pre}(\Delta) = u + v + \frac{\Delta}{2} + \frac{\Delta^2}{36} - \frac{5}{2}$$

Contrary to what happened in the baseline model where only some consumers were purchasing the premium service, here the pre-merger allocation distributes both services to every consumer. The pre-merger allocation of premium services is efficient, while the allocation of the generalist services is not. Efficiency gains can be created by expanding the consumption of the generalist service with a content advantage without distorting too much the allocation of premium services. Whether this can happen and whether merging firms and consumers can both benefit, as always, depends on what are the changes in the competitive structure.

We examine the impact of the merger between firms A and C. This is still neutral under a commitment to separate pricing.[18] Therefore we focus on pure and mixed bundling strategies in what follows.

[18] When there is a positive cost per customer c, the merger with separate prices is profitable and always beneficial for consumers as in the baseline framework. For instance, with $\Delta = 0$, we have $p_j^{SP} = \frac{6+3c-c^2}{6-c} < p_j^{pre} = 1 + c$ for the generalist providers $j = A, B$ and $p_k^{SP} = \frac{6+4c-c^2}{6-c} < p_k^{pre} = 1 + c$ for the premium providers $k = C, D$. The merger is also profitable with bundling for a large enough c, but the qualitative results that follow remain unaffected.

3.2 Merger with pure bundling

Under a commitment to *pure bundling* by the merged entity, consumers have a reduced choice between a bundle AC with value $v_A + u - x - y - P$ and a mix & match combination BD with value $v_B + u - (1-x) - (1-y) - p_B - p_D$, with the bundle purchased by types:

$$y < \frac{\Delta + 2 - P + p_B + p_D}{2} - x \tag{11}$$

As shown in Fig. 2 b, the demand of the mix & match combination BD corresponds to the area of a triangle in the upper right hand corner of the unit square, with sides of length $1 - \frac{\Delta - P + p_B + p_D}{2}$. The residual demand is destined to the bundle of the merged entity. The equilibrium prices derived below are consistent with positive demands in this region for any $\Delta \in (-1, 3)$, therefore the equilibrium demand geometry is internally consistent when there is a positive content advantage.[19]

This allows us to express the relevant profits of the three firms as:

$$\pi_M^{PB}(\Delta) = P\left[1 - \frac{(2 - \Delta + P - p_B - p_D)^2}{8}\right], \pi_j^{PB} = p_j \frac{(2 - \Delta + P - p_B - p_D)^2}{8} \tag{12}$$

for $j = B, D$. The profit function of each firm is strictly concave in its own price. Notice that while the bundle exhibits substitutability with the services of the non-merging firms, these services exhibit perfect complementarity between themselves, which gives rise to a double marginalization problem.

The Bertrand equilibrium involves the price of the bundle:

$$P^{PB} = \frac{3(\Delta - 2) + 2\sqrt{\Delta^2 - 4\Delta + 44}}{5}$$

and the prices of the non-merging firms:

$$p_B^{PB} = p_D^{PB} = \frac{2 - \Delta + \sqrt{\Delta^2 - 4\Delta + 44}}{10}$$

When the content advantage of the merging firm is small enough, these prices are below the corresponding pre-merger levels and generate lower profits for the merging firms. For instance, in the case with $\Delta = 0$ we have $P^{PB} = 1.45 < 2$ and $p_B^{PB} = p_D^{PB} = 0.86 < 1$, with profits $\pi_M^{PB}(0) = 0.91 < 1$ and $CS^{PB} = u + v - 2.24 > CS^{pre} = u + v - 2.5$.

[19] In the residual region with $\Delta \in (-3, -1)$ the demand of the bundle corresponds to a smaller area in the lower left hand side corner of the unit square and the equilibrium prices deliver an unprofitable merger.

However, when the content advantage increases, the merged entity increases the price of the bundle until it overcomes the joint pre-merger prices $p_A^{pre} + p_C^{pre}$ for $\Delta > 6(\sqrt{2} - 1) \approx 2.49$. Evaluating equilibrium profits shows that pure bundling becomes profitable only when the content advantage is sufficiently large. The merged firm's equilibrium profit $\pi_M^{PB}(\Delta)$ exceeds the joint pre-merger profits of firms A and C for a content advantage above a threshold $\tilde{\Delta}$ such that $\pi_M^{PB}(\tilde{\Delta}) = \pi_A^{pre}(\tilde{\Delta}) + \pi_C^{pre}$, which identifies $\tilde{\Delta} \approx 0.70$. This allows the merger to become profitable for any $\Delta \in (0.70, 3)$.

Changes in consumer surplus cannot be inferred from prices alone because pure bundling eliminates the mixed consumption configurations AD and BC. Integrating utility over the equilibrium allocations it emerges that the merger increases consumer welfare $CS^{PB}(\Delta)$ for a low content advantage, but the softening of competition due to an increase in content advantage reduces the benefit gains for consumers until $CS^{PB}(\bar{\Delta}) = CS^{pre}(\bar{\Delta})$ for $\bar{\Delta} \approx 1.76$. For a higher content advantage the merger harms consumers by softening competition and restricting variety. Therefore, there is a wide range of intermediate levels of content advantage $\Delta \in (0.70, 1.76)$ where the merger with pure bundling is both profitable and beneficial for consumers.[20]

3.3 Merger with mixed bundling

We finally turn to the equilibrium under the full set of pricing instruments for the merged entity, which takes the form of *mixed bundling*. This can emerge either as a restricted mixed bundling where only one standalone service is provided by the merging firms (Fig. 2c) or as an unrestricted mixed bundling where both standalone services are provided (Fig. 2d).

