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Modelling Minskyan financial cycles with fundamentalist and extrapolative price strategies: An empirical analysis via the Kalman filter approach

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Abstract

In this paper we empirically analyse Minskyan financial cycles in asset prices, where the cycles are driven by the presence of two unobserved evaluation price strategies: the fundamentalist and the extrapolative price strategy. To achieve this, we construct a model, that incorporates the two behavioural equations and we investigate the financial cycles via a state space model. Using the Kalman filter, the conditions for the existence of cycles can be evaluated empirically. The model is estimated for four OECD countries using the times series of equity and housing prices over the period 1970-2017 for annual data. We find evidence of cycles in the equity market for the UK, France, Germany and the USA. Regarding housing prices, we find evidence of cyclical fluctuations in the UK, France and the USA but not in Germany. For both the equity market and the housing market, we find the highest price overshooting in the UK and the USA. Our results provide empirical support for the Minskyan theory, highlighting the role of the evaluation effect for an endogenous generation of cyclical phenomena in asset prices.

Keywords: Minsky cycles, asset prices, financial instability hypothesis, state space model, Kalman filter.

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1 Introduction

During the "Great Moderation" period, limited attention has been paid by mainstream economics to financial cycles. Finance has been seen as a veil, a factor that could be ignored in the studies of business fluctuations (Borio, 2014). Conversely, since the global financial crisis of 2007-8, the role of financial factors is back at the center of economic studies and Minsky's theory, the financial instability hypothesis (FIH) (Minsky, 1985), has started to receive considerable attention in the academic discourse. This theory emphasizes the role of financial factors in a capitalist economy, characterized by the gradual emergence of endogenous financial fragility which ultimately causes the demise of the boom phases. Due to no agreement on the formal presentation of Minsky's argument, FIH has been formalized and interpreted in different ways. Nikolaidi and Stockhammer (2017) and Nikolaidi (2017), two recent surveys on Minsky theory, identify two families of models. In the first, the dynamics of debt or interest rate is central in the analysis. In the second, asset prices play a fundamental role. Also, while some Minsky models analyse the cyclical dynamics or instability as the interaction between the real and financial markets, other models consider these phenomena primarily in the financial markets themselves.

Regarding the financial cycles in asset prices, the boom phase is characterized as a period during which, as the asset prices tend to increase, financial fragility increases as a consequence of endogenous actions adopted by the economic agents. In other words, the theory suggests that asset price dynamics models can generate pure financial cycles or instability, which depends endogenously on the mood of the agents. Furthermore, after the financial crisis, it is recognized that economic agents do not form their expectations as rational agents having superior cognitive capacities causing no bubbles and crashes. The agents, unable to observe all the information about the state of the economy, take their decisions following extrapolative actions thus feeding the boom and bust in the financial cycles. These not observable self-fulfilling beliefs produce waves of optimism and pessimism making the economy more systemically fragile and possibly giving rise to cycles (De Grauwe, 2012; Dieci & Westerhoff, 2012; Franke and Westerhoff, 2017). In this sense, a non-stationary economy must experience at least some fleeting moments of disequilibrium so that observed prices depend on "the state of the market" (Beja and Goldman, 1980). This argument is in line with the behavioural finance theory where changes in price occur not for fundamental reasons but because of such things as "animal spirits". This theory emphasizes the psychological and behavioural elements of traders in the determination of stock prices so that short-run momentum can be consistent with psychological feedback mechanisms (Schleifer and Summers, 1990; Shiller, 2003).

In this context, despite the high number of theoretical studies, there exist few empirical literature on the financial instability hypothesis. The existing empirical literature tries to establish a link between the theoretical models and the empirical analysis on Minsky focusing on the distinction between hedge, speculative and Ponzi states of a firm's condition for different countries and economic sectors[2] (Schroeder, 2009; Mulligan, 2013; Nishi, 2016; Davis et al., 2017). Other studies have explored the impact of financial variables on aggregate demand or their effect on the probability of a financial crisis (Palley, 1994; Kim, 2013, 2016). Only recently, Stockhammer et al. (2019a) and Stockhammer et al. (2019b) have contributed to the estimation of the endogenous interaction mechanism to highlight the causal mechanism driving financial-real interactions. As financial variables, these works include the interest rate as well as business and household debt. Conversely, there are no empirical Minsky models which incorporate an active role for asset prices.

[2]The over-indebtedness of firms is expressed via Minsky's categorization of firms into the hedge, speculative and Ponzi ones. Based on the relationship between cash flow and debt service requirements, firms gradually shift from hedge to speculative and Ponzi regimes generating a higher financial fragility and cycles.

In the light of these considerations, the present paper aims to provide empirical evidence of financial cycles in asset prices, where the cycles can be explained by the presence of two unobserved evaluation price strategies, the fundamentalist (price's evaluation based on fundamental price) and the extrapolative price strategy (price's evaluation based on past prices).[3] To achieve this, we construct a model, which incorporates the two unobserved price strategies and we investigate the financial cycles using the Kalman model-based filter via the state-space model where the conditions for the existence of financial cycles can be evaluated empirically. We estimate the parameters associated with the two price strategies to analyse the presence of financial cycles, the relative shares of the two economic agents in the market and at the same time the value of extrapolative price overshooting. The parameters are estimated by maximum likelihood utilizing the prediction error decomposition approach where the one-step prediction and updating equations are calculated via the state space form using the Kalman filter. This econometric methodology seems to be the most appropriate for its statistical characterization, in fact, it aims to model latent factors that cannot be measured directly but cause the responses on observed data. To the best of our knowledge, ours is the first examination of Minskyan cycles with the Kalman filter based model in a context of unobserved price strategies.

The model is estimated for four OECD countries (UK, France, Germany and USA) using the times series of equity and housing prices over the period 1970-2017 for annual data. We focus on equity prices both because they play a key role in Minsky models and because they are predominantly used as asset price indicators for macroeconomic analysis. The choice of housing prices is due to the increasing interest of real estate prices in the Minskyan framework after the global financial crisis (Ryoo, 2016). The obtained results provide evidence of financial fluctuations in the equity market for the UK, France, Germany and the USA, with the highest price overshooting in the financial asset-oriented market economies, respectively the UK and the USA. Regarding the housing prices, we find evidence of cyclical fluctuations in the UK, France and the USA with the highest price overshooting in the USA. For Germany, we find the lowest price overshooting and there is no evidence of financial fluctuations.

The contribution of this article is threefold. First, in the light of Minskyan theory, we introduce the role of asset prices for an empirical analysis of endogenous financial cycles. In this sense, we go beyond the existing empirical literature on Minsky (Nikolaidi and Stockhammer, 2017; Nikolaidi, 2017). Second, we propose a model in which the financial cycles are driven by the evaluation price strategies of speculative agents. The use of Kalman filter via state space model represents an important extension of the empirical Minskyan literature in terms of methodology and evidence. In this regard, the contribution of the present paper is to estimate the effect of this endogenous mechanism within the proposed analytical framework. Third, our modelling strategy combines the Minskyan with the behavioural argument. From that perspective, this work matches the increasing behavioural theoretic works where heuristic decisions of agents are considered as the first source of instability and fluctuations in the economy (Shiller, 2003; De Grauwe, 2012; Dieci & Westerhoff, 2012; Franke and Westerhoff, 2017).

The paper is organized as follows. Section II reviews both the related theoretical and empirical works. In section III we present the model with the conditions for the cycles. Section IV presents data and our econometric approach. In section V we discuss the estimation results. Section VI concludes with final considerations and directions for future research.

[3] We concentrate on the emergence of endogenous cycles in isolation, without taking into consideration the interaction of the real and financial markets as a source of instability. In other words, we investigate if the asset prices dynamics in a context of unobserved price strategy is the driver of cyclical behaviour.

2 Review of the related literature

In this section we review both the theoretical and empirical related literature. A revision of the theoretical works on Misnky's theory is presented in Nikolaidi and Stockhammer (2017). In the following section, we concentrate on the theoretical contributions in which the role of heuristic strategies is highlighted for the presence of instability and fluctuations both in the real and financial sector as well as in the housing sector. These theoretical studies range from the behavioural finance theory (Schleifer and Summers, 1990; Shiller, 2003), where the speculative thinking among investors plays a fundamental role in the determination of asset prices to the behavioural New-Keynesian Models, with the presence of heterogeneous agents for an endogenous explanation of bubbles and crash (De Grauwe (2012); Bofinger et al. (2013)).[4] In addition, we can find linear and non-linear dynamic models of speculative asset market in a disequilibrium setting (Beja and Goldman (1980); Westerhoff (2006); Lines and Westerhoff (2006); Dieci & Westerhoff (2012)). These theoretical works take in considerations the heterogeneity between agents and some of them employ one of the most widely analysed heterogeneous agent models developed by Brock and Hommes (1998). A brief review of the empirical literature on heterogeneous agents based model is presented in section 2.2.

Finally, in light of Minsky's contribution, in section 2.3 we review the empirical papers in the literature that have explored the impact of financial variables on aggregate demand or their effect on the probability of a financial crisis (Palley (1994); Kim (2013); Kim (2016)). Further, empirical papers on endogenous real-financial interaction mechanism are reviewed (Stockhammer et al. (2019a); Stockhammer et al. (2019b)).

2.1 Theoretical contributions

Beja and Goldman (1980) represents a pioneering theoretical work. They present a continuous dynamic model of the security prices process in a disequilibrium setting. The main idea is that financial theory has to consider the possibility that financial traders speculate in the financial market influencing the dynamics of the stock prices. They distinguish between fundamentalist and speculative traders acting on their perception of the current price trend and making it possible to observe phenomena which are unrelated to economic fundamentals. A final relevant observation can be grasped from the model: in the dynamic system presented, the speculation on the price-trend causes endogenous instabilities and oscillations in the asset prices.

Beja and Goldman's investigation study was opposite to the efficient market theory where, even if in the short term the unexpected exogenous shock can shift prices producing a financial crisis, in the long term the rational investors are winning out so that asset prices reflect all the relevant information.[5] At the same time, the work by Beja and Goldman (1980) was related to the behavioural theory. In behavioural finance, security price dynamics is unrelated to news stories about fundamentals and may attract other investors, promoting and increasing expectations for further price increases or decreasing under the feedback mechanism. If this process is not interrupted, it may produce both a positive or negative speculative bubble, in which extrapolative price strategy feeds for further price movements until the market reaches an unsustainable level (Schleifer and Summers, 1990; Shiller, 2003). Within the noise traders literature, a behavioural finance segments, two categories of traders can be present

[4]In the the rational representative agent models (including the DSGE-model), fully informed agents ensure that bubbles and crashes in asset prices do not occur. In these models, financial markets are efficient and asset prices are uniquely determined by underlying fundamentals. Large booms and busts phenomena are the consequence of exogenous shocks to the system.

