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Textbook Estimators of Multiperiod Optimal Hedging Ratios: Methodological Aspects and Application to the European Wheat Market

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This work deals with methodological and empirical issues related to multiperiod optimal hedging OLS estimators. We propose an analytical formula for the multiperiod minimum variance hedging ratio starting from the triangular representation of a cointegrated system DGP. Since estimating the hedge ratio matching the frequency of data with the hedging horizon leads to a sample size reduction problem, we carry out a Monte Carlo study to investigate the pattern and hedging efficiency of OLS hedging ratio based on overlapping vs non-overlapping observations exploring a range of hedging horizons and sample sizes. Finally, we applied our approach to real data for a cross hedging related to soft wheat.

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Keywords
Future prices, Hedging, Monte Carlo, Soft wheat

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Introduction

The first decade of the new millennium has been characterized by a sustained volatility of world cereal prices. Although it is not clear whether the rise in volatility is short-lived or instead reveals a structural change (Gilbert and Morgan 2011), its impact on the farming industry is relevant for the time being as it has increased the risk producers face. The rationale behind this article stems from a practical problem faced by the food industry and cereal producers in a particular EU region when implementing forward contracts along the bread food chain. The contracts offered by the industry provide farmers with a hedging instrument through which they can reduce the price risk they face. In its own turn, the food industry needs to hedge with futures the price risk it assumes when issuing forward contracts. As the forward contracts with farmers need to be signed in October to expire the following June, the hedging horizon spans more than 6 months. We hypothesize that the food industry can cross-hedge its long term buying commitment by selling soft wheat futures contracts exchanged on the French MATIF (Marché a Terme Internationale de France). The described risk managing problem calls for an estimation of the optimal hedging ratio (OHR) based on the available weekly prices. Most of the studies on hedging deal with the derivation of OHR under different utility functions or the methodological aspects of OHR estimation under different hypotheses about the data generation processes (DGP) behind the observed price series. However, as noted by Chen, Lee and Shrestha (2004, p.360) many empirical studies "ignore the effect of hedging horizon length on the optimal hedging ratio and hedging effectiveness".

We contribute to this literature in three ways. Firstly, we derive an analytical formula for the multiperiod minimum variance hedging ratio (MVHR) starting from the triangular representation of a co-integrated system, a representation that has not yet been explored in this context. Secondly, using both overlapping and non-overlapping price changes we investigate the properties of OLS or textbook MVHR estimators and discuss the sample reduction problem for longer hedging horizons. Thirdly, we resort to simulated data to investigate the performance of MVHR estimators. In contrast to previous studies, we do not use real data whose DGP is unknown; instead we run a Monte Carlo exercise to investigate our estimators and compare them with the theoretical measures. Only after that do we estimate the hedging ratio on real data and discuss results in the light of the outcome of the previous steps.

The article is set out as follows: first we offer a brief literature review of multiperiod hedging ratios and we discuss the issues still to be explored and state the research objectives. The section that follows is about the development of a triangular representation of a prototypical error correction model (ECM) from which an analytical formula for multiperiod MVHR is derived. Next, we report results of the Monte Carlo exercise investigating OLS estimators and comparing them with theoretical measures. In addition, we report results of the empirical analysis conducted using real rather than simulated data, by evaluating a cross-hedge for soft wheat. Finally, we report our main conclusions.

Multiperiod hedging: literature review, unsolved problems and unexplored issues

Theoretical and empirical issues related to the determination and estimation of optimal hedge ratio (OHR) (the size of the futures contract relative to the hedged cash transaction) and hedging effectiveness (HE) (the extent to which hedging actually reduces portfolio risk) have received considerable interest in the futures literature[1].

In spite of such interest on broad hedging issues, literature concerning the relationship between the hedge ratio and the hedging horizon has been scanty, even though it appears to have important implications for the use of derivatives in commodity hedging.

Few studies consider the effect of longer horizons on optimal hedge ratio and hedge effectiveness. One such study, Ederington (1979), empirically compares the effectiveness of MVHRs for different hedging horizons. The issue is re-examined in Malliaris and Urrutia (1991), who use an autoregression (AR) model, instead of simple OLS, in order to deal with autocorrelation among residuals. Other empirical studies include Benet (1992), Chou, Fan and Lee (1996) and Lee et al. (2009).

A second group of studies such as Howard and D'Antonio (1991), Lien (1992), Geppert (1995) and Juhl, Kawaller and Koch (2012) try to model the relationship between the hedge horizon and the OHR and HE deriving analytical formulae that are valid for the assumed DGP.

Generally, both MVHR and HE have been found to be increasing in the hedging horizons with the former tending to the naïve hedging ratio of 1[2]. As the hedging horizon increases, the short-term noises are smoothed out, and spot and futures prices are closer to each other, resulting in a larger hedge ratio. Moreover, the spot price series become noisier as the hedging horizon increases and a bigger spot-price risk tends to increase the HE.