Vincent (2024) has analyzed the case with $\Delta = 0$ showing that what emerges is always the unrestricted form of mixed bundling, with two standalone services provided by the merged entity, which strengthens competition and makes the merger unprofitable. This is also the case for negative and positive but small values of Δ. However, for a larger enough content advantage the provision of the standalone premium service is not part of a mixed bundling equilibrium (see Appendix B), and the relevant mixed bundling equilibrium is a restricted regime in which the merged entity offers the bundle and the standalone generalist service, but not the standalone premium service (which, in practice, could be

[20] One may wonder whether such a merger incentivizes a countermerger of firms B and D adopting a pure bundling strategy. This is not the case because the countermerger would solve a double marginalization problem, but would also strengthen bundle-to-bundle competition reducing the profits of firms B and D.

offered at a prohibitively high price). Such a strategy is the equilibrium one in this case and can increase the profits of the merged entity compared to the pre-merger situation. Therefore it is our focus here.[21]

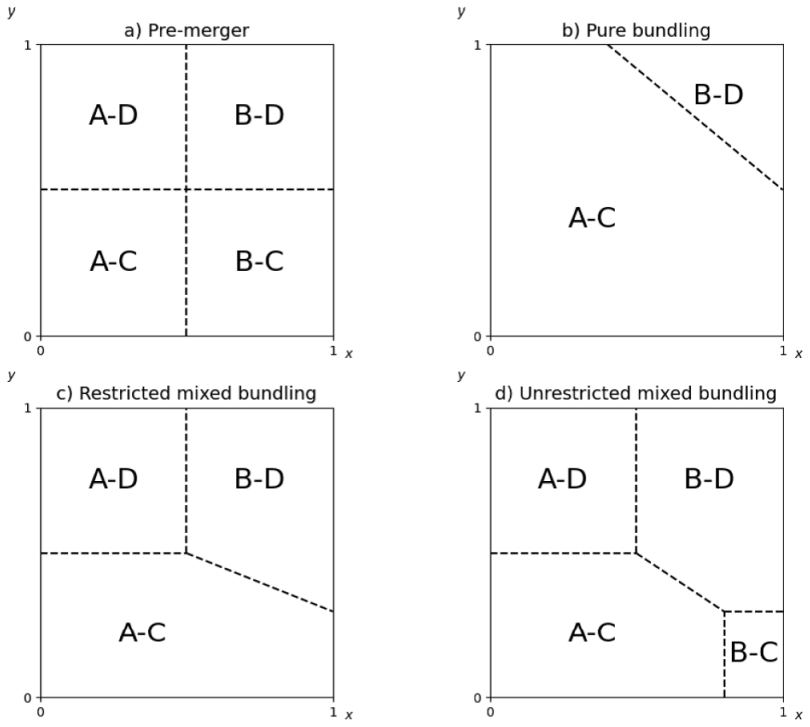


Fig. 2: Demands in the double Hotelling model

We consider the restricted mixed bundling regime where the merged entity supplies the bundle AC and its standalone generalist service, which can be combined with the other specialist service in the mix & match combination AD. The last option available for consumers is the combination BD with the two services of the non-merging firms (see Fig. 2c).

[21] Interestingly, over an intermediate range of content advantages, the identity of the standalone service A provided with the bundle is reversed relative to the baseline model of Section 2. With a premium monopoly, the merged entity could monetize on consumers purchasing the rival generalist service B by supplying the premium service C standalone. With premium competition, however, these consumers can purchase the rival premium service D, reducing the strategic value of a standalone premium service of the merged entity. Therefore, the merger provides the generalist service standalone to monetize on consumers who select the mix & match option AD. In practice, competition in the premium market can change which component the merged entity makes exclusive to the bundle.

The choice of the bundle AC over the mix & match combination BD requires the same condition (11) as under pure bundling. Moreover, the mix & match combination AD is preferred to the bundle AC by any type y above the cutoff:

$$\hat{y}^* \equiv \frac{1 + p_A + p_D - P}{2}$$

and to the alternative combination BD by any type x below the cutoff:

$$\hat{x} \equiv \frac{1 + \Delta + p_B - p_A}{2}$$

Given this, the profits of the three firms become:

$$\pi_M^{MB} = PD_{AC} + p_A D_{AD}, \quad \pi_B^{MB} = p_B D_{BD} \text{ and } \pi_D^{MB} = p_D(1 - D_{AC}) \quad (13)$$

where the merged entity monetizes on purchases of the bundle and its standalone service, the rival generalist firm monetizes on its standalone service, and the rival premium provider monetizes on all consumers who do not purchase the bundle.