[5]A detailed description and different interpretations of the efficient market theory can be found in Fama (1991) and Malkiel (2003).

in the market: information traders and noise traders. The first act on the basis of fundamental information and process information rationally. The second do not use fundamental information but heuristic technique such as extrapolative expectations and positive feedback dynamics. This theory analyses the conditions under which the noise traders may dominate in the market. The presence of these agents directly affects the prices, which can diverge significantly from fundamental value and so generate market inefficiency (Vikash et al., 2015).

More recently, De Grauwe (2012) highlights the role of "animal spirits" in output movements. A Behavioural Neo-Keynesian theoretical model (BNK) which consists of an aggregate demand equation, an aggregate supply equation and a Taylor rule is presented. In the model, agents are supposed to have limited cognitive abilities and expectations are not formed rationally. Agents can use fundamentalist or heuristic rules to form their expectations on output and inflation. In contrast to the paper by Beja and Goldman (1980), a time-variant selection mechanism is introduced, thanks to which agents evaluate the performance of the rule to perform. De Grauwe (2012) shows by numerical simulation how the heuristic rules of agents produce waves of optimism and pessimism in an endogenous way thus providing a good explanation of the observed oscillations in output. When agents who extrapolate positive output gaps dominate, it is possible to observe an increase in output growth. When agents who extrapolate negative output gaps dominate the growth rate of output tends to decline. Moreover, in De Grauwe and Macchiarelli (2015), preserving the financial accelerator approach, the existence of the financial sector intensifies these movements, amplifying the presence of animal spirits.

Westerhoff (2006) and Lines and Westerhoff (2006), relying on the seminal work of Samuelson (1939) and the multiplier-accelerator model, show how economic activity endogenously depends on the mood of the agents, thus emphasizing the role of heuristic strategies in the generation of the business cycle. Supposing that induced investment depends on the difference between expected national income at two different times, the cyclical dynamics in output is the result of extrapolative and regressive expectations of output. Always, in a context of heuristic expectations, they show that persistent oscillations are a generic possibility of the non-linear model. The same results are obtained in Westerhoff (2008) where agents hold optimistic or pessimistic beliefs concerning their long-run average income. In this case, however, the fraction of agents who follow one or the other predictor is fixed in time. In Westerhoff and Schmit (2016) real cycle is substituted with the financial cycle. The interactions between heterogeneous agents who rely on heuristics forecasting rules, not always based on careful and rational calculations, may stimulate endogenous oscillations in the financial market dynamics.

A theoretical mechanism, similar to the De Grauwe (2012), is presented in Bofinger et al. (2013) where they incorporate the Behavioural New Keynesian mechanism into a housing price model. Differentiating between patient and impatient households, in the light of financial accelerator mechanism, an expansionary shock increases the economic activity, which increases housing prices, relaxing at the same time the collateral constraint on impatient household's borrowing. The effects of the initial shock are amplified in a self-sustaining boom by the limited cognitive abilities of agents.

Leaving the Neo-Keynesian paradigm, the central role of house prices is highlighted by Dieci & Westerhoff (2012). Total demand for housing is created as an interaction between real and speculative demand, where the real demand is decreasing in price while the speculative demand is driven by price dynamics and depends on extrapolative and mean-reverting speculative strategies. It should be noted that an endogenous switching rule between different behavioural rules is considered as in the paper of De Grauwe (2012). Considering a two-dimension discrete non-linear difference equations, Dieci & Westerhoff (2012) study, with bifurcation theory, how bubbles and crashes can arise from the presence of different price strategies.

2.2 Heterogeneous agents: a brief review of the empirical literature

Theoretical models with the presence of heterogeneous agents (fundamentalist vs. extrapolative traders) can explain important observed stylized facts in financial time series, such as excess volatility, temporary bubbles and boom and bust phases. These models generate significant phenomena which are consistent with empirical observations. Shiller (1981) was one of the first that highlights how movements in stock prices are much larger than movements in underlying economic fundamentals. Statistical tests for excess volatility were developed, but the power of these tests are still debated (Malkiel, 2003).

In this context, studies that empirically estimate "animal spirits" models are less frequently encountered in the literature and few attempts have been made to estimate these type of models in search of direct empirical support. Regarding empirical testing, there are two different ways to try it, a direct and a more indirect way (Franke and Westerhoff, 2017). The first method treats a sentiment equation as a single-equation estimation, where the variable is proxied by an economic survey: this works collect information about sentiments of a specific group in the economy. Nowadays, there is a vast amount of empirical evidence from survey data stating that agents typically rely on relatively simple heuristics such as the use of extrapolative trading rules. References for comprehensive literature on this type of works can be found in Franke and Westerhoff (2017).

The second method considers a model as a whole and seeks to estimate its parameters in one effort. In this second case, the literature is increasing but we do not observe any consensus on the estimation methodology (Hommes, 2006). In recent years, such evidence is usually of analytical and simulation-based nature and computational agent-based models have the goal of replicating empirical features of actual market data (LeBaron, 2006). The paper by Kukacka and Barunik (2016), summarises all the main works which attempt to statistically estimate the parameters of various financial agents based model with the empirical data. The choice of estimated parameters, which range from less to more complicated direct techniques is affected by various model designs. We can list the non linear least squares, maximum and quasi-maximum likelihood, the method of simulated moments and in some specific simplified cases even by ordinary least squares among others. Moreover, different specification of behavioural strategies is considered in these type of models. We can find a fundamentalist - chartist setup where the switching between groups is permitted or not and where fractions of the two types can change over time, so passing from highly simplified model to more complicated models (Frankel and Froot, 1990; Chiarella et al. 2012).

These models are estimated for different financial market data. For example, models with heterogeneous agents have been applied both to stock prices market (Boswijk et al., 2007; Chiarella et al., 2014; Lof, 2012), and to the exchange rates market (Westerhoff and Reitz, 2003; De Jong et al., 2010).

These type of works suggest that sentiment dynamics is important in explaining the dynamics of the data but much more work is needed to investigate empirical findings. Our paper attempts to bring the heterogeneous between agents to the data within the proposed analytical framework in search of empirical support of endogenous financial cycles in the light of Minsky's theory.

2.3 Empirical contributions on Minsky's theory

Pioneering work is undoubtedly due to Palley (1994). In this paper, that has both a theoretical and an empirical part, Palley (1994) analyses three linear multiplier-accelerator models of the business cycle which incorporate the effect of a change in the stock of debt. Modifying a simple multiplier-accelerator model, it shows that an increase of debt flows produce at first an increase of the aggregate demand through consumption (and thus output) but in the long run, it decreases aggregate demand by the rising of the debt accumulation. In the empirical part, with a single-equation distributed lag model and a 3-dimensional Vector Autoregressive model (VAR), the paper shows that a shock to the change in consumer debt generates an initial positive and a subsequent negative GNP damped cyclical response.

The papers by Kim (2013) and Kim (2016) go in the same directions. The two contributions investigate the relationship between U.S. output, the household net worth and the consumer debt for the period 1951 to 2009. Kim (2013), with an Autoregressive Distributed Lag model (ARDL), provides evidence to support debt-driven cycles: the short change in the level of debt has a short positive effect on the gross domestic product (GDP), while the change in the level harms the real variable. A structural break test shows the evidence of a structural change in the relationship of U.S. output with the household debt. For pre-1982, there is no evidence that household debt variables had any adverse effect on output; the opposite, for the post-1982 period. In Kim (2016), the econometric analysis is different. Firstly, a VAR model is used to capture the short feedback effects: it provides a bidirectional positive feedback process between aggregate income and debt. In the second part of the analysis, the consumption variable is introduced in the system finding a first and a second order of cointegration. A Vector Error Correction model (VEC) is used to study the long-run analysis between variables of interest, finding a negative long-run relationship between household debt and output.

Ma and Zhang (2016) confirm the fundamental role played by the financial factors in macroeconomic dynamics. Based on the New Keynesian theoretic framework, they estimate an output gap equation, a Phillips curve, a monetary policy function with an equation with a composite financial cycle index for four major economic. They find that the positive coefficients of the financial index reveal that the financial cycle tends to exhibit pro-cyclical fluctuations for the business cycle. Moreover, exogenous shocks to the financial cycle index explain the volatility of real fluctuations.

Greenwood-Nimmo and Tarassow (2016), with a policy-oriented Minsky model, examine the implications of monetary and macro-prudential shocks for aggregate financial fragility using a sign restricted VAR model estimated with US quarterly data spanning the period 1960-2007. They show that when the interest rate is free to accommodate with macro-prudential shock, both the credit to GDP ratio and the financial ratio decline, indicating a reduction of financial fragility.

Passing to the endogenous real-financial interaction mechanism analysis, Stockhammer et al. (2019a) start from a reduced form system of simultaneous equations in which the real variable and the financial variable interact with each other. Two conditions guarantee endogenous oscillations in a debt-burdened growth: complex eigenvalues and negative sign of the coefficient's product of the Jacobian matrix. This means that from the interaction between the two state variables of the system an increase in one variable (the real one) induces an acceleration of the second variable (the financial one) which in turn drags down the first. Estimating the model for seven OECD countries over the period 1970 to 2015, Stockhammer et al. (2019a) find evidence for financial-real interactions at high frequencies between GDP and interest rate and a low frequency between GDP and business debt. No evidence between GDP and household debt interaction are found. Stockhammer et al. (2019b), using historical macroeconomic data for the USA (1889-2015) and the UK (1882-2010), estimate a Vector Autoregressive Moving Average model (VARMA), to investigate whether business cycles are driven by corporate debt or by mortgage debt. Similar to the precedent paper, US economy has indeed experienced corporate debt-driven Minsky business cycles over the sample period. Concerning the UK economy, its leverage ratio is pro-cyclical, but no robust evidence for debt-burdened growth is found. On the other hand, estimation using mortgage debt yields no evidence for mortgage debt-driven Minsky cycles.

From Table 1, it is relevant to notice that in all these works, the interaction of the goods market and financial markets is the main source of instability or cyclical phenomena. This empirical literature on Minskyan tradition does not take into consideration both the role of asset prices and the different evaluation strategies of economic agents that can have a direct effect on the dynamics of the financial asset. We intend to examine in depth this relationship in a context of unobserved price strategies.

Table 1: Empirical works on real-financial cycles

Year	Authors	Est. Strategy	Variables	Data
(1994)	*Palley*	VAR model	Per capita GNP Consumer debt	USA Quarterly (1975-1991)
(2013)	*Kim*	OLS/ ARDL models	GDP Consumer and household debt Mortgage debt	USA Quarterly (1951-2009)
(2016)	*Kim*	OLS/ VAR/ VEC models	GDP Consumer and Household debt Mortgage debt	USA Quarterly (1951-2009)
(2016)	*Ma and Zhang*	OLS/ VAR models	Real and financial index	USA/UK/JAPAN/CHINA Quarterly
(2016)	*Greenwood-Nimmo* et al.	VAR model	GDP, financial indexes	USA quarterly (1960- 2007)
(2019a)	*Stockhammer et al.*	VAR model	GDP Interest rates Business debt Household debt	7 OECD Countries Annual (1970-2015)
(2019b)	*Stockhammer et al.*	VARMA model	GDP Real investment Business debt Mortgage debt	USA Annual (1889-2015) UK Annual (1882-2010)

3 The model

In this section we describe the proposed modelling strategy. Let the observed asset price be P_t, for equity asset and housing price, dependent on the weighted sum of two unobserved stochastic dynamic components, respectively the fundamental price strategy P_t^f and the extrapolative (momentum) price strategy P_t^e

$$P_t = \gamma P_t^f + (1-\gamma) P_t^e \qquad 0 \leq \gamma \leq 1 \tag{1}$$

where the weights γ and $1-\gamma$ are the proportions of fundamentalists and extrapolative agents in the housing and equity market. In the context of the unobserved component model, fundamental and extrapolative price strategies are unobserved state variables that have to be specified parametrically.