The first issue that emerges from the literature is the sensitivity of results to how the relationship between spot and futures prices is modeled. Geppert (1995) works on the Stock-Watson common trend representation, whereas Lien (1992) and Juhl, Kawaller and Koch (2012) use the error correction representation. Geppert (1995) finds that the limit of the hedge ratio as the hedging horizon goes to infinity is equal to the futures price coefficient in the co-integrating vector (which is not necessarily 1) while the degree of HE tends to 1.
Lien (1992) derives the expression for his multiperiod hedging ratio by solving a dynamic programming problem for a finite time series, therefore he does not provide the limit of the hedging ratio for *k* - the number of periods - going to infinity. However, the multiperiod hedging ratio calculated with empirically estimated parameters shows a cyclical pattern as the horizon lengthens.
Juhl, Kawaller and Koch (2012) find that, as the hedge horizon lengthens OHR and HE come close to one for co-integrated series. Noticeably, the hedging ratio in Juhl, Kawaller and Koch (2012) is based on conditional (on the error correction term), rather than unconditional variances.
In order to validate the models, theoretical measures are often compared with empirical estimates on real spot and futures series. However, unless we are sure that our model truly represents the DGP underlying the observed real data, we cannot use real data to compare or validate alternative estimates of the hedging ratio against the results from the formula analytically derived within our model. The question of which estimator of the multiperiod OHR provides the best unbiased and efficient estimates should be addressed only when working with data generated by the very model which provides a benchmark measure for the hedging ratio.
Hypothesizing a given GDP which is not supported by the data leads to misspecified estimators of the OHR even though this approach may prove useful to get an idea of how MVHR and HE evolve with the hedging horizon. To this purpose the model should be as simple as possible whilst still accounting for the basic features of real data such as co-integration.
The use of real data gives rise to a second methodological issue: to get empirical estimates it is necessary to match the frequency of data with the hedging horizon, whereupon we face the problem of sample size reduction. Indeed, a trade-off arises between working on overlapping observations thereby maintaining an adequate sample size as the hedge horizon lengthens, but inducing a moving average process in OLS residuals, or else resorting to non-overlapping observations, thus facing a

dramatic reduction in sample size for longer hedging horizon. Authors differ widely in the solutions they propose.

Geppert (1995) estimates OHR by the classical Ederington (1979) OLS method. He resorts to using overlapping observations to account for the induced autocorrelation in residuals with a two stage GLS estimator.

Lien and Luo (1993) calculate OHR with the Lien (1992) formula, drawing on single period estimates of an ECM, thus bypassing the problem incurred with degrees of freedom.

Other researchers use wavelet multiscaling techniques in order to overcome the sample reduction issue[3]. In a minimum-variance framework, In and Kim (2006) and Lien and Shrestha (2007) show that wavelets allow the determination of a unique hedge ratio associated with different hedging horizons and that S&P index hedgers achieve greater effectiveness at longer horizons.

Chen, Lee and Shrestha (2004) compare OLS and a modified ECM (where both short and long term OHR are simultaneously estimated) estimate of OHR across a number of time horizons using non-overlapping data. They state that having quite a large sample size available they don't need to resort to overlapping data.

Juhl, Kawaller and Koch (2012) compare OLS and modified (as in Ghosh (1993)) ECM estimates of the hedging ratios using non-overlapping data for price changes of growing length (from 1 day to 6 months). Not unexpectedly, with a sample size of 27 months, the authors state that the "paucity of data at long horizons represents the major limitation for a hedger conducting this analysis" (p.865).

Surprisingly, after Geppert (1995), the simplest way to address this problem, which is the use of overlapping data, has been overlooked by the multiperiod hedging literature. Actually, according to Hansen and Holdrick (1980) and Hansen (1992), the overlapping OLS estimator is more efficient than the corresponding non-overlapping one.

Finally, there is scant literature on hedging horizons applied to agricultural commodity markets even though farmers need to hedge for long (generally lasting several months) horizons depending on the crop produced (see, e.g., See Chen, Lee and Shrestha (2003) and Lien and Shrestha (2007)). Mathews and Holthausen (1991) and Myers and Hanson (1996) provide a dynamic strategy for multiperiod hedging and estimate dynamic optimal hedging ratios for a number of agricultural commodities, however they do not touch on the issue of a relationship between hedging horizon and MVHR. A notable exception is Revoredo-Giha and Zuppiroli (2013), who provide an empirical comparison of MVHR across different time horizons and futures contracts for the European cereal sector.

In the next section we propose a Monte Carlo study to investigate the pattern and hedging efficiency of both overlapping and non-overlapping OLS hedging ratios for different hedging horizons and different sample sizes[4]. Our work differs from previous studies since it consistently explores the behaviour of hedging ratio estimators within the hypothesized DGP which provides benchmark measures. In addition, Monte Carlo simulations allow us to explore the performance of hedging ratio estimators when the sample size lengthens without the interference of confounding factors which may appear in real data.