The demands of the three regions can be computed geometrically as:

$$D_{AC} = \hat{y}^* - \frac{(1-\hat{x})^2}{2}, \quad D_{AD} = \hat{x}(1-\hat{y}^*) \text{ and } D_{BD} = (1-\hat{x})(1-\hat{y}^*) + \frac{(1-\hat{x})^2}{2}$$

The four first order conditions for the price of the bundle and the standalone services can be expressed as:

$$P^{MB} = 2\hat{y}^* - (1-\hat{x})^2 + \frac{2\hat{x} - (1-\hat{x})^2\hat{x}}{1 - \hat{x} + \frac{1-\hat{y}^*}{\hat{x}}}$$

$$p_A^{MB} = \frac{2 - (1-\hat{x})^2}{1 - \hat{x} + \frac{1-\hat{y}^*}{\hat{x}}}$$

$$p_B^{MB} = \frac{(1-\hat{x})(3 - 2\hat{y}^* - \hat{x})}{2 - \hat{x} - \hat{y}^*}$$

$$p_D^{MB} = 2\left[1 - \hat{y}^* + \frac{(1-\hat{x})^2}{2}\right]$$

Solving this system provides the four prices of the Bertrand equilibrium as long as $0 \leqslant \hat{x} \leqslant 1$ and $0 \leqslant \hat{y}^* \leqslant 1$. Along the relevant equilibrium branch, the merged entity's Hessian with respect to (P, p_A) is negative definite, while the profits of firms B and D are strictly concave in their respective prices. These conditions, as well as the restrictions defining the demand geometry, hold throughout the

relevant restricted mixed bundling range.[22]

While closed form solutions are not available for a positive content advantage, a numerical computation in function of the content advantage (as the only parameter) shows that the bundle is always offered at a discount, while the standalone service of the merged entity is offered at a higher price to increase the total price of the mix & match combination.[23] More important, such a mixed bundling strategy is always more profitable in equilibrium than the pure bundling strategy ($\pi_M^{MB} \geqslant \pi_M^{PB}$). A higher content advantage of the merging firm tends to soften competition with the rival generalist service, and the merger with mixed bundling becomes privately profitable for a large enough advantage:

$$\pi_M^{MB} > \pi_A^{pre} + \pi_C^{pre} \quad \text{for } \Delta \in (0.58, 3).$$

It is useful to recall that in this parametric range, this form of restricted mixed bundling is the only feasible mixed bundling equilibrium (see Appendix B: for $\Delta > 0.48$ the unrestricted solution no longer satisfies the second-order conditions, while restricted mixed bundling does and is immune from profitable unilateral deviations).[24]

Finally, consumer welfare increases under a merger with mixed bundling for a small advantage of the merging firms, but its equilibrium level $CS^{MB}(\Delta)$ reaches the pre-merger level when $CS^{MB}(\hat{\Delta}) = CS^{pre}(\hat{\Delta})$ for $\hat{\Delta} \approx 1.61$. For a lower content advantage, the merger diverts a large portion of industry rents toward both the merging firms and consumers attracted from their services

[22] If the merged entity used this restricted mixed bundling menu at $\Delta = 0$ we would obtain a simple solution with equilibrium prices:

$$P^{MB} = \frac{3}{2} < 2, \quad p_A^{MB} = \frac{7}{6} > 1, \quad p_B^{MB} = \frac{5}{6} < 1 \text{ and } p_D^{MB} = \frac{8}{9} < 1$$

and thresholds $\hat{x} = 1/3$ and $\hat{y}^* = 7/9$. In this case the bundle AC is sold at a discount compared to pre-merger levels, the combo of the rivals BD is also sold at a lower price to the majority of consumers, and the mix & match combination AD is sold at a price above the pre-merger level. The profits of the merged entity can be computed as $\pi_M^{MB} = 0.92$, which is below the unitary profits of firms A and C before the merger, and consumer surplus increases to $CS^{MB} = u + v - 2.25$ in this example. Therefore, both restricted and unrestricted mixed bundling are not privately profitable in the symmetric setup of Vincent (2024) without content advantage for the merging firms.

[23] The positive static effects of mixed bundling emerge also in standard models with complement goods (Choi, 2008). See also Baranes, Podesta and Poudou (2016) for other positive dynamic effects.

[24] Also in this case one may wonder if firms B and D would have an incentive to merge and adopt a mixed bundling strategy. As already noted by Vincent (2024) for the symmetric benchmark, this is not the case because the strengthening of competition would further reduce the profits of firms B and D.

through a low price of the bundle. For a higher content advantage consumers are harmed by the merger.