Regarding the fundamentalists, following the efficient market hypothesis, fundamental prices strategy cannot be based on past prices information and consequently, historical prices are of no value. In this sense, the fundamental price strategy is based on the fundamental price which is known by fundamentalists and updated with time. The fundamental value is intrinsic to the asset and to the income stream, which in the mind of the fundamentalists the same asset can generate. So the fundamentalist strategy can be defined in the following way

$$P_t^f = \varphi P_{t-1}^f + \varepsilon_t \qquad \varphi = 1, \quad \varepsilon_t \sim \mathrm{N}\left(0, \sigma_\varepsilon^2\right) \tag{2}$$

where ε_t is the individual disturbance term which is normally distributed with mean zero and variance σ_ε^2. With $\varphi = 1$, Eq. (2) leads to the concept of a random walk. We assume that the fundamentalists believe that the price follows a random walk pattern where it is not possible to earn excess investment profit and so giving no incentive for speculation.

Passing to the momentum traders, we define their price strategies in the following way

$$P_t^e = P_{t-1} + \beta \left(P_{t-1} - P_{t-2}\right) + \eta_t \qquad \beta \geq 0, \quad \eta_t \sim \mathrm{N}\left(0, \sigma_\eta^2\right) \tag{3}$$

where β denotes the actual extrapolation parameter which captures the agent's price overshooting and η_t is the individual disturbance term which is normally distributed with mean zero and variance σ_η^2. From Eq. (3), when the asset price is above (below) its value at previous time, it follows that the economic agent optimistically (pessimistically) believe in a further price increase (decrease). Accordingly, its extrapolative strategy is positive (negative). This form of price strategy can be defined as a form of speculation on the current price trend affected by extrapolation beliefs that are not fully justified by fundamental news.

Given Eq. (1), the extrapolative price strategy can be rewritten in the following way

$$\begin{aligned} P_t^e &= P_{t-1} + \beta\left(P_{t-1} - P_{t-2}\right) + \eta_t = \\ &= \gamma P_{t-1}^f + (1-\gamma) P_{t-1}^e + \beta\left[\gamma P_{t-1}^f + (1-\gamma) P_{t-1}^e - \left(\gamma P_{t-2}^f + (1-\gamma) P_{t-2}^e\right)\right] + \eta_t = \\ &= \gamma P_{t-1}^f + (1-\gamma) P_{t-1}^e + \gamma\beta P_{t-1}^f + \beta(1-\gamma) P_{t-1}^e - \gamma\beta P_{t-2}^f - \beta(1-\gamma) P_{t-2}^e + \eta_t \end{aligned}$$

obtaining

$$P_t^e = \gamma(1+\beta) P_{t-1}^f + (1-\gamma)(1+\beta) P_{t-1}^e - \gamma\beta P_{t-2}^f - \beta(1-\gamma) P_{t-2}^e + \eta_t \tag{4}$$

We set

$$\begin{aligned} a_{21} &= \gamma(1+\beta) \\ a_{22} &= (1-\gamma)(1+\beta) \\ a_{23} &= -\gamma\beta \\ a_{24} &= -\beta(1-\gamma) \end{aligned} \tag{5}$$

so that

$$P_t^e = a_{21} P_{t-1}^f + a_{22} P_{t-1}^e + a_{23} P_{t-2}^f + a_{24} P_{t-2}^e + \eta_t$$

Now we write our model in the state space form for the econometric estimation. With state space models, the dynamic system is described by changes in the state of its components. The observed variable is represented as dynamic function of the unobserved state components, also called state variables, which are driven by a stochastic process. With this modelling strategy, we can reveal the nature and the cause of dynamic movement of observed variables in an effective way. In fact, it is possible to explain the behaviour of an observed variable by examining the internal dynamic properties of the unobserved components. An essential feature of any state space model is such that the state equation must be a first-order stochastic difference equation (Commandeur and Koopman, 2007; Enders, 2016). Passing to our model, the observation equation of the state space model is

$$P_t = \begin{pmatrix} \gamma & 1-\gamma & 0 & 0 \end{pmatrix} \begin{pmatrix} P_t^f \\ P_t^e \\ P_{t-1}^f \\ P_{t-1}^e \end{pmatrix} \tag{6}$$

Taking into account Eq. (5) and Eq. (2) with $\varphi = a_{11} = 1$, we have the transition equation of the state space model

$$\begin{pmatrix} P_t^f \\ P_t^e \\ P_{t-1}^f \\ P_{t-1}^e \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} P_{t-1}^f \\ P_{t-1}^e \\ P_{t-2}^f \\ P_{t-2}^e \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \eta_t \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

In a compact form, we define

$$P_t = HZ_t \tag{8}$$

$$Z_t = AZ_{t-1} + \delta_t \qquad \delta_t \sim \mathrm{N}\left(0, Q\right) \tag{9}$$

where P_t is the observable asset price,

$$Z_t = \begin{pmatrix} P_t^f \\ P_t^e \\ P_{t-1}^f \\ P_{t-1}^e \end{pmatrix}$$

is the state vector,

$$H = \begin{pmatrix} \gamma & 1-\gamma & 0 & 0 \end{pmatrix}$$

is the measurement matrix,

$$A = \begin{pmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

is the transition matrix and δ_t is the vector containing the state disturbance of unobserved components, normally distributed with mean zero and variances collected in the diagonal matrix Q.

The dynamic of the system is given by the transition equation which describes the evolution of the vector of unknown latent variables. Eigenvalues analysis can be performed to study the conditions for oscillations in our two-dimension discrete dynamic system associated with the two unobserved price strategies.[6] We obtain the associated characteristic equation considering the following determinant of the transition matrix:

$$\begin{vmatrix} a_{11}-\lambda & 0 & 0 & 0 \\ a_{21} & a_{22}-\lambda & a_{23} & a_{24} \\ 1 & 0 & -\lambda & 0 \\ 0 & 1 & 0 & -\lambda \end{vmatrix} = 0$$

from which

$$\begin{vmatrix} a_{11}-\lambda & 0 & 0 & 0 \\ a_{21} & a_{22}-\lambda & a_{23} & a_{24} \\ 1 & 0 & -\lambda & 0 \\ 0 & 1 & 0 & -\lambda \end{vmatrix} = \left(a_{11}-\lambda\right) \begin{vmatrix} a_{22}-\lambda & a_{23} & a_{24} \\ 0 & -\lambda & 0 \\ 1 & 0 & -\lambda \end{vmatrix} = -\left(a_{11}-\lambda\right)\left(\lambda\right) \begin{vmatrix} a_{22}-\lambda & a_{24} \\ 1 & -\lambda \end{vmatrix} = 0$$

so that, first of all we have the following two eigenvalues

$$\lambda_4 = a_{11} = 1 \in \Re \qquad \lambda_3 = 0$$

In addition, regarding the other two eigenvalues, they must satisfy

[6]See Appendix A

$$\begin{vmatrix} a_{22} - \lambda & a_{24} \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - a_{22}\lambda - a_{24} = 0$$

from which

$$\lambda_{1,2} = \frac{a_{22} \pm \sqrt{a_{22}^2 + 4a_{24}}}{2}$$

In order to have an oscillating behaviour, these two last eigenvalues have to be complex, so that we require

$$\Delta = a_{22}^2 + 4a_{24} < 0$$

$$a_{24} < -\frac{a_{22}^2}{4} \tag{10}$$

When this is the case:

$$\lambda_{1,2} = \frac{a_{22}}{2} \pm i\frac{\sqrt{-\left(a_{22}^2 + 4a_{24}\right)}}{2} = a + ib$$

where i is the imaginary unit and a and b are real numbers. a is called the real part of the complex number and ib is the imaginary part. The complex number in the cartesian form $a \pm ib$ can be written in the equivalent trigonometric form $\rho\left(\cos\omega \pm i\sin\omega\right)$. The positive number $\rho = \left(a^2 + b^2\right)^{\frac{1}{2}}$ is called the modulus of the complex number (Gandolfo, 2009).

In order to have oscillations of constant amplitude we require

$$\rho = 1$$

$$\sqrt{\left(\frac{a_{22}}{2}\right)^2 + \frac{-\left(a_{22}^2 + 4a_{24}\right)}{4}} = 1$$

$$\sqrt{-a_{24}} = 1$$

$$a_{24} = -1$$

Inserting in Eq. (10)

$$-4 < -a_{22}^2$$

$$-2 < a_{22} < 2$$

Then, the conditions to have oscillating behaviour of constant amplitude are

$$a_{24} = -1$$

$$-2 < a_{22} < 2$$

If the condition in Eq. (10) is respected, with $-1 < a_{24} < 0$ (length of eigenvalues < 1) we have damped oscillations. With $a_{24} < -1$ (length of eigenvalues > 1) we have explosive oscillations. Summarizing we have an oscillating system if

$$|a_{11}| \leq 1 \qquad \forall a_{21}, \forall a_{23} \qquad a_{24} < -\frac{a_{22}^2}{4} \tag{11}$$

4 Data and Econometric approach

The dataset, with annual frequency, consists of four OECD countries: USA, UK, Germany and France. We consider the time series of equity prices and housing prices with a sample size ranges from 1970 to 2017. The main source for our time series is the OECD database. We use deflated series for all the variables. Housing prices and equity prices series are deflated by the GDP deflator which is taken from the Federal Reserve Economic Database for all the countries.[7]

Once the model is in state space form, the recursive Kalman filter algorithm is used in calculating the optimal estimator of the state variables and in estimating the model parameters. Precisely, the parameters of the model are estimated by maximum likelihood using the prediction error decomposition approach where the one-step prediction and updating equations are calculated via the state space form using the Kalman filter.[8] Given the vector prediction errors and the variance-covariance matrix of the system, the log likelihood can be maximized.[9]

In the econometric analysis we set $a_{11} = \varphi = 1$ for the fundamentalists. For the extrapolative agents, the coefficients a_{21}, a_{22}, a_{23} and a_{24} are estimated. To obtain oscillations, conditions in Eq. (11) have to be respected. Moreover, we estimate γ to obtain the proportion of fundamentalists and extrapolative agents both in equity and housing market. Once we obtain our estimation results, it is possible to obtain β from Eq. (5). From Eq. (5) it follows that

$$\begin{cases} a_{21} + a_{23} - \gamma = 0 \\ a_{22} + a_{24} + \gamma = 1 \end{cases}$$

These linear equality constraints parameter transformer for constrained likelihood objective function maximization have been imposed to obtain for β two values that differs for the sign. Considering Eq. (3), for the economic meaning, the positive value for the price overshooting has been chosen.