The model: a prototypical ECM

As in Jhul, Kawaller and Koch (2012), we consider a cross hedge with a price y_t for a commodity at the time it will be delivered to a given location and the corresponding futures price x_t for future delivery of the same commodity to a different location. We assume that the two prices are co-integrated with x_t weakly exogenous. To make analytical derivation simpler, we also assume that the futures price is a purely random walk. The vector error correction representation of the co-integrated system is thus given by:

$$(1) \quad \begin{aligned} \Delta y_t &= \alpha(y_{t-1} - \beta x_{t-1}) + v_t \\ \Delta x_t &= \varepsilon_t \end{aligned}$$

where ε_t and u_t are jointly white noise, possibly contemporaneously correlated. Differently from Lien (1992) and Juhl, Kawaller and Koch (2012) we reparameterize the vector ECM in (1) to the Phillips's (1990) triangular representation of the cointegrated system:

$$(2) \quad \begin{aligned} y_t &= \beta x_t + u_t \\ x_t &= x_{t-1} + z_t \end{aligned}$$

with u_t and z_t $I(0)$ processes given by:

$$(3) \quad \begin{aligned} u_t &= \phi u_{t-1} + \eta_t \\ z_t &= \varepsilon_t \end{aligned}$$

with $\phi = 1 + \alpha$ and $\eta = v_t - \beta\varepsilon_t$.

Note that:

$$\sigma_u^2 = \sigma_\nu^2 + \beta^2\sigma_\varepsilon^2 + 2\beta\sigma_{\varepsilon\nu}$$
$$\sigma_z^2 = \sigma_\varepsilon^2$$
$$\sigma_{uz} = \sigma_{\varepsilon\nu} - \beta\sigma_\varepsilon^2$$

The triangular representation facilitates both simulations of the cointegrated system (Zivot 2006) and derivation of a formula for the MVHR. The resulting model is close to the one employed by Jhul, Kawaller and Koch (2012) although under a different parameterization and without the restriction of the co-integration coefficient β being equal to 1.

According to Ederington (1979) also known as the "textbook solution", the MVHR, is given by:

(4) $$MVHR(k) = \frac{Cov(\Delta_k y, \Delta_k x)}{Var(\Delta_k x)}$$

where Δ_k is the difference operator over *k* periods[5].

We are interested in comparing the performance of simple OLS estimators of MVHR(*k*) using the Ederington unconditional HE measure. However, we note that when x_t follows a random walk, conditional and unconditional hedge ratios are equal (Lien 2005).

MHVHR can be easily expressed as a function of the triangular representation parameters for different values of *k*.

After some relatively straightforward algebra (see Appendix A) we get:

(5) $$MVHR(k) = \frac{\sigma_{uz}}{\sigma_z^2} + \beta - \frac{\sigma_{uz}\left[\left(K - 1 + \phi^k\right) - \frac{\phi\left(1-\phi^k\right)}{1-\phi}\right]}{k\sigma_z^2}$$

which for *k*=1 gives the short horizon hedge

$$MVHR(1) = \frac{\sigma_{uz}}{\sigma_z^2} + \beta = \frac{\sigma_{\varepsilon\nu}}{\sigma_\varepsilon^2}$$

while for $k \to \infty$ the limit of (5) is β, that is the long term hedge ratio (as in Chen, Lee and Shrestha (2004)).

The Ederington HE measure is given by (see appendix A):

(6) $$HE = \frac{k\beta\sigma_z^2 + \sigma_{uz}\left[\left(\frac{\phi\left(1-\phi^k\right)}{1-\phi}\right)(1-2\phi)\right]}{2\left(1-\phi^k\right)\sigma_u^2 + k\beta^2\sigma_z^2 + 2\frac{1-\phi^k}{1-\phi}\beta\sigma_{uz}} MVHR(k)$$

whose limit for $k \to \infty$ is 1[6], as in Geppert (1995) and Juhl, Kawaller and Koch (2012).

Summing up, the analytical derivation of MVHR(k) for co-integrated prices under the triangular representation parallels the results of models developed under other representations (ECM or common trend) with a relatively more straightforward algebra. In addition, the triangular representation is particularly convenient to simulate the series as it will be illustrated in the next section.

Monte Carlo Analysis

We run Monte Carlo simulations from the triangular representation of a co-integrated system given in equations (2) and (3). We start from the ECM parameterization and generate a bivariate error series from a bivariate normal with parameters:

$$\begin{matrix} \nu \\ \varepsilon \end{matrix} \sim N\left(\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \Sigma = \begin{bmatrix} 10 & 6 \\ 6 & 30 \end{bmatrix} \right)$$

Then, we generate the error series η in the triangular representation as:

$$\eta = \nu_t - \beta \varepsilon_t$$

with the parameter β set to 0.92. Next, the error u_t in equation (3) is simulated as an AR(1) process with innovations η and parameter $\phi = 1 + \alpha$ set to 0.88. All parameter values are chosen in the proximity of values obtained by estimating the system equations (1) with real data on Italian wheat spot and French MATIF futures prices.

We then take both the overlapping and the non-overlapping k-period differences of the simulated series and estimate the linear model:

$$\Delta_k y_t = \alpha + \beta \Delta_k y_t + \varepsilon_t \quad \text{for } t=1 \ldots T \qquad (7)$$

We run Monte Carlo simulations using a variety of values for hedging horizon (k) and for the sample period (T) resulting in different scenarios[7].

The sample period T ranges from 360 to 2880. Series of double that length are actually generated (from 720 to 5760), the first half being employed to estimate β, and the second to recover an out-of-sample HE measure[8].

The hedging horizon ranges from 1 to 36 weeks.