In conclusion, for $\Delta < 0.58$ the merger is never profitable (as in Vincent, 2024), but there is an intermediate range of content advantage:

$$\Delta \in (0.58, 1.61)$$

where the merger with restricted mixed bundling is profitable, is implemented through the provision of the bundle sold at a discount and the standalone generalist service sold at a higher price than the pre-merger level, and is also beneficial for consumers. We summarize as follows our findings:

Proposition 2. *Consider a Hotelling duopoly between generalist services and a Hotelling duopoly between premium services with independent demand. A merger of the generalist provider with a content advantage and a premium provider is profitable under mixed bundling and beneficial for consumers for intermediate levels of content advantage.*

At this point it is important to note that our focus has been on static price effects generated by the merger for a given market structure, namely one with a premium monopoly in Section 2 and one with a premium duopoly in this Section 3. However, one may consider an endogenous entry choice in the premium service sector with a positive entry cost. When the entry cost is high enough that there is an incentive to enter only for firm C we have the market structure of Section 2, and when the entry cost is lower we have the market structure of Section 3 with a premium duopoly between firms C and D. Then, it is interesting to note that a merger with bundling may be privately optimal even when unprofitable conditional on entry, because it can deter entry of firm D into the premium market. While interesting in this framework with mixed bundling, the possibility of a merger for entry deterrence purposes has already been emphasized by Church and Gandal (2000) and others.

Since the static impact of the merger depends on the size of the content advantage, we briefly reconsider endogenous investments in content value.

3.4 Endogenous content advantage

We endogenize content advantage by analyzing preliminary investments of the generalist providers.

In the pre-merger scenario, the Nash equilibrium involves the same investments for the generalist providers without creating any content advantage. For

illustrative purposes we use again the example of a quadratic cost function $C(e) = e^2/2$ that provides $e^{pre} = 1/3$ with $\Delta = 0$, and implies the profits of the merging firms $\Pi_A^{pre} + \pi_C^{pre} = \frac{4}{9} + \frac{1}{2} = 0.94$ and consumer welfare $CS^{pre} = u + \bar{v} - 2.17$.

In the post-merger scenario the gross profit functions in (13) depend on Δ directly and through the prices of the rivals as shown in the static analysis above. A Nash equilibrium implies a positive content advantage if the marginal profitability of investment is higher for the merged entity, which is not necessarily the case under unrestricted mixed bundling. For instance, our example with a quadratic cost delivers a unique equilibrium with unrestricted mixed bundling where:

$$e_A^{MB} = 0.30 \quad \text{and} \quad e_B^{MB} = 0.34 \quad \text{with } \Delta^{MB} = -0.04$$

which reduces both investments and profits for the merging firms. However, it is interesting to note that in this example a commitment to pure bundling or even restricted mixed bundling would lead the merged entity to invest more than the rival and deliver a merger that is both profitable and beneficial for consumers.[25] All of this confirms that the static effects of the merger, associated with price changes for given content, are only part of the merger effects: dynamic effects can complement them through the creation of additional content or reduce them by reducing the static benefits.

4 Merger with homogeneous premium services and an ecosystem theory of harm

The qualitative results of our main analysis are robust to changes in the degree of differentiation between services that affect the market power of their providers. However, genuinely different effects of the merger can emerge when the merged entity can adopt a commitment topure bundling and premium services are highly substitutable. These effects resonate well with what are commonly described as

[25] For instance, under commitment to pure bundling we obtain a unique equilibrium with the following investments:

$$e_A^{PB} = 0.50 \quad \text{and} \quad e_B^{PB} = 0.12 \quad \text{with } \Delta^{PB} = 0.38$$

This delivers net profits for the merged entity $\Pi_M^{PB} = 0.97$ and consumer welfare $CS^{PB} = u + \bar{v} - 1.96$, which are both above the pre-merger levels in spite of a marginal reduction in aggregate investment. This is not the case if one takes the same content advantage as exogenous.

ecosystem effects of conglomerate mergers (Chen and Rey, 2025) and a simple version of them is discussed in this section.

When competition between premium services is strong due to limited differentiation, a commitment to pure bundling can become the most profitable strategy for the merged entity because it softens competition in a substantial way with the non-merging rivals. In such a case a conglomerate merger with a credible commitment to pure bundling can create consumer harm. This perverse effect derives from the fact that the adoption of pure bundling shifts new market power to the remaining provider of premium services, which becomes the only available add-on for the rival generalist service. The services of the non-merging firms are perceived as perfect complements, generating a double marginalization problem that increases their prices and softens competition even more with the bundle of the merged entity.

To verify this outcome, we focus on the extreme case with two differentiated generalist services and two perfectly substitutable premium services. Consumers are characterized by a type $x \sim U[0,1]$ representing the consumer's distance cost from the generalist service of firm A, with $1-x$ as the cost of distance from the rival generalist service of firm B. Firm A has the advantage Δ in content value compared to firm B. Instead, firms C and D provide a homogenous premium service. We always assume zero costs for all firms. Firms set prices simultaneously.

In the pre-merger scenario, competition between generalist services works as a standard Hotelling competition, while Bertrand competition between homogeneous premium services leads to marginal cost pricing. This implies pre-merger prices:

$$p_A^{pre} = 1 + \frac{\Delta}{3}, \quad p_B^{pre} = 1 - \frac{\Delta}{3} \quad \text{and} \quad p_C^{pre} = p_D^{pre} = 0$$

and pre-merger profits:

$$\pi_A^{pre} = \frac{1}{2}\left(1 + \frac{\Delta}{3}\right)^2, \ \pi_B^{pre} = \frac{1}{2}\left(1 - \frac{\Delta}{3}\right)^2 \ \text{and} \ \pi_C^{pre} = \pi_D^{pre} = 0 \tag{14}$$

The associated consumer welfare can be computed as:

$$CS^{pre} = u + v + \frac{\Delta}{2} + \frac{\Delta^2}{36} - \frac{5}{4}$$

In this environment efficiency gains can be created by expanding the consumption of the generalist service with a content advantage, and there is no risk of distorting the allocation of premium services because they are regarded as perfect substitutes. Nevertheless, changes in the competitive structure may not create benefits for consumers.