An assumption is made about the standard deviation of the error term of the two-state equations in the equity market, respectively σ_ε for the fundamentalists and σ_η for the extrapolative traders. While in principle the Kalman filter allows estimating the two parameters together with the other parameters of the model, the standard deviations can be fixed in empirical works. In this sense, the error variance ratio $\lambda^2 = \frac{\sigma_\varepsilon^2}{\sigma_\eta^2}$ can be usefully imposed a priori. We restrict our standard deviations such that $\lambda = 0.2$, indicating a higher standard deviation for the extrapolative stochastic equation respect to the fundamentalist one.

The adopted hypothesis inferred from the conceptual meaning of the standard deviation and the effect of momentum traders in the financial market. The standard deviation corresponds to the volatility, which is a measure of the percentage of the price change over time. In other words, the standard deviation indicates the magnitude of the fluctuations with which the variable moves around the average. In general, volatility in financial markets refers to a period of price turbulence characterized by large fluctuations so that, the higher this indicator, the more significant is the share of fluctuations over time.[10] Furthermore, it is the investors with their actions and reactions that determine the volatility

[7] For the econometric analysis all the series are transformed in log levels.

[8] See Appendix B.

[9] The estimation procedure has been implemented with Matlab programming codes.

[10] In Ma and Zhang (2016), when there is a drop in the standard deviations of the variables, both macroeconomic and financial fluctuations tend to decrease. In Greenwood-Nimmo and Tarassow (2016), asset prices volatility is explicitly driven by bubbles.

of the stock market. Following Shiller (2003), investors' operations are influenced by emotional and psychological forces that cause prices to deviate systematically from their fundamental value. For this reason, extrapolative expectations are worse in terms of macroeconomic volatility. The evidence regarding excess volatility seems to imply that changes in prices occur because of the presence of "animal spirits" and not for fundamental reasons. However, as the choice of λ is relatively arbitrary, the results of a robustness analysis of the effects of changing λ over the interval [0.1 - 0.3] is reported in Appendix C. Section 5.1 discusses in detail.

5 Estimation results

Tables 2 and 3 report the maximum likelihood estimates of a_{22}, a_{24}, γ and β for equity prices and housing prices in the UK, France, Germany and the USA. The estimate of the model's parameters with the cyclical conditions, the log-likelihood with the sample size and Akaike information criterion are given in the four columns headed by the country name.

Regarding the equity asset (Table 2) in all the countries there is significant evidence of the presence of financial cycles. The signs of a_{22} and a_{24} respect conditions in Eq. (11) for oscillatory phenomena. In particular, we have damped fluctuations ($-1 < a_{24} < 0$) with a_{22} inside the allowed range size ($a_{22}^2 < -4a_{24}$). Moreover, all the estimated coefficients are statistically significant at 1% statistical level.

Looking at the percentage of the two different types of agents in the financial market, for all the countries considered in the estimation analysis, we notice that the fundamentalists (γ) are the majority in comparison with the extrapolative agents ($1 - \gamma$). Nevertheless, the percentage of the latter is high enough to be considered significant in the movement of the observed price. In the UK, 82% of the agents are estimated to be fundamentalists while the remaining 18% are extrapolators. In France and Germany, the extrapolative agents represent 29% and 26% respectively while the fundamentalists are estimated to be 71% and 74%. In the USA, 88% of agents are estimated to be fundamentalists and the remaining 12% are extrapolators. It is worth noting that the percentage of fundamentalists and extrapolative agents results to be significant at 1% for all the countries considered.

Once we obtain these results, from a_{22}, a_{24} and $1-\gamma$, it is possible to obtain the value of β to analyse the price overshooting of the extrapolative agents. In the UK and the USA, even if the percentage of momentum traders is inferior in comparison with Germany and France, the price overshooting is higher. The highest price overshooting is in the UK ($\beta = 3.5$), followed by the USA ($\beta = 2.2$), Germany ($\beta = 1.8$) and France ($\beta = 1.5$). For example, in the UK, when the asset price is above (below) its value at the time before, extrapolative behaviour implies that the economic agent optimistically (pessimistically) believes in a further price increase (decrease) of 3.5 times.

Overall, in all the countries considered, the obtained results provide empirical support of Minsky's hypothesis for the existence of univariate financial cycles in equity prices as a consequence of the different price strategies defined in our model. The percentage of extrapolators is lower compared to the fundamentalists, especially in the UK and the USA, even if the price overshooting is higher in these two countries in comparison with Germany and France. From the obtained results we notice similarities in the equity market across countries. These results seem to confirm the idea that the expectation forces which move the equity prices come from a global financial interconnected market linking the cyclical phenomena between countries. Namely, it is much more a global, rather than national, phenomenon. These results seem to be coherent with the observation of a similar dynamics movement in the observed asset prices for the four countries (see Appendix D).

Passing to the housing prices, maximum likelihood estimate results via the Kalman filter for the UK, France, Germany and the USA are summarized in Table 3. For all the countries we find that

Table 2: Estimation via Kalman filter for equity prices

	Countries			
	UK	France	Germany	USA
a_{22}	0.8083***	0.7189***	0.7347***	0.3988***
	(0.1878)	(0.1308)	(0.1052)	(0.0868)
a_{24}	-0.6316***	-0.4324***	-0.4773***	-0.2743***
	(0.2148)	(0.1476)	(0.1107)	(0.0941)
γ	0.82***	0.71***	0.74***	0.88***
	(0.0489)	(0.0297)	(0.0286)	(0.0107)
$1-\gamma$	0.18***	0.29***	0.26***	0.12***
	(0.0489)	(0.0297)	(0.0286)	(0.0107)
β	3.5	1.5	1.8	2.2
	Cyclical Conditions			
	UK	France	Germany	USA
$[-1 < a_{24} < 0]$	*yes*	*yes*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*yes*	*yes*	*yes*
Log-Likelihood	19.7509	7.84399	14.0683	24.5548
Akaike	-27.5019	-3.68798	-16.1367	-37.1095
Sample size	48	48	48	48
Years	(1970-2017)	(1970-2017)	(1970-2017)	(1970-2017)

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

both the sizes and the signs of a_{22} and a_{24} respect conditions number (11) for damped fluctuations. However, although the conditions for a cycling mechanism are formally satisfied, compare with the equity case, these results are not statistically significant for Germany. For the UK, France and the USA, both a_{22} and a_{24} are statistically significant at the 1% level.

Regarding the percentage of economic agents in the market from the estimation results we notice that in the UK, Germany and the USA the percentage of fundamentalists (γ) is higher in contrast to the percentage of extrapolative agents ($1-\gamma$). For the UK, the 69% of agents are fundamentalists and the remaining 31% are extrapolators. In Germany, the extrapolative agents represent 30% while the fundamentalists are estimated to be 70%. In the USA, 74% of agents are estimated to be fundamentalists and the remaining 26% are extrapolators. In France, the proportions of agents is very similar with the extrapolative agents representing the 51% while the fundamentalists are estimated to be 49%. Similar to equity asset, these percentages are statistically significant at 1% for all the countries considered. As before it is possible to obtain the positive values of β to analyse the price overshooting of the extrapolative agents from a_{22}, a_{24} and $1-\gamma$. We find the higher price overshooting in the USA with a value of β equal to 3.7. This value is followed by the price overshooting in the UK with France ($\beta = 1.9$) and in Germany ($\beta = 0.2$).

Summarizing the obtained results for the housing market, we find empirical evidence of Minsky's cycle in the UK, France and the USA. For Germany, we obtain no statistically reliable evidence of cyclical phenomena from the proposed model. Qualitatively speaking, these differences seem to be confirmed by the observed price's series of the four countries: unlike the UK, France and the USA, the house price fluctuation in Germany is less evident (see Appendix D).

Comparing these results from the house market to those for the equity market, we find similarities between the two markets. With the exception of Germany, we find empirical evidence of Minsky's hypothesis for the existence of univariate financial cycles in a context of different price strategies in the two asset prices. In general, we notice a lower percentage of extrapolative agents compare with the fundamentalists with the highest price overshooting in the UK and the USA, the two advanced financial asset market-oriented economies. In this sense, the speculative position is primarily taken from beliefs that are not shared by the majority of the market.

Moreover, the obtained results confirm the importance of considering the housing prices affected by the presence of speculative forces that can generate cyclical fluctuations. The same forces of behavioural strategy that drive international financial markets also have the potential to affect other markets, like the housing market. In fact, it does not appear possible to explain the boom and bust in terms of fundamentals such as construction costs (Shiller, 2005; Shiller, 2007). The qualitative differences between equity asset and housing price can be detected by the smoothed estimate of the state variables, always obtained via Kalman filter (see Appendix E).

However, while our model suggests that there is evidence of financial cycles in asset prices, more complex model specifications could be considered. At the theoretical level, different unobserved price strategies and the inclusion of an external variable into the fundamentalist equation could be considered. For example the equity market profit, or household income for the housing prices. Regarding the extrapolative price strategy, a credit shock for the extrapolators could be included to highlight the presence of fluctuations. In fact a credit shock for the momentum traders would increase the volume of money that can be mobilized, generating and amplifying cyclical phenomena. However, these modifications require an extension of the model proposed. These extensions should be explored in future research.

Table 3: Estimation via Kalman filter for housing prices

	Countries			
	UK	France	Germany	USA
a_{22}	0.8991***	1.5102***	0.3580*	1.2195***
	(0.0036)	(0.0894)	(0.1935)	(0.0220)
a_{24}	-0.5924***	-0.9968***	-0.0583	-0.9599***
	(0.0004)	(0.0009)	(0.1928)	(0.0083)
γ	0.69***	0.49***	0.70***	0.74***
	(0.0032)	(0.0885)	(0.0016)	(0.0252)
$1-\gamma$	0.31***	0.51***	0.30***	0.26***
	(0.0032)	(0.0885)	(0.0016)	(0.0252)
β	1.9	1.9	0.2	3.7
σ_ε	0.0830***	0.0621***	0.0305***	0.0374***
	(0.0074)	(0.0018)	(0.0025)	(0.0032)
σ_η	0.0670***	0.0376***	0.0000	0.0673***
	(0.0172)	(0.0037)	(0.0000)	(0.0061)
	Cyclical Conditions			
	UK	France	Germany	USA
$[-1 < a_{24} < 0]$	*yes*	*yes*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*yes*	*yes*	*yes*
Log-Likelihood	56.297	67.8361	111.475	77.1056
Akaike	-96.594	-119.672	-206.951	-138.211
Sample size	48	48	48	48
Years	(1970-2017)	(1970-2017)	(1970-2017)	(1970-2017)

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

5.1 Robustness Checks

Our estimation results for equity asset seem to be independent on a single a priori choice of the variance ratio. The results appear to be robust to the change of λ, so we consider this to be a useful approach to estimation. Robustness checks on the changes in estimates caused by varying the parameter λ over the interval [0.1,0.3] are reported in the Tables C1 and C2, Appendix C.