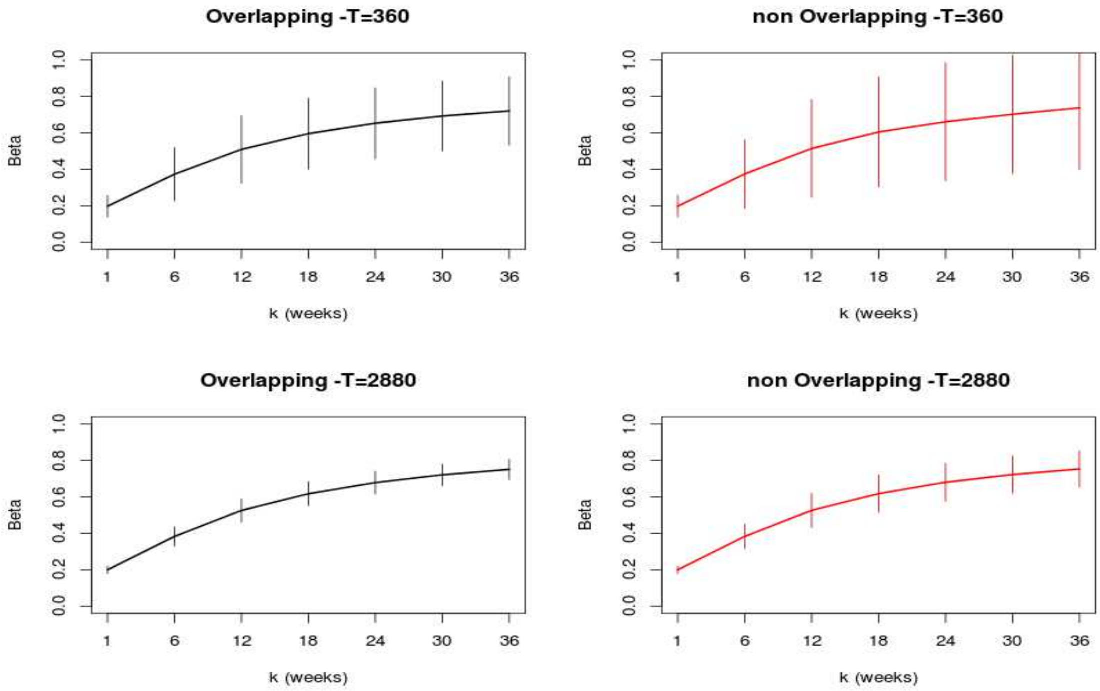
For each scenario and for each draw, we then take both the overlapping and the non-overlapping k-period differences of the simulated series and estimate the linear model (7

We report the bias, standard deviation and root mean squared error (RMSE) of OLS estimates of β, as well as the related OLS standard error and R^2 estimates. For overlapping observations we also report the truncated kernel autocorrelation consistent standard errors (HAC)[9] in order to take into account the autocorrelation of OLS residuals.

In table 1 we report estimates for overlapping observations. As expected, both hedge ratio and R^2 increase with *k*, the hedging horizon. Estimates exhibit a small downward bias, which grows with *k* but decreases to negligible values with *T*, the sample size. As expected, OLS standard errors are not appropriate because of the induced serial correlation of the OLS residuals. HAC standard errors, in this case, provide a better approximation. R^2 and out-of-sample HE show close values as expected given the same DGP underlying both in sample and out-of-sample observations.

Table 2 reports values for non-overlapping observations that basically show the same pattern seen in the overlapping case with respect to *k* and *T*. However, non-overlapping observations exhibit higher β values and R^2 values although the difference shrinks with higher values of *T*. Interestingly, non-overlapping observations exhibit a lower bias, this time upwardly. Finally, the difference between the overlapping and non-overlapping estimates of β and R^2 decreases as the sample period *T* increases.

The most striking difference between OLS estimates with overlapping and non-overlapping observations is the strong rise in the Monte Carlo standard deviation of β that we observe with non-overlapping observation. For higher values of *k* and lower *T* standard deviation of β is more than double when estimated with overlapping observations. Figure 1 illustrates both the pattern of β mean estimates and variability as *k* varies in the case of the smaller and the larger sample sizes employed in the Monte Carlo exercise. It is clear that the OLS estimates of MVHR on non-overlapping observations are far less efficient when *T* is small and *k* large. This comes as no surprise since Hansen and Holdrick (1980) demonstrate that the asymptotic standard errors for β_{ov} are smaller than those for $\beta_{non\text{-}ov}$.



Note: Vertical bars are at 2 standard deviation above and below the mean values.