Let us consider the merger of firms A and C. Under separate pricing the merger is always neutral because the premium services can only be provided at marginal cost and competition between generalist services is unaffected by this.

Under a merger with a commitment to pure bundling, instead, the competitive arena is completely reshaped. The merged entity offers the bundle AC with value $u + v + \Delta - x - P$, while the alternative synthetic bundle BD has value $u + v - (1 - x) - p_B - p_D$. Accordingly, the bundle AC is purchased by types $x < \frac{1}{2}(1 + \Delta - P + p_B + p_D)$, and the synthetic bundle BD is purchased by all the other types. The profits of the three firms become:

$$\pi_M^{PB} = P\left(\frac{1 + \Delta - P + p_B + p_D}{2}\right) \quad \text{and} \quad \pi_j^{PB} = p_j \frac{1 - \Delta + P - p_B - p_D}{2} \tag{15}$$

with $j = B, D$, and they are strictly concave in the respective prices. The first order conditions for profit maximization can be derived as:

$$P = \frac{1 + \Delta + p_B + p_D}{2}$$

$$p_B = \frac{1 - \Delta + P - p_D}{2}$$

$$p_D = \frac{1 - \Delta + P - p_B}{2}$$

The Bertrand equilibrium delivers a price of the bundle:

$$P^{PB} = \frac{5 + \Delta}{4}$$

and a total price of the rivals:

$$p_B^{PB} + p_D^{PB} = \frac{3 - \Delta}{2}$$

and none of the firms obtains higher profit from a unilateral price deviation inducing a corner allocation. The equilibrium prices of the bundle of the merged entity and of the synthetic bundle of the rivals are both higher compared to the corresponding levels before the merger, namely $p_A^{pre} + p_C^{pre} = \frac{3+\Delta}{3}$ and $p_B^{pre} + p_D^{pre} = \frac{3-\Delta}{3}$.

Assuming a symmetric scenario with the same price of the non-merging firms, we can compute firms's profits as:

$$\pi_M^{PB} = \frac{(5 + \Delta)^2}{32}, \quad \pi_B^{PB} = \pi_D^{PB} = \frac{(3 - \Delta)^2}{32} \tag{16}$$

These profits increase for the merging firms, as well as for the rival premium provider, compared to the pre-merger scenario.[26] The high differentiation between the two alternative offers generated by the commitment to pure bundling softens competition and increases prices, especially for the merging firms. Consumers are unambiguously harmed by higher prices, with consumer welfare:

$$CS^{PB} = u + v + \frac{42\Delta + \Delta^2 - 103}{64}$$

which is below the pre-merger level for any Δ.[27]

In this environment with homogeneous premium services consumer harm requires an effective commitment to pure bundling by the merged entity. Without a commitment to withdraw the standalone premium service of the merged entity, mixed bundling leads back to the pre-merger outcome. To verify this, notice that Bertrand competition between homogeneous premium providers implies zero prices for both of them: any positive standalone premium price can be profitably undercut. But given this, offering the bundle cannot alter consumers' effective relative prices between generalist services, and the equilibrium therefore reproduces the pre-merger outcome. We summarize our main findings as follows:

Proposition 3. *Consider a Hotelling duopoly between generalist services and a Bertrand duopoly between homogeneous premium services with independent demand. A merger of the generalist provider with a content advantage and a premium provider is neutral under mixed bundling, but is always profitable and harmful for consumers under a commitment to pure bundling.*

The merger impact under a commitment to pure bundling provides a particularly transparent special case of ecosystem-type concerns, that resonates well

[26] One may wonder whether such a merger would incentivize the non-merging firms to agree on a countermerger to provide a bundle of their services and avoid the double marginalization inefficiency. Somewhat surprisingly, the answer is no. To verify this, notice that the countermerger would lead back to the initial Hotelling competition between generalist providers where each one would offer also a homogeneous and free premium service. So the countermerger would deliver the same joint profits that firms B and D were obtaining in the pre-merger scenario, but:

$$\pi_B^{pre} + \pi_D^{pre} = \frac{(3-\Delta)^2}{18} < \pi_B^{PB} + \pi_D^{PB} = \frac{(3-\Delta)^2}{16}$$

Accordingly, there is no space for a profitable countermerger.

[27] Nevertheless, when content advantage is high enough, the reallocation of consumers toward higher quality content allows industry profits to increase more than the losses in consumer welfare (increasing total welfare when $\Delta > 3/17$).

with some of the findings of Chen and Rey (2025) in an environment with consumption synergies or one-stop shopping benefits from bundling.[28] We should remark that the adverse effects of the merger with bundling rely on the fact that this merger leaves only one firm as a standalone premium provider, which contributes to softening competition, but they would vanish with entry of new providers of the homogenous premium service (as already emphasized in the model of Chen and Rey, 2025). However, such providers would expect zero profits from entry, and a small entry cost would be sufficient to prevent also this option.