For the equity asset, as is evident from the results of Tables C1 and C2, the estimations do not change significantly for $\lambda = 0.1$ and $\lambda = 0.3$. In particular for the cyclical conditions, we have always damped fluctuations ($-1 < a_{24} < 0$) with a_{22} inside the allowed range size ($a_{22}^2 < -4a_{24}$). Moreover, all the estimated coefficients remain statistically significant in the majority of the cases at 1% statistical level. As before, the percentage of extrapolators is lower compared to the fundamentalists with the highest price overshooting in the UK and the USA.

It is worth noticing a tendency of the point estimates for the extrapolators to increase as λ increases. A possible explanation for the simultaneous increase of the percentage of extrapolators with the error variance ratio is that if the extrapolative unobservable state variable is more volatile (the error variance ratio decreases), the dispersion around the average tends to increase, reaching some values for which becomes less influential in the formation of the observed time series. The change of the percentage of the extrapolators, from Eq. (5), directly affects the value of price overshooting which in turn is indirectly affected by the error variance range size. For this reason, the lower bound of the range size is chosen to generate empirically plausible values of the price overshooting (for $\lambda = 0.1$, Appendix C, $\beta = 6.3$ in the USA and $\beta = 8.1$ in the UK seem to be implausible values respect to the baseline model with $\lambda = 0.2$). The upper bound reflects the standpoint that volatility associated with the extrapolators should be significantly higher than fundamentalist volatility rate.

6 Conclusions

Empirical evidence for the presence of financial cycles in asset prices that arise from the endogenous interaction of the fundamentalist and the extrapolative price strategies has been presented. The proposed model is formulated in a state space form and the parameters are estimated using standard maximum likelihood via Kalman filter. We find evidence of financial cycles in the equity market for the UK, France, Germany and the USA with a high statistical significance. At the same time, there is statistical evidence for such an interaction mechanism in the housing market for the UK, France and the USA, but not in Germany. Further, we find evidence for the highest extrapolative price overshooting in the financial asset-oriented market economies, namely the UK and the USA.

The obtained results, having both theoretical and empirical implications, contribute to the literature in two main aspects. Firstly, concerning the discussion in the Minskyan literature, the results of the proposed model supports empirically the theory of Minskyan univariate financial cycles in asset prices. In this sense, our work goes beyond the existing empirical literature which does not consider the asset prices (Nikolaidi and Stockhammer, 2017).

Secondly, our empirical results lend support to models in which behavioural strategies play a key role for the dynamic of the economy, highlighting the role of heuristics as drivers of endogenous cyclical behaviour (De Grauwe, 2012; De Grauwe and Macchiarelli, 2015; Franke and Westerhoff, 2017). The obtained results shed further light on the relevant aspect of the different price strategies, suggesting the fundamental role of extrapolative strategies and positive feedback in generating fluctuations and instability both in the equity market (Beja and Goldman, 1980; Westerhoff and Schmit, 2016) and in the housing market (Dieci & Westerhoff, 2012; Bofinger et al., 2013). In other words, our results contrast with the standard theoretical approach to asset price fluctuations, based on rational expectations and market "fundamentals". Conversely, these findings appear to be in line with the idea that price changes are not explained by an economic fundamentals variations but by the use of heuristics (Shiller, 2003).

Future research could aim to integrate other mechanisms in the framework proposed so as to improve the approximation of the asset price dynamics. Natural extensions of the baseline model proposed can be considered. The model can be modified with time-varying coefficients of the measurement matrix and the transition matrix. At the same time, other price strategies can be introduced in the model, for example the mean-reverting price strategy, the adaptive price strategy among others. In this sense, an external exogenous variable representing the fundamental variable can be taken into account. Moreover, in the model proposed, financial cycles are not linked to the real sector of the economy so future studies could be direct to the analysis of the relationship between the real and financial sector in a multivariate state space model setting.

Finally, even if this task is beyond the scope of our paper, policy implications could be found if the extrapolative traders affect the rest of society by causing a cost of their actions. It is necessary to understand how to stabilize or control the financial fluctuations to avoid a negative repercussion on the rest of the economy. In conclusion, we have to consider the role of the financial cycle and possible instability moving away from the idea that price changes always reflect rational and precise information in a permanent efficient financial market. Evidence from Minsky's theory can help us to go in this direction.

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Appendix A

Let us consider a discrete system

$$U = [u_i(t)] = \begin{bmatrix} u_1(t) \\ . \\ u_i(t) \\ . \\ u_r(t) \end{bmatrix} \in \Re^{rx1}$$

where

$$u_i(t) : \Re \to \Re \qquad i = 1, ..., r \quad t \in [0, T]$$

We assume that:

Hp.1) functions $u_i(t)$ can be thoroughly described by their values assumed in discrete time.

Introducing the vector

$$U_j = [u_i(t_j)] \quad t_j = j\Delta t \quad j = 1, 2, ..., n \quad n\Delta t = T$$

Hp.2) the values at time t_j can be expressed by the values assumed at previous times $t_{j-1}, ..., t_{j-R}$ where R is the memory's degree.

Introducing the vector

$$[\mathbf{U}_j] = \begin{bmatrix} U_j \\ U_{j-1} \\ . \\ U_{j-(R-1)} \end{bmatrix}$$

the condition assumed by the second hypothesis can be expressed by

$$[\mathbf{U}_j] = \begin{bmatrix} U_j \\ U_{j-1} \\ . \\ U_{j-(R-1)} \end{bmatrix} = [\mathbf{A}] \begin{bmatrix} U_{j-1} \\ U_{j-2} \\ . \\ U_{j-R} \end{bmatrix} = [\mathbf{A}] [\mathbf{U}_{j-1}] \qquad j = R+1, ..., N$$

where

$$[\mathbf{U}_k] \in \Re^N \quad [\mathbf{A}] \in R^{NxN} \quad N = rR$$

It should be noted that it is necessary to know the state vector at the first R-times to activate the recursive law. Assuming in the previous equation $j = 1, ..., N$ (that correspond to assume that the state vector is known at R previous times), the previous recursive law can be expressed by

$$\begin{aligned}
&\mathbf{U}_2 = \mathbf{A}\mathbf{U}_1 \\
&\mathbf{U}_3 = \mathbf{A}^2\mathbf{U}_1 \\
&... \\
&\mathbf{U}_j = \mathbf{A}^j\mathbf{U}_1
\end{aligned}$$

Let be $\mathbf{V}$ and $\mathbf{D}$ the matrix of the eigenvectors and eigenvalues of the matrix $\mathbf{A}$

$$\mathbf{A} = \mathbf{V}\mathbf{D}\mathbf{V}^{-1} \quad \mathbf{V}\mathbf{V}^{-1} = \mathbf{I}$$

so that

$$\mathbf{U}_j = \mathbf{V}\mathbf{D}^j\mathbf{V}^{-1}\mathbf{U}_1$$

also, the behaviour of the recursive law is entirely described by the values of the eigenvalues

$$\lambda_i \quad i = 1, ..., N$$

When $\lambda_i \in \Re$, $i = 1, ..., N$, the system is constant if $\lambda_i = 1 \quad \forall i$, monotonic increasing (explosive oscillations) if $\lambda_i > 1$ for one i, monotonic decreasing (damped oscillations) if $\lambda_i < 1$ for one i.

In order to have an oscillating behaviour it is necessary that

$$\lambda_i \in C \quad i = 1, ..., N$$

Moreover, the behaviour is depending on the modulus ρ of the complex eigenvalues. Amplitude will be increasing, constant or decreasing if, respectively, ρ is greater than equal or smaller than unity.

Now let us consider $r = 1$ and $R = 2$

$$u_j = \alpha u_{j-1} + \beta u_{j-2}$$

so that

$$\begin{bmatrix} u_j \\ u_{j-1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} u_{j-1} \\ u_{j-2} \end{bmatrix}$$

with

$$\mathbf{A} = \begin{bmatrix} \alpha & \beta \\ 1 & 0 \end{bmatrix}$$

We consider

$$\det \begin{bmatrix} \alpha - \lambda & \beta \\ 1 & -\lambda \end{bmatrix} = \lambda^2 - \alpha\lambda - \beta = 0$$

so that the eigenvalues are

$$\lambda_{1,2} = \frac{\alpha \pm \sqrt{\alpha^2 + 4\beta}}{2}$$

In order to have an oscillating behaviour, the eigenvalues have to be complex so that

$$\Delta = \alpha^2 + 4\beta < 0$$

$$\beta < -\frac{\alpha^2}{4} \tag{A.1}$$

When this is the case:

$$\lambda_{1,2} = \frac{\alpha}{2} \pm i\frac{\sqrt{-\left(\alpha^2 + 4\beta\right)}}{2} = a + ib$$

where i is the imaginary unit and a and b are real numbers. a is called the real part of the complex number and ib is the imaginary part. The complex number in the cartesian form $a \pm ib$ can be written in the equivalent trigonometric form $\rho\left(\cos\omega \pm i\sin\omega\right)$. The positive number $\rho = \left(a^2 + b^2\right)^{\frac{1}{2}}$ is called the modulus or absolute value of the complex number (Gandolfo, 2009).

In order to have oscillations of constant amplitude we require

$$\rho = 1$$

$$\sqrt{\left(\frac{\alpha}{2}\right)^2 + \frac{-\left(\alpha^2 + 4\beta\right)}{4}} = 1$$

$$\sqrt{-\beta} = 1$$

$$\beta = -1$$

Inserting in Eq. (A.1)

$$-4 < -\alpha^2$$

$$\alpha^2 - 4 < 0$$

$$-2 < \alpha < 2$$

Then, the conditions to have oscillating behaviour of constant amplitude are

$$\beta = -1$$

$$-2 < \alpha < 2$$

If the condition in Eq. (A.1) is respected, with $-1 < \beta < 0$ (length of eigenvalues < 1) we have damped oscillations. With $\beta < -1$ (length of eigenvalues > 1) we have explosive oscillations.