Figure 1. Monte Carlo simulation: means and standard deviations of betas

Table 1. OLS with overlapping observations

T	*k*	Mean β	Std. Dev β	Std. β	HAC Std. β	Bias β	RMSE β	R^2	Out of Sample Hedg. Effectiv.
360	1	0.198	0.031	0.031	0.031	-0.002	0.031	0.104	0.099
	6	0.373	0.060	0.032	0.073	-0.011	0.061	0.284	0.269
	12	0.509	0.073	0.030	0.090	-0.018	0.076	0.448	0.429
	18	0.596	0.077	0.029	0.093	-0.024	0.081	0.559	0.539
	24	0.653	0.079	0.027	0.091	-0.028	0.084	0.633	0.615
	30	0.693	0.079	0.026	0.088	-0.031	0.085	0.686	0.668
	36	0.721	0.079	0.024	0.085	-0.034	0.086	0.723	0.706
720	1	0.199	0.022	0.022	0.022	-0.001	0.022	0.104	0.102
	6	0.379	0.043	0.022	0.053	-0.005	0.043	0.289	0.282
	12	0.519	0.052	0.021	0.065	-0.009	0.052	0.457	0.449
	18	0.609	0.053	0.020	0.067	-0.011	0.055	0.572	0.564
	24	0.669	0.053	0.018	0.065	-0.013	0.054	0.651	0.644
	30	0.710	0.051	0.017	0.062	-0.014	0.053	0.707	0.701
	36	0.740	0.050	0.016	0.059	-0.015	0.052	0.747	0.742
1440	1	0.200	0.016	0.015	0.015	0.000	0.016	0.104	0.103
	6	0.382	0.030	0.016	0.038	-0.002	0.030	0.292	0.288
	12	0.523	0.036	0.015	0.046	-0.004	0.036	0.463	0.458
	18	0.614	0.037	0.014	0.047	-0.006	0.037	0.581	0.576
	24	0.675	0.036	0.013	0.046	-0.006	0.036	0.662	0.657
	30	0.718	0.034	0.012	0.044	-0.007	0.035	0.718	0.714
	36	0.748	0.033	0.011	0.041	-0.007	0.034	0.759	0.756
2880	1	0.200	0.011	0.011	0.011	0.000	0.011	0.104	0.103
	6	0.383	0.021	0.011	0.027	-0.002	0.021	0.293	0.291
	12	0.526	0.025	0.011	0.033	-0.002	0.025	0.466	0.463
	18	0.617	0.026	0.010	0.033	-0.003	0.026	0.584	0.581
	24	0.679	0.025	0.009	0.032	-0.003	0.025	0.666	0.663
	30	0.721	0.024	0.008	0.031	-0.003	0.024	0.723	0.721
	36	0.751	0.023	0.008	0.029	-0.004	0.024	0.765	0.762

Note: *T* is the length of the simulated series, the number of available observations for OLS is given by *T/k*.

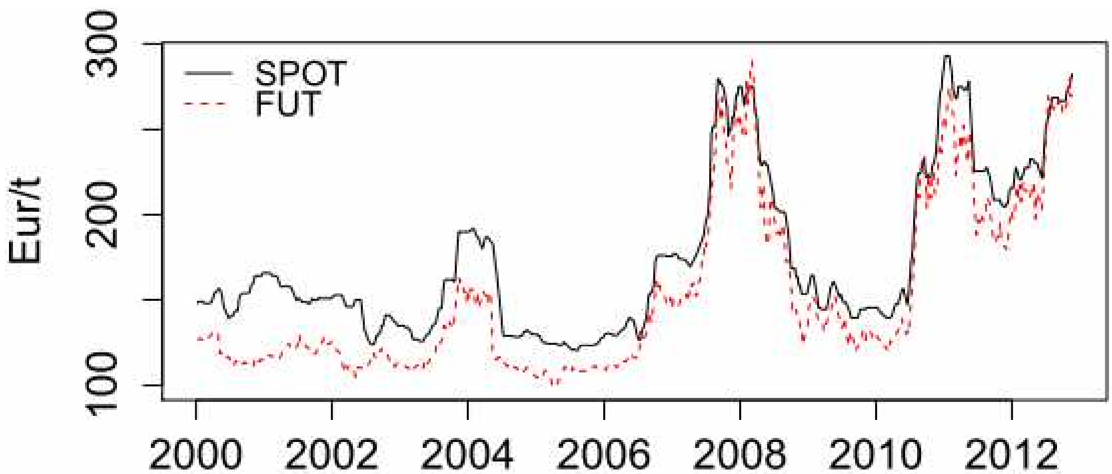
Table 2. OLS with non overlapping observations