We can finally evaluate the dynamic effects of the merger in the current framework endogenizing the content value of the generalist providers. Let firms A and B undertake preliminary investments e_A and e_B in content creation at a common increasing, strictly convex and twice differentiable cost $C(e_i)$ so that $\Delta = e_A - e_B$.

Before the merger, the two providers choose the same investment so that there is no content advantage - as in (7). After the merger, using the gross profits from pure bundling (16) we obtain the net profits:

$$\Pi_M^{PB} = \frac{(5 + e_A - e_B)^2}{32} - C(e_A), \quad \Pi_B^{PB} = \frac{(3 - e_A + e_B)^2}{32} - C(e_B) \tag{17}$$

The first order conditions for an interior investment equilibrium are:

$$\frac{5 + \Delta}{16} = C'(e_A) \quad \text{and} \quad \frac{3 - \Delta}{16} = C'(e_B)$$

with second order conditions $C''(e_i) > 1/16$, which imply strategic substitutability. The best response functions satisfy:

$$e_A(\Delta) = \phi\left(\frac{5 + \Delta}{16}\right) \quad \text{and} \quad e_B(\Delta) = \phi\left(\frac{3 - \Delta}{16}\right)$$

for an increasing function $\phi(z) \equiv C'^{-1}(z)$, and the equilibrium content advantage is a fixed point of:

$$\Delta = \phi\left(\frac{5 + \Delta}{16}\right) - \phi\left(\frac{3 - \Delta}{16}\right) \equiv \Omega(\Delta)$$

If $C''(e) > 1/8$ over the relevant range, we have $\Omega'(\Delta) \in (0, 1)$, therefore $\Omega(\Delta)$ is a contraction, implying a unique and stable equilibrium. Moreover, $\Omega(\Delta) > 0$ for any $\Delta \geqslant 0$ because the marginal profitability of investment for the merging

[28] Chen and Rey (2025) demonstrate that consumer harm can arise even when a commitment to pure bundling is infeasible or prohibited.

firm is higher than for the rival, therefore the equilibrium must involve a positive content advantage $\Delta^{PB} > 0$ for the merged entity.

As an example, consider the quadratic cost function $C(e) = e^2/2$. Before the merger we have investments:

$$e^{pre} = \frac{1}{3} \quad \text{with } \Delta^{pre} = 0$$

and $\Pi_A^{pre} + \pi_C^{pre} = 0.44$. After the merger we immediately obtain the best response functions $e_A = \frac{5-e_B}{15}$ and $e_B = \frac{3-e_A}{15}$, and a Nash equilibrium characterized by:

$$e_A^{PB} = \frac{9}{28} \quad , \quad e_B^{PB} = \frac{5}{28} \quad \text{with } \Delta^{PB} = \frac{1}{7}$$

which reduces the investment of both firms and preserves the profitability of the merger ($\Pi_M^{PB} = 0.77 > 0.44$) and its negative impact on consumer surplus. Accordingly, in this context a commitment to pure bundling not only delivers a harmful merger in a static perspective, but reduces (and reallocates) investment in content amplifying the negative effects on consumers in a dynamic perspective.

It is important to remark that the harmful effects of a conglomerate merger with pure bundling apply not only with two homogeneous premium providers, but also in case of low differentiation (i.e. imperfect but high substitutability) between the premium services. In particular, retaining our Hotelling competition between firms A and B, and adopting Hotelling competition between firms C and D with a transport cost $d \in [0, 1]$, we obtain a double Hotelling model that nests the model with undifferentiated providers of this section for $d = 0$ and the baseline model with differentiated providers of the previous section for $d = 1$. What is important is that the results of the former case (including demands, prices and profits), hold as long as $d < (3 - \Delta)/4$, confirming that a merger with a commitment to pure bundling softens competition and harms consumers.

More generally, the double Hotelling model can be extended with different content values for all the four firms and different degrees of differentiation between the two services at the cost of becoming analytically intractable. The underlying symmetry of the model implies that the equilibrium merger with mixed bundling remains profitable for a large enough content advantage of the merging firms and increases consumer welfare for a small enough advantage. Accordingly, the merger is both profitable and beneficial in an intermediate region of the content advantage, although endogenous investments can either expand or reduce this region depending on the post-merger pricing regime. Instead, the

harmful effects of a merger with a commitment to pure bundling emphasized in this section are more likely to emerge when the two sectors are characterized by different degrees of market power.

5 Conclusion

We have shown that conglomerate mergers between platforms serving overlapping sets of customers can generate benefits for consumers by strengthening competition between the bundle and the offers of the rivals. In these cases mixed bundling emerges and the merger tends to be at the same time privately profitable and beneficial for consumers when the content advantage of the merging firm is not too large.

However, we have also emphasized that these kinds of conglomerate mergers can harm consumers when the content advantage is large or there is a credible commitment to restrict the supply of standalone services. Unlike foreclosure-based theories of harm, the mechanism underlying our theory is based on a softening of competition between the bundle of the merged entity and the complement offers of the non-merging rivals.