Connecting to our model with $r = 2$ and $R = 2$, where $u_1 = p^f$ and $u_2 = p^e$, we have

$$\begin{pmatrix} u_{1,j} \\ u_{2,j} \\ u_{1,j-1} \\ u_{2,j-1} \end{pmatrix} = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} u_{1,j-1} \\ u_{2,j-1} \\ u_{1,j-2} \\ u_{2,j-2} \end{pmatrix}$$

$$\begin{vmatrix} a_{11}-\lambda & 0 & 0 & 0 \\ a_{21} & a_{22}-\lambda & a_{23} & a_{24} \\ 1 & 0 & -\lambda & 0 \\ 0 & 1 & 0 & -\lambda \end{vmatrix} = (a_{11}-\lambda) \begin{vmatrix} a_{22}-\lambda & a_{23} & a_{24} \\ 0 & -\lambda & 0 \\ 1 & 0 & -\lambda \end{vmatrix} = -(a_{11}-\lambda)(\lambda) \begin{vmatrix} a_{22}-\lambda & a_{24} \\ 1 & -\lambda \end{vmatrix} = 0$$

The first two eigenvalues are

$$\lambda_4 = a_{11} \in \Re \qquad \lambda_3 = 0$$

Regarding the other eigenvalues, it should be noted that the problem is equivalent to the precedent case so that the system is oscillating if

$$|a_{11}| \leq 1 \qquad \forall a_{21}, \forall a_{23} \qquad a_{24} < -\frac{a_{22}^2}{4} \tag{A.2}$$

Appendix B

The Kalman filter is a recursive dynamic procedure for calculating the optimal estimator of the state vector (Enders, 2016). The goal is to minimize the mean square prediction error of the unobserved state vector conditional of the observation of P_t.

The optimal forecasting rule has the form

$$Z_{t\,|\,t} = Z_{t\,|\,t-1} + K_t \left(P_t - P_{t\,|\,t-1}\right)$$

where K_t is a weight that changes as new information becomes available, $Z_{t\,|\,t}$ denotes the forecast of state variable once P_t is realized while $Z_{t\,|\,t-1}$ and $P_{t\,|\,t-1}$ denote respectively the forecast of variables Z_t and P_t before P_t is realized.

Now we can select the optimal value of K_t to minimize the mean square prediction error at time t

$$\min_{k_t} E_t\big(Z_t - Z_{t\,|\,t}\big)^2 = \min_{k_t} E_t\big[Z_t - \big(Z_{t\,|\,t-1} + K_t\left(P_t - P_{t\,|\,t-1}\right)\big)\big]^2$$

using the equation (8) for the observable asset price, we obtain

$$\min_{k_t} E_t\big[Z_t - \big(Z_{t\,|\,t-1} + K_t\left(HZ_t - HZ_{t\,|\,t-1}\right)\big)\big]^2$$

$$\min_{k_t} E_t\big[(I - HK_t)\left(Z_t - Z_{t\,|\,t-1}\right)\big]^2$$

$$\min_{k_t} (I - HK_t)^2 E_t\big(Z_t - Z_{t\,|\,t-1}\big)^2$$

so optimizing with respect to K_t we get

$$-2H\,(I - HK_t)\,E_t\big(Z_t - Z_{t\,|\,t-1}\big)^2 = 0$$

calling $E_t\big(Z_t - Z_{t\,|\,t-1}\big)^2 = \Gamma_{t\,|\,t-1}$ we obtain

$$-2H\,(I - HK_t)\,\Gamma_{t\,|\,t-1} = 0$$

solving for K_t we obtain

$$K_t = \frac{H\ \Gamma_{t\,|\,t-1}}{H\ \Gamma_{t\,|\,t-1} H'}$$

Regrouping the equations, we obtain that

$$Z_{t\,|\,t-1} = AZ_{t-1\,|\,t-1} \tag{B.1}$$

$$\Gamma_{t\,|\,t-1} = A\,\Gamma_{t-1\,|\,t-1}A' + Q \tag{B.2}$$

$$P_{t\,|\,t-1} = HP_{t-1\,|\,t-1}$$

Equations (B.1) and (B.2) are the so-called prediction equations in the Kalman filtering.

The other equation we need are the three updating equations which are

$$K_t = \Gamma_{t\,|\,t-1}H'(\psi_t)^{-1} \tag{B.3}$$

with

$$\psi_t = H\Gamma_{t\,|\,t-1}H'$$

$$Z_{t\,|\,t} = Z_{t\,|\,t-1} + K_t\left(P_t - P_{t\,|\,t-1}\right) \tag{B.4}$$

$$\Gamma_{t\,|\,t} = (I - K_t H)\,\Gamma_{t\,|\,t-1} \tag{B.5}$$

In this case, the inference about Z_t is updated using the observed value of P_t.

We start with a specification information set with initial conditions $Z_{0\,|\,0}$ and $\Gamma_{0\,|\,0}$. Then we use the prediction equations (B.1) and (B.2) to obtain $Z_{1\,|\,0}$ and $\Gamma_{1\,|\,0}$. Once we observe P_1 we use the updating equations (B.3), (B.4), and (B.5) to obtain $Z_{1\,|\,1}$, $\Gamma_{1\,|\,1}$ and $P_{1\,|\,1}$. We next use this information to form $Z_{2\,|\,1}$ and $\Gamma_{2\,|\,1}$, then forecasts are updated and we continue to repeat this process until the end of the dataset.

Given the vector prediction errors $\mu_t = P_t - P_{t\,|\,t-1}$ and the variance-covariance matrix ψ_t we can form the log-likelihood to be maximized and to estimate our parameters.

$$\log l = -\tfrac{T}{2}\ln(2\pi) - \tfrac{1}{2}\sum_{t=1}^{T}\ln\left(\left|\psi_{t\,|\,t-1}\right|\right) - \tfrac{1}{2}\sum_{t=1}^{T}\mu_t{}'\left(\psi_{t\,|\,t-1}\right)^{-1}\mu_t$$

Appendix C

In this Appendix we report the results of a robustness analysis of the effects of changing λ over the interval [0.1 - 0.3]. Tables C1 and C2 present the estimation results via Kalman filter for equity asset respectively with $\lambda = 0.1$ and $\lambda = 0.2$.

Table C1: Estimation via Kalman filter for equity prices with $\lambda = 0.1$

	Countries			
	UK	France	Germany	USA
a_{22}	0.7314***	0.6137***	0.6369***	0.3688***
	(0.1627)	(0.1853)	(0.1099)	(0.0792)
a_{24}	-0.6546***	-0.4845**	-0.5174***	-0.3151***
	(0.1703)	(0.2018)	(0.1147)	(0.0821)
γ	0.92***	0.87***	0.88***	0.95***
	(0.0176)	(0.0208)	(0.0140)	(0.0058)
$1-\gamma$	0.08***	0.13***	0.12***	0.05***
	(0.0176)	(0.0208)	(0.0140)	(0.0058)
β	8.1	3.7	4.3	6.3
	Cyclical Conditions			
	UK	France	Germany	USA
$[-1 < a_{24} < 0]$	*yes*	*yes*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*yes*	*yes*	*yes*
Log-Likelihood	19.423	6.30767	12.7206	25.1227
Akaike	-26.8461	-0.61534	-13.4413	-38.2454
Sample size	48	48	48	48
Years	(1970-2017)	(1970-2017)	(1970-2017)	(1970-2017)

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Table C2: Estimation via Kalman filter for equity prices with $\lambda = 0.3$

	Countries			
	UK	France	Germany	USA
a_{22}	0.8761***	0.8225***	0.8141***	0.4470***
	(0.1975)	(0.1167)	(0.1278)	(0.1138)
a_{24}	-0.5668**	-0.3426***	-0.4031***	-0.2223*
	(0.2594)	(0.1196)	(0.1359)	(0.1178)
γ	0.69***	0.52***	0.59***	0.78***
	(0.0932)	(0.0334)	(0.0424)	(0.0286)
$1-\gamma$	0.31***	0.48***	0.41***	0.22***
	(0.0932)	(0.0334)	(0.0424)	(0.0286)
β	1.8	0.7	0.9	1
	Cyclical Conditions			
	UK	France	Germany	USA
$[-1 < a_{24} < 0]$	*yes*	*yes*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*yes*	*yes*	*yes*
Log-Likelihood	20.1239	9.3738	15.0567	23.9316
Akaike	-28.2477	-6.7476	-18.1134	-35.8632
Sample size	48	48	48	48
Years	(1970-2017)	(1970-2017)	(1970-2017)	(1970-2017)

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Appendix D

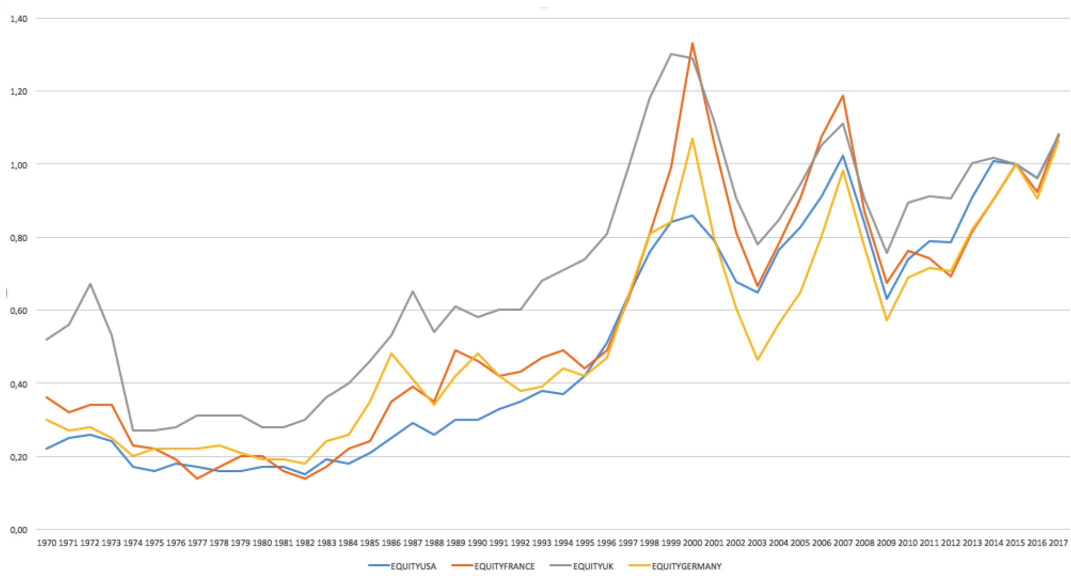

Figure D1: Real Equity Prices Index (1970-2017).

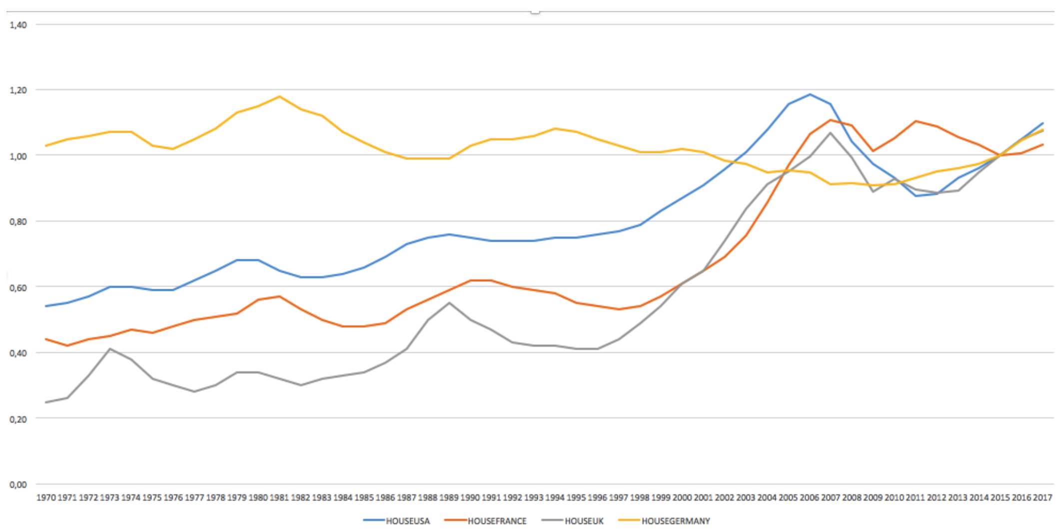

Figure D2: Real Housing Prices Index (1970-2017).