T	k	Mean β	Std. Dev β	Std. Err. β	HAC Std. Err. β	Bias β	RMSE β	R^2	Out of Sample Hedg. Effectiv.
360	1	0.198	0.031	0.031		0.001	0.001	0.104	0.099
	6	0.375	0.078	0.079		0.006	0.009	0.289	0.264
	12	0.514	0.107	0.108		0.012	0.018	0.458	0.418
	18	0.605	0.126	0.126		0.016	0.026	0.572	0.522
	24	0.661	0.139	0.138		0.019	0.031	0.649	0.595
	30	0.702	0.152	0.149		0.023	0.038	0.701	0.642
	36	0.738	0.167	0.158		0.028	0.051	0.742	0.675
720	1	0.199	0.022	0.022		0.000	0.001	0.104	0.102
	6	0.379	0.056	0.055		0.003	0.004	0.290	0.280
	12	0.520	0.074	0.075		0.005	0.008	0.461	0.444
	18	0.611	0.085	0.085		0.007	0.011	0.578	0.558
	24	0.674	0.095	0.092		0.009	0.014	0.659	0.635
	30	0.715	0.098	0.097		0.010	0.015	0.714	0.691
	36	0.744	0.103	0.101		0.011	0.017	0.752	0.730
1440	1	0.200	0.016	0.015		0.000	0.000	0.104	0.103
	6	0.382	0.038	0.039		0.001	0.002	0.293	0.287
	12	0.525	0.052	0.052		0.003	0.004	0.466	0.456
	18	0.618	0.059	0.059		0.003	0.005	0.585	0.572
	24	0.679	0.063	0.063		0.004	0.006	0.666	0.653
	30	0.721	0.066	0.066		0.004	0.007	0.721	0.710
	36	0.751	0.068	0.068		0.005	0.007	0.764	0.751
2880	1	0.200	0.011	0.011		0.000	0.000	0.104	0.103
	6	0.383	0.027	0.027		0.001	0.001	0.293	0.290
	12	0.526	0.036	0.037		0.001	0.002	0.467	0.462
	18	0.618	0.041	0.041		0.002	0.002	0.585	0.580
	24	0.680	0.045	0.044		0.002	0.003	0.668	0.661
	30	0.723	0.045	0.046		0.002	0.003	0.726	0.719
	36	0.753	0.047	0.047		0.002	0.003	0.767	0.760

Note: T is the length of the simulated series, the number of available observations for OLS is given by T/k .

An Empirical application: Italian spot price and MATIF wheat futures

We also conduct an empirical analysis using real rather than simulated data. We consider a cross hedge for soft wheat: the two locations for spot and for the commodity specification underlying the futures contracts exchanged on the MATIF[10] market being respectively Bologna (Italy) and Rouen (France) (see figure 2).



Source: Authors' elaboration of data from Bloomberg and ISMEA.

Figure 2. Spot and future series

The futures contract used in the article is the nearest-to-maturity contract and it is rolled over to the next contract on the first day of the month when the contract expires. In order to see the impact of the length of hedging horizon, the same data frequencies used in the previous section (ranging from 1 to 36 weeks) are examined. Data are weekly prices from the 2nd week of 2000 to the 47th week of 2012 amounting to 670 observations.

Co-integration

Both spot and futures series are first tested for unit roots. We fail to reject the null with Augmented Dickey-Fuller test (ADF)[11] and we reject it with the Kwiatkowsky, Phillips, Schmidt e Shin test (KPSS) suggesting that the series in level are non stationary at the 5% significance level (see table 3). The same test on the series in differences provides rejection of the null of non stationarity with ADF and failure to reject the null of stationarity with KPSS suggesting that both the futures and spot prices are $I(1)$.

Table 3. Unit Root and Cointegration Tests on Futures and Cash Prices

	ADF test			
	level		differences	
	ADF	5% CV	ADF	5% CV
Futures (MATIF)	-2.831	-3.41	-23.352	-3.41
Spot (Bologna)	-2.917	-3.41	-12.741	-3.41
	KPSS test			
	level		differences	
	mu	5% CV	mu	5% CV
Futures (MATIF)	5.591	0.463	0.107	0.463
Spot (Bologna)	5.709	0.463	0.122	0.463

Co-integration is tested by carrying out the Johansen trace test for the null hypothesis of the presence of at least *r* co-integration vector[12]. Trace test is performed for the presence of 1 or 0 co-integration vectors including a constant in the co-integration relationship. The trace test (table 4) suggests the presence of one co-integrating vector.

Table 4. Johansen Cointegration Tests on Futures and Cash Prices

		Johansen Trace test		
Series	Lags	Trace Test	5% CV	H_0
Spot (BO)-	11	2.00	9.24	r<=1
Futures		20.42	19.96	r=0

Table 5 provides estimates of the co-integrating vector and loading coefficients (that is the coefficients of the error correction term in the spot and futures price changes equations) with their standard errors. According to these estimates, futures prices seem to be weakly exogenous as previously hypothesized.

Table 5. Long Term Relationship and Loadings

Cointegration Vector			**Loadings**		
const	31.57	(6.92)	α (Spot eq.)	-0.05	(0.01)
Futures	0.93	(0.04)	α (Fut eq.)	-0.01	(0.02)

We then go on to estimate the prototypical single equation ECM for the spot price change. Not surprisingly, the model, being very simple, turns out to be misspecified. In particular, residuals serial correlation is severe and the log-likelihood is far lower in comparison with a model that includes 10 lagged values for both spot and futures price changes. However, we retain an estimate of -0.12 for the coefficient of the lagged error correction term to be used in the Monte Carlo exercise.