The main policy implication emerging from our analysis is that conglomerate mergers are less likely to harm consumers when standalone services remain available, while concerns become more serious when bundling leads to the discontinuation of pre-merger standalone options or when the content advantage is sufficiently large to soften competition. Several extensions could deepen our analysis of mergers in the unit square. First, relaxing the assumption of independent valuations could clarify how correlation in preferences or substitutability/complementarity between generalist and premium services affects the effects of mixed bundling. Second, allowing for richer pricing menus or multi-homing consumers would connect the model more directly to digital platform competition. Third, endogenous entry or innovation in premium markets would allow the study of dynamic leverage and entry deterrence. These directions may help bridge the gap between static bundling models and ecosystem competition.

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Appendix A

In this Appendix we derive the equilibrium with unrestricted mixed bundling for the model of Section 2.

We allow the merged entity to offer the bundle at price P, together with standalone services at prices p_A and p_C, while the rival generalist provider sets the price p_B. Consumers choose between the bundle AC, the standalone generalist services A and B, and the mix & match combination BC whose value is $v_B + 1 - (1 - x) - y - p_B - p_C$.

To derive the demand of the four alternative options, notice that all consumers with $y < 1 - p_C$ purchase the premium service, and their relevant cutoff for the choice of the bundle over the mix & match combo is:

$$\hat{x}^* = \frac{1 + \Delta + p_B + p_C - P}{2}$$

For the other consumers, the relevant cutoff for the choice of the bundle over the standalone generalist service of the rival becomes:

$$\hat{x}(y) = \frac{2 + \Delta + p_B - P - y}{2}$$

Finally, as long as there are consumers with $y > 1 + p_A - P$, they prefer standalone A to the bundle and standalone B to the mix & match option, selecting standalone A when $x < \hat{x}$ where:

$$\hat{x} = \frac{1 + \Delta + p_B - p_A}{2}$$

and standalone B otherwise.

This allows us to compute the demand for the bundle as:

$$D_{AC} = (1 - p_C)\hat{x}^* + \int_{1-p_C}^{1+p_A-P} \hat{x}(y)dy,$$

the demand of standalone A as:

$$D_A = (P - p_A)\hat{x},$$

and the demand for the mix & match combination as:

$$D_{BC} = (1 - p_C)(1 - \hat{x}^*)$$

with the residual demand in the unit square for standalone B. Accordingly, the post-merger profits of the two firms can be expressed as:

$$\pi_M^{MB} = PD_{AC} + p_A D_A + p_C D_{BC} \quad \text{and} \quad \pi_B^{MB} = p_B \left(1 - D_{AC} - D_A\right)$$

where the merged entity monetizes on the bundle and the standalone services, while the rival B serves all consumers that do not purchase the generalist service A.

The merged entity selects three prices. The first order condition for the price of the standalone generalist service p_A provides:

$$p_A = \frac{2(1+\Delta+p_B)}{3}$$

and the first order conditions for the other two prices can be obtained as quadratic expressions. The rival firm has the first order condition for the price p_B:

$$p_B = \frac{2(1-D_{AC}-D_A)}{1+P-p_A}$$

with second order condition always satisfied.

For a given price $p_C^{MB} = p$ of the premium service, the first order conditions provide:

$$P^{MB} = \frac{1+6\Delta p-3\Delta-11p+13p^2}{13p-9}$$

$$p_A^{MB} = \frac{2(9\Delta p-6\Delta+23p-16)}{3(13p-9)}$$

and

$$p_B^{MB} = \frac{3\Delta+10p-4\Delta p-7}{13p-9}$$

where p solves the equation:

$$2197+210\Delta+9\Delta^2 = 8(1049+39\Delta)p+4(1014p-2587)p^2$$

satisfying the interior demand restrictions and the second order conditions. We have verified that the merger firm's Hessian is negative definite at the equilibrium prices and there are no profitable deviations at the candidate equilibrium prices.

This equilibrium involves a positive demand for the standalone generalist service of the merged entity for a small enough advantage Δ. When Δ increases, the difference $P^{MB} - p_A^{MB}$ becomes smaller, and for $\Delta = 1/9$ it vanishes with:

$$P^{MB} = p_A^{MB} = \frac{4}{3} < 1.5, \quad p_B^{MB} = \frac{8}{9} < 1 \quad \text{and} \quad p_C^{MB} = \frac{2}{3} > 0.5$$

Accordingly, the unrestricted mixed bundling equilibrium applies for $\Delta \in (-3, 1/9]$. For $\Delta > 1/9$ the regime of restricted mixed bundling analyzed in the text becomes the post-merger equilibrium.

Appendix B

In this Appendix we briefly summarize the equilibrium with unrestricted mixed bundling for the double Hotelling model of Section 3, extending the characterization of Vincent (2024) with a content advantage for the merging generalist firm.

We consider a merged entity that supplies the bundle AC and its standalone services, which can be combined with the other generalist service in the mix & match combination BC and with the other specialist service in the mix & match combination AD, while the last option available for consumers is the combination BD with the services of the non-merging firms. The four regions are shown in the unit square of Fig. 2d.