Appendix E

The smoothed estimate of the state variables has been obtained via Kalman filter. Smoothed states are estimated states at period t, which are updated using all available information. The results relative to the equity asset are reported in Figures E.3, E.4, E.5 and E.6. The results relative to housing price are reported in Figures E.7, E.8, E.9 and E.10.

In the figures below we have the smoothed state variable of the fundamentalists (red), the smoothed state variable of the extrapolative traders (blue), the observed asset prices (black) and the union of the three-time series. On the x-axis for the smoothed states, we have the time period from 1972 to 2017, because the first two years of the sample period correspond to the observations required to initialize the Kalman filter and for which the smoothed states assume a value equal to zero. For the housing prices in the UK we have the time period from 1973 to 2017.

E.1 Equity Asset

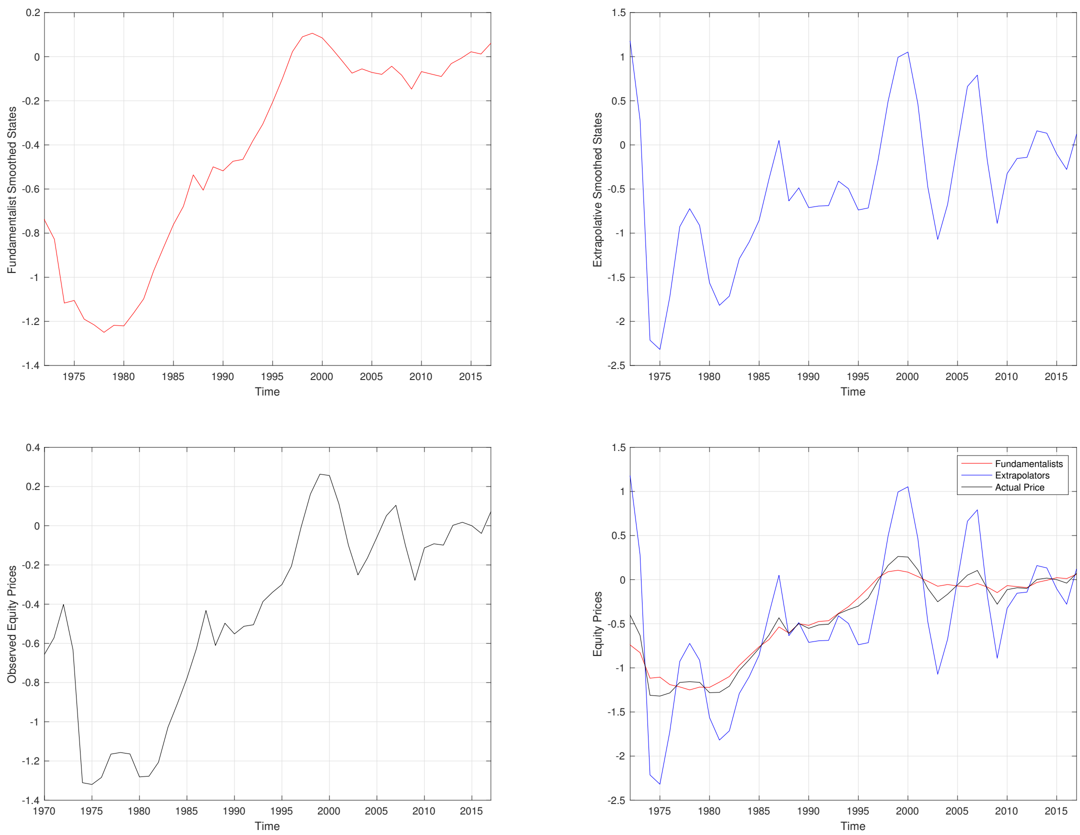

Figure E3: Smoothed state variables (UK)

Figure E4: Smoothed state variables (France)

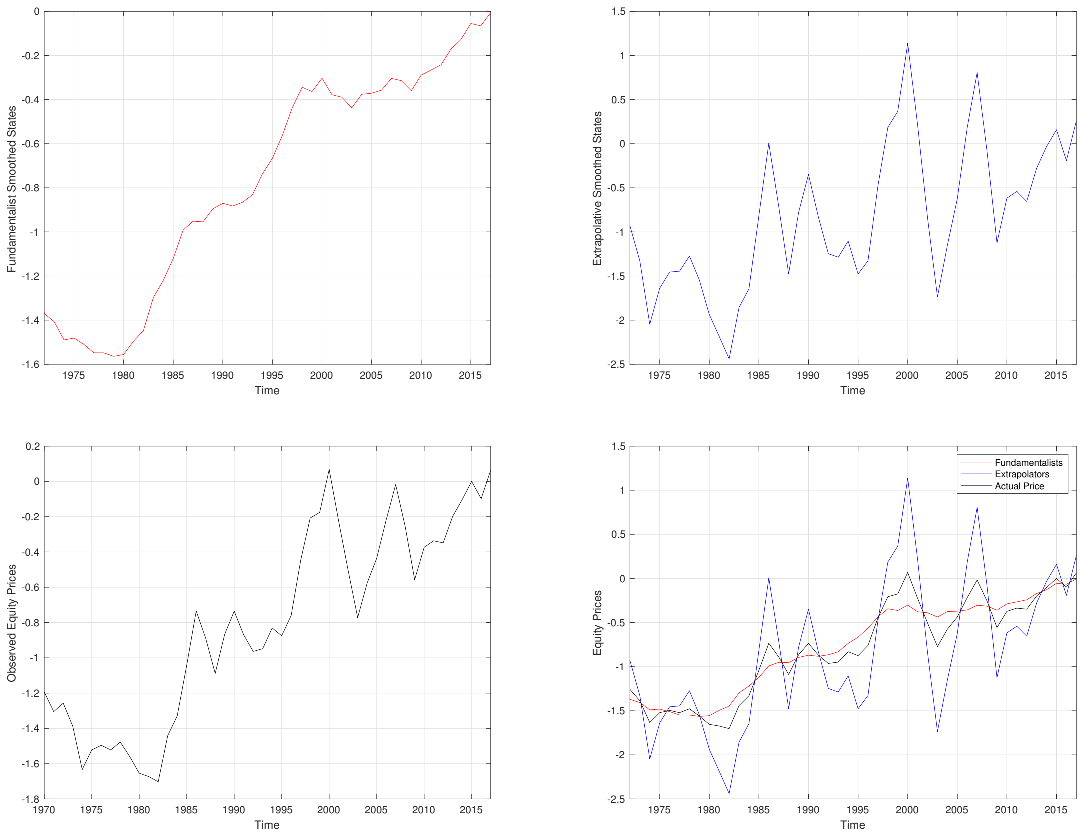

Figure E5: Smoothed state variables (Germany)

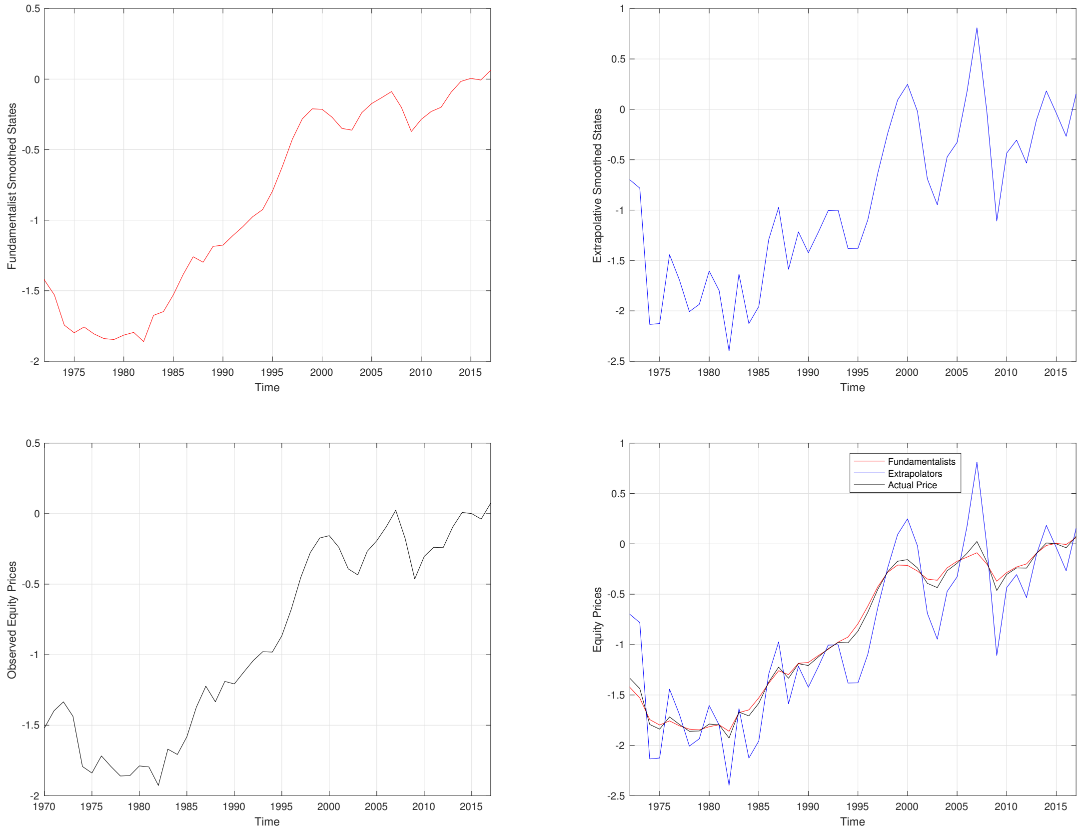

Figure E6: Smoothed state variables (USA)

E.2 Housing Price

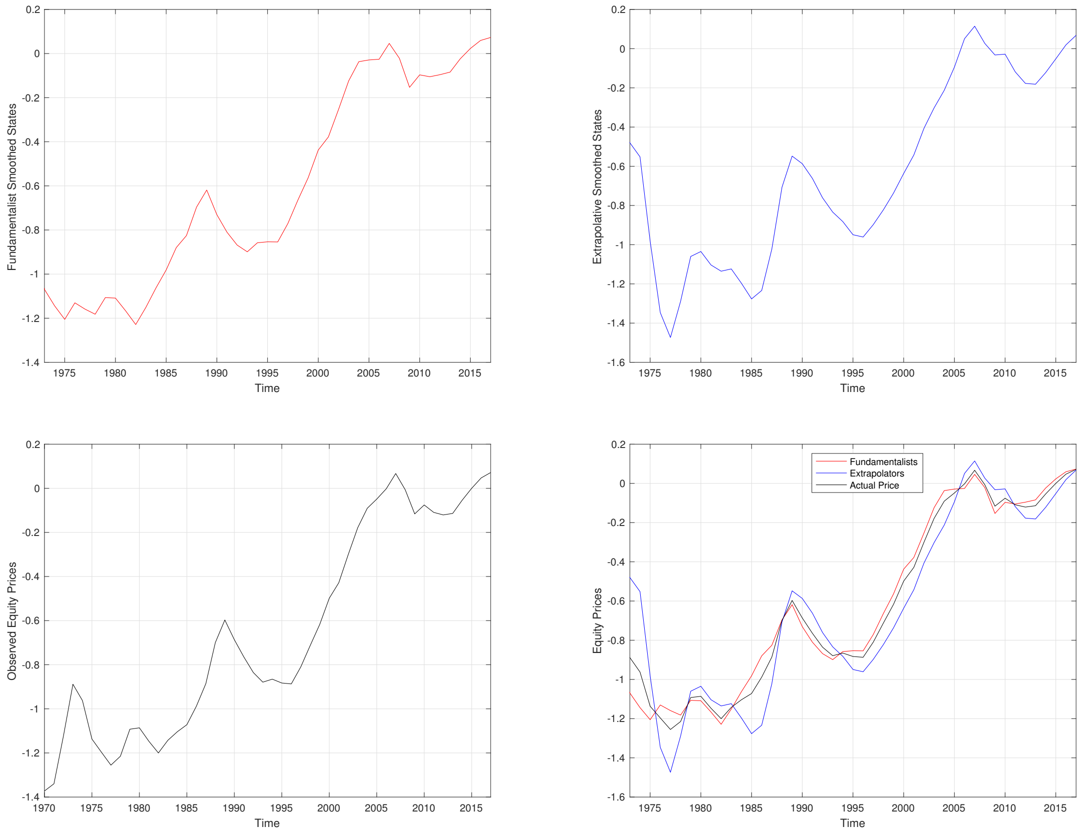

Figure E7: Smoothed state variables (UK)

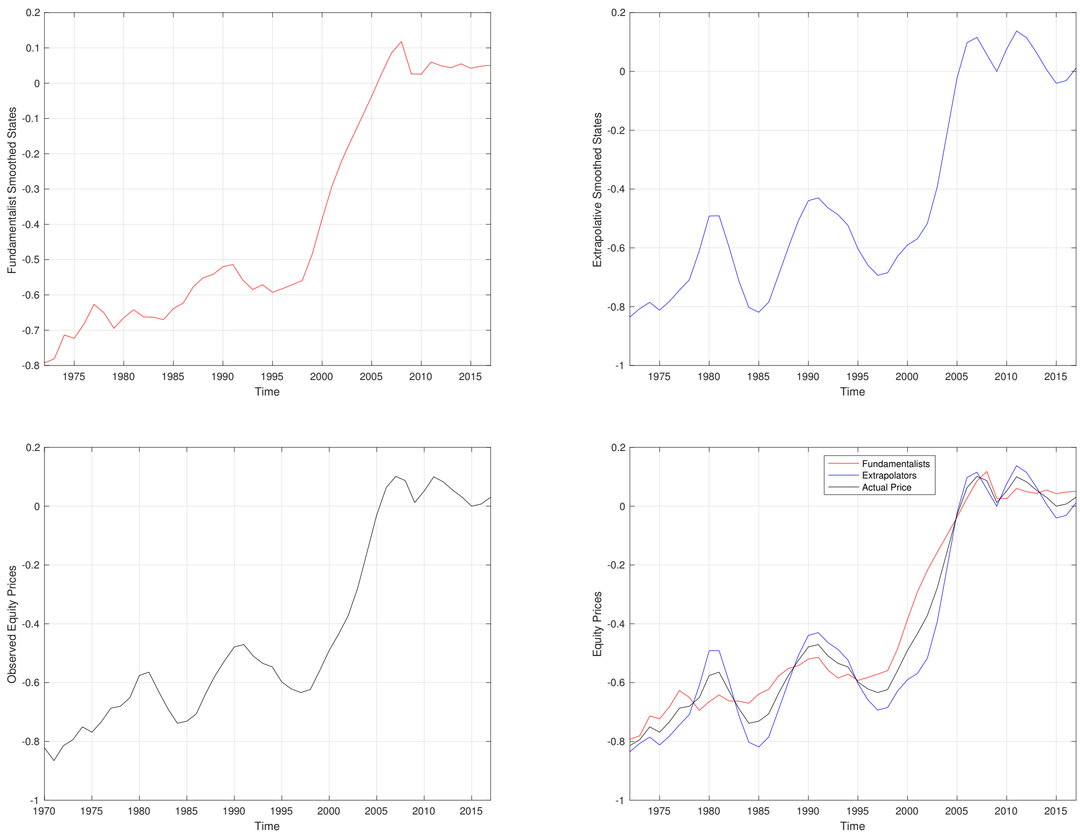

Figure E8: Smoothed state variables (France)

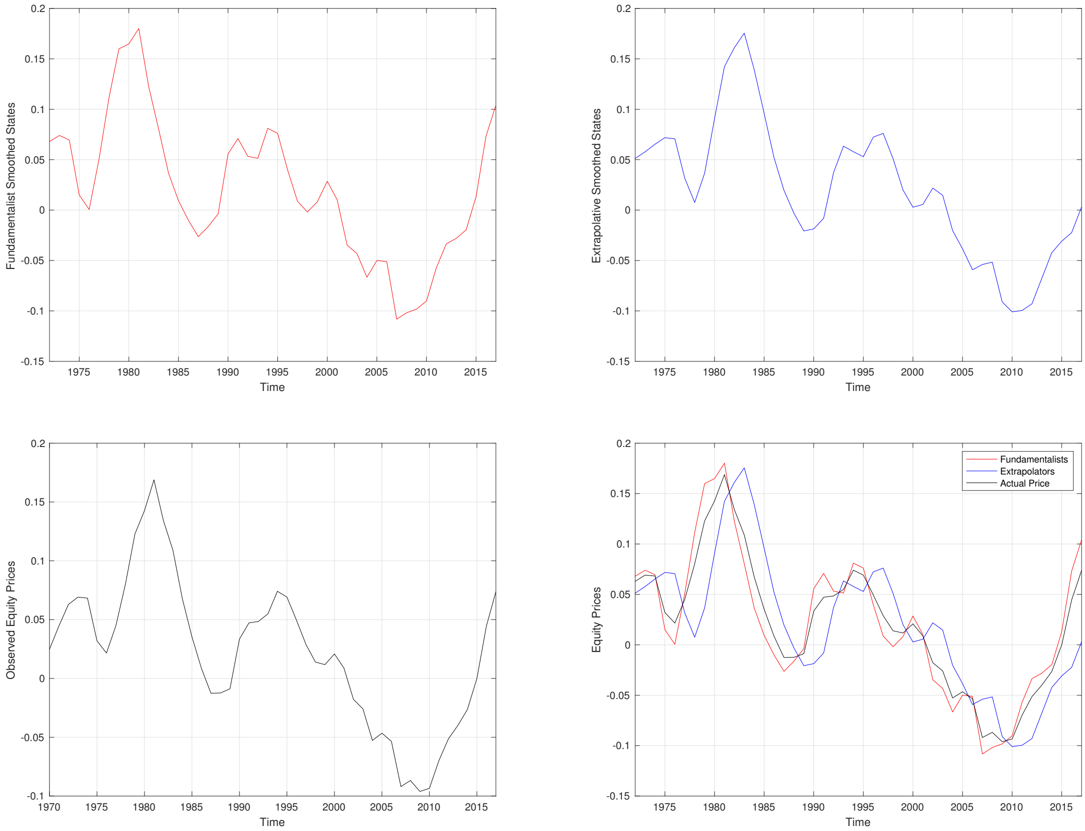

Figure E9: Smoothed state variables (Germany)

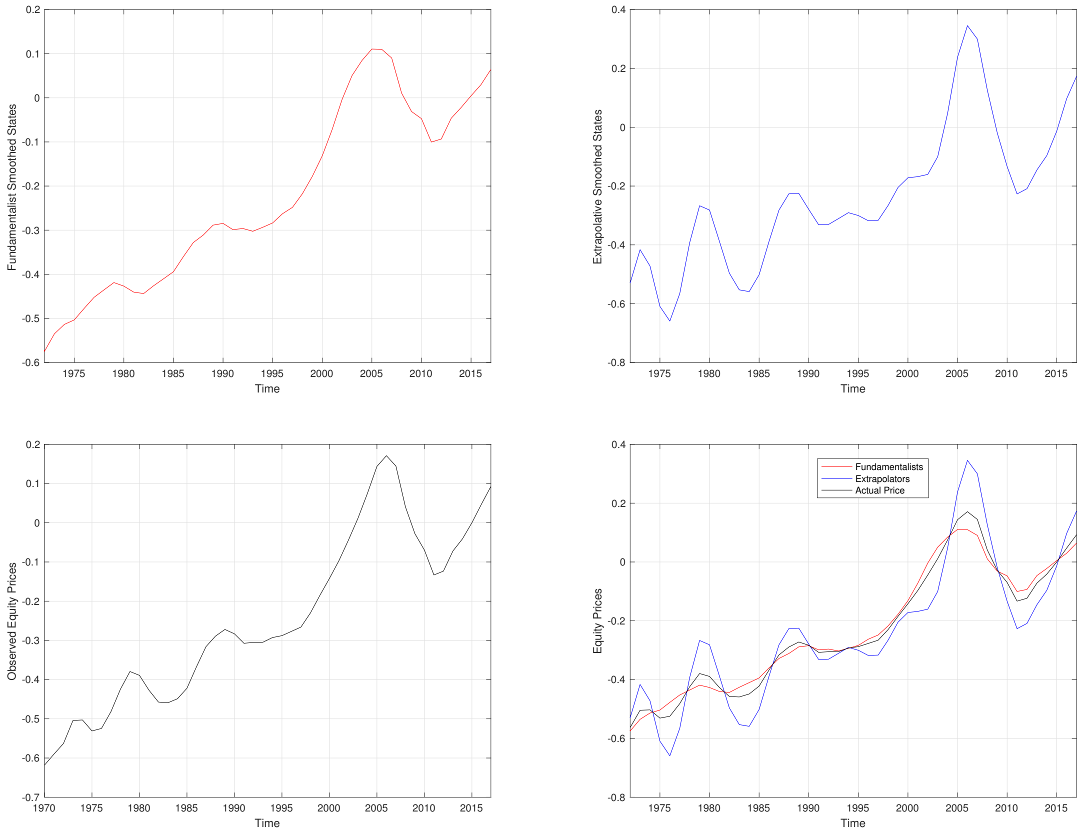

Figure E10: Smoothed state variables (USA)