Table 6 ECM estimation

	ECM		**Prototypical ECM**	
	Coef	Std. Err	Coef	Std.Err.
ECT(-1)	-0.0454	0.0110	-0.12	0.01
Δspot(-1)	0.3222	0.0415		
Δspot(-2)	0.0763	0.0435		
Δspot(-3)	-0.1333	0.0434		
Δspot(-4)	0.1582	0.0437		
Δspot(-5)	0.0047	0.0440		
Δspot(-6)	-0.1228	0.0439		
Δspot(-7)	0.1291	0.0433		
Δspot(-8)	-0.0643	0.0427		
Δspot(-9)	-0.0732	0.0416		
Δspot(-10)	0.1346	0.0354		
Δfut(-1)	0.1798	0.0179		
Δfut(-2)	0.1069	0.0208		
Δfut(-3)	0.0454	0.0213		
Δfut(-4)	-0.0453	0.0212		

Δfut(-5)	0.0010	0.0212	
Δfut(-6)	0.0406	0.0211	
Δfut(-7)	0.0120	0.0207	
Δfut(-8)	-0.0208	0.0207	
Δfut(-9)	-0.0386	0.0205	
Δfut(-10)	-0.0157	0.0201	
	-		-
LogLik	1519.64		1730.67
DW	1.99		0.94

Hedge Ratios and Hedging Effectiveness

The results of the hedge ratios for various hedging horizon lengths are shown in table 7. The hedge ratio (the β coefficient) increases with the hedging horizon[13]. Noticeably, the hedge ratios for longer horizons are larger than the futures coefficient in the long term relationship which is 0.92. This contradicts the findings of the modelling section concerning the limit of the MVHR when the hedging horizon grows to infinity. This evidence points out how misleading it can be to validate a theoretical model for MVHR with data that do not strictly follow the assumed DGP, a danger we avoided by using data from a Monte Carlo simulation.

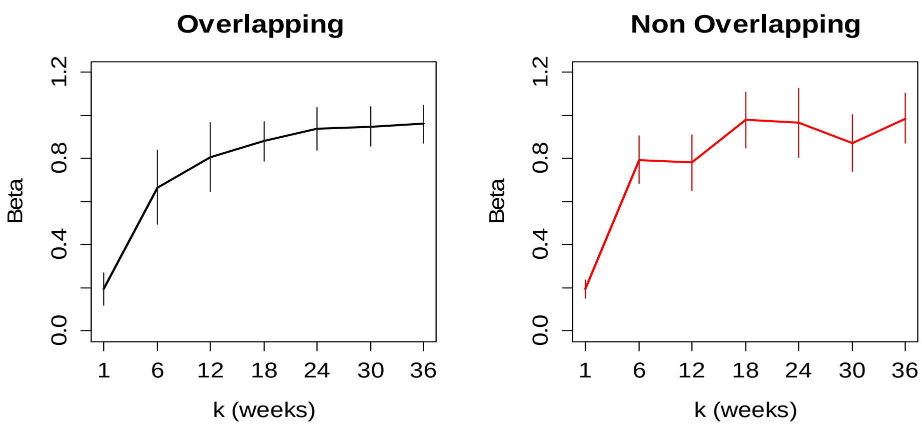
Non-overlapping hedge ratios are slightly higher than overlapping hedge ratios and do not grow monotonically with *k* (figure 3): a feature already observed by Geppert (1995). Non overlapping standard errors are sensibly higher confirming the findings by Hansen and Hodrik (1980) about sample size reduction associated with non overlapping observations reducing efficiency in OLS estimates.

In addition, R^2 statistics for longer horizons is well above 0.80, which is considered by the US accounting standards to be a condition for hedging to be effective in offsetting a particular exposure (see Juhl, Kawaller and Koch 2012).

At least in the overlapping case, the overall pattern of evolution of MVHR estimates as the hedging horizon grows is similar for both real and simulated data. In particular, MVHR shows a monotonic increase in both cases, with diminishing increments as *k* grows.

Table 7. Estimates of MVHR: MATIF Wheat Future and Bologna Spot Wheat

		overlapping obs.				non ovelapping obs.		
T	*k*	β	Std. Err.	HAC Std. Err.	R^2	β	Std. Err.	R^2
670	1	0.195	0.022	0.038	0.106	0.195	0.022	0.106
	6	0.667	0.022	0.085	0.575	0.794	0.056	0.651
	12	0.806	0.020	0.081	0.722	0.781	0.064	0.736
	18	0.881	0.018	0.046	0.788	0.979	0.064	0.870
	24	0.938	0.016	0.049	0.842	0.965	0.080	0.855
	30	0.949	0.015	0.046	0.868	0.874	0.065	0.899
	36	0.960	0.013	0.044	0.895	0.986	0.057	0.949



Note: Vertical bars are at 2 standard errors above and below the point estimates

Figure 3. Overlapping vs non overlapping OLS estimates of MVHR: real data

Conclusions

This article has dealt with both methodological and empirical issues concerning the multiperiod OHR. Hypothesizing a given GDP, when the data do not actually follow it, leads to misspecified estimators of the OHR even though this approach can be useful to have an idea of how MVHR and hedging effectiveness evolve with hedging horizon. To this purpose, we have proposed analytical formula for the multiperiod MVHR starting from the triangular representation of the co-integrated system DGP.

In addition, empirically estimating the OHR matching the frequency of data with the hedging horizon leads to the problem of sample size reduction. Thus, we have proposed a Monte Carlo study to investigate the pattern and hedging efficiency of both overlapping and non-overlapping OLS hedging ratios for different hedging horizons and different sample sizes.
Finally, we have carried out our empirical estimation of the hedging ratio by considering a cross hedge scheme for soft wheat, with futures contracts exchanged on the French MATIF (Marche a Terme International de France) market.
The Monte Carlo exercise shows that, as expected, both hedge ratio and R^2 increase with the hedging horizon. OLS standard errors with overlapping observations are not appropriate because of the induced serial correlation of the OLS residuals. Interestingly, the difference between the overlapping and non-overlapping estimates of beta and R^2 increases as the sample period decreases.
The most striking difference between OLS estimates with overlapping and non-overlapping observations is the strong rise in the Monte Carlo standard deviation of hedge ratios that we observe with non-overlapping observation. Thus, it becomes clear that the OLS estimates of MVHR on non-overlapping observations are far less efficient when sample sizes are small and the hedging horizon long.
Non-overlapping hedge ratios are slightly higher than overlapping hedge ratios, while the variance is slightly lower in the latter case. It is also found that hedging effectiveness increases with the length of hedging horizon.
Empirical application with real data reveals again that the variance of the hedge ratios with non-overlapping observations is slightly higher than in the case of overlapping observations, confirming that OLS estimates of MVHR with robust standard error on overlapping observations are more efficient when the hedging horizon is long and the sample size not sufficiently large.
As our article differs majorly from previous studies, since it consistently explores the behaviour of MVHR estimators within the hypothesized data generating process, not only eliminating the sample size reduction problem but actually exploring the performance of hedging ratio estimators when the sample size varies, we think that this article succeeds in highlighting and clarifying some methodological and empirical issues related to multiperiod hedging.
Indeed, the double estimate with simulated and real data have highlighted how misleading it could be to validate a theoretical model for MVHR with data that do not

strictly follow the assumed DGP. The Monte Carlo exercise enabled us to avoid such danger.

However, we think that this article mainly provides a practical contribution to hedging activities. Indeed, the results we have obtained allow us to give preference to results with robust variance and covariance estimates for overlapping observations instead of estimates for non-overlapping ones. The problem of sample size reduction, indeed, appears to be a major issue. Thus, we think we have provided a real compromise between sophisticated methods against sample reduction (e.g. wavelet analysis) and straightforward estimations: we think that such a compromise could be particularly useful for hedgers.

Future research could focus on providing more complex DGPs (for example, by inserting the lagged price differences) in order to see how different DGPs impact results. Moreover, future efforts could be directed towards comparing results obtained through other estimators, such as the ECM *à la* Gosh (1993).

Notes

[1] For a detailed review of theoretical and empirical issues about the optimal hedge ratio see, for example, Chen, Lee and Shrestha (2003).

[2] An exception is Lien and Luo (1993).

[3] Wavelets allows to decompose a time-series in both time and frequency.

[4] We limit ourselves to the investigation of OLS estimators since we are interested in unconditional hedging effectiveness, against this criterion OLS hedge ratios always perform better than other measures. For a discussion of this point see Lien (2005).

[5] For example: $\Delta_3 y_t = y_t - y_{t-3}$ or, equivalently, $\Delta_3 y_t = \Delta y_t + \Delta y_{t-1} + \Delta y_{t-2}$.

[6] It is sufficient to divide by *k* both numerator and denominator of the first r.h.s. term of 6 to see that its limit tends to $1/\beta$ which multiplied by the limit of MVHR (that is β) gives unity.

[7] Monte Carlo results are obtained running 5000 simulations.

[8] The hedging effectiveness (HE) is computed as follows: $HE = 1 - \frac{Var(\text{hedged portfolio})}{Var(\text{unhedged portfolio})}$.

[9] This estimator was proposed by Hansen and Hodrik (1980).

[10] Futures series are obtained from Bloomberg, while spot series are obtained from ISMEA.

[11] We have performed the ADF test with drift and trend.

[12] Lags have been selected according to the higher value provided by the Aikake's AIC, Hannan and Quinn, Schwarz's BIC, and Lutkepohl final prediction error.

[13] This result is confirmed, for example, in Chen, Lee and Shrestha (2004), Juhl, Kawaller and Koch (2012).

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Appendix A

From equation (1) in the article we get:

(A1) $\Delta y_t = u_t - u_{t-1} + \beta z_t$

Hence

(A2) $Var(\Delta y) = 2\sigma_u^2 + \beta^2\sigma_z^2 + 2\beta\sigma_{uz} - 2\phi\sigma_u^2$

since $Cov(u_{t-1}, z_t) = 0$

Similarly

(A3) $Var(\Delta x) = \sigma_z^2$

and

(A4) $Cov(\Delta y, \Delta x) = \sigma_{uz} + \beta\sigma_z^2$

Now define:

(A5) $\Delta_k x_t = \sum_{i=0}^{k-1} \Delta x_{t-1}$

For k=2 we have:

(A6) $Var(\Delta_2 x) = Var(\Delta x_t + \Delta x_{t-1}) = 2\sigma_z^2$ (in the general case $Var(\Delta_k x) = k\sigma_z^2$)

while the corresponding expression for y is:

(A7) $Var(\Delta_2 y) = 2(1-\phi)\sigma_u^2 + 2\beta^2\sigma_z^2 + 2\beta\sigma_{uz}$

which generalizes to:

(A8) $Var(\Delta_k y) = 2(1-\phi^k)\sigma_u^2 + k\beta^2\sigma_z^2 + 2\dfrac