The choice of the bundle of the merged entity AC over the synthetic bundle BD requires the same condition as under pure bundling, namely:

$$y < \frac{\Delta + 2 - P + p_B + p_D}{2} - x$$

Moreover, the mix & match combination AD is preferred to the bundle AC by a type y above the cutoff:

$$\hat{y}^* \equiv \frac{1 + p_A + p_D - P}{2}$$

and to the synthetic bundle BD by a type x below the cutoff:

$$\hat{x} \equiv \frac{1 + \Delta + p_B - p_A}{2}$$

The mix & match combination BC is preferred to the combination BD by a type y below:

$$\hat{y} \equiv \frac{1 - p_C + p_D}{2}$$

and to the bundle AC by a type x above:

$$\hat{x}^* \equiv \frac{1 + \Delta + p_B + p_C - P}{2}$$

Given this, the profits of the three firms become:

$$\pi_M^{MB} = PD_{AC} + p_A D_{AD} + p_C D_{BC},$$

$$\pi_B^{MB} = p_B(D_{BD} + D_{BC}), \text{ and}$$

$$\pi_D^{MB} = p_D(D_{BD} + D_{AD})$$

where the demands of the four regions can be computed geometrically as:

$$D_{AC} = \hat{x}^*\hat{y}^* - \frac{(\hat{x}^* - \hat{x})(\hat{y}^* - \hat{y})}{2},$$

$$D_{AD} = \hat{x}(1 - \hat{y}^*),$$

$$D_{BC} = (1 - \hat{x}^*)\hat{y} \quad \text{and}$$

$$D_{BD} = (1 - \hat{x})(1 - \hat{y}) - \frac{(\hat{x}^* - \hat{x})(\hat{y}^* - \hat{y})}{2}$$

The five first order conditions for the prices of the bundle and the standalone services can be expressed as:

$$P = 2\left[\frac{2\hat{x}^*\hat{y}^* - (\hat{x}^* - \hat{x})(\hat{y}^* - \hat{y}) + \hat{x}p_A + \hat{y}p_C}{\hat{y} + \hat{y}^* + \hat{x} + \hat{x}^*}\right]$$

$$p_A = 2\left[\frac{\hat{x}(1 - \hat{y}^*) + \left(\frac{\hat{x}^* + \hat{x}}{4} - \frac{\hat{y}^* - \hat{y}}{4}\right)P}{1 + \hat{x} - \hat{y}^*}\right]$$

$$p_B = 2\left[(1 - \hat{x}^*)\hat{y} + (1 - \hat{x})(1 - \hat{y}) - \frac{(\hat{x}^* - \hat{x})(\hat{y}^* - \hat{y})}{2}\right]$$

$$p_C = 2\left[\frac{(1 - \hat{x}^*)\hat{y} + \left(\frac{\hat{y}^* + \hat{y}}{4} - \frac{\hat{x}^* - \hat{x}}{4}\right)P}{1 + \hat{y} - \hat{x}^*}\right]$$

$$p_D = 2\left[\hat{x}(1 - \hat{y}^*) + (1 - \hat{x})(1 - \hat{y}) - \frac{(\hat{x}^* - \hat{x})(\hat{y}^* - \hat{y})}{2}\right]$$

whose solution provides the equilibrium with unrestricted mixed bundling as long as $0 \leqslant \hat{x} < \hat{x}^* \leqslant 1$ and $0 \leqslant \hat{y} < \hat{y}^* \leqslant 1$. The equilibrium corresponds to the one of Vincent (2024) with $\Delta = 0$ for the symmetric case with unitary transport costs in both services, and prices:

$$P^{MB} = 1.63 < 2, \quad p_A^{MB} = p_C^{MB} = 1.21 > 1, \quad p_B^{MB} = p_D^{MB} = 0.89 < 1$$

Computations for the general case with a non-null Δ show that the bundle is always provided at a discount, and the standalone services are provided by the merged entity at prices above the pre-merger levels, while the non-merging firms reduce their prices. In this regime of unrestricted mixed bundling the merger is never profitable, but is always beneficial for consumers.

For moderate levels of the content advantage Δ, along the relevant unrestricted mixed bundling equilibrium branch, the merged entity's Hessian with respect to (P, p_A, p_C) is negative definite, while the profits of firms B and D

remain strictly concave in their respective prices. These second order conditions and the restrictions defining the four region demand geometry hold for $\Delta \in (-3, 0.48)$. At $\Delta \approx 0.48$ the determinant of the merged entity's Hessian becomes zero and for larger values the unrestricted stationary solution no longer satisfies the second order conditions. Just below that threshold the merged entity starts having a profitable deviation that drops the standalone premium service with a high enough price p_C, as under the regime of restricted mixed bundling. For any higher value of the content advantage, the restricted mixed bundling solution analyzed in the text represents a valid equilibrium: it satisfies the interior demand restrictions and second order conditions and is immune from profitable unilateral deviations to either unrestricted mixed bundling or pure bundling. However, under this restricted mixed bundling the merger is profitable only for $\Delta \in (0.58, 3)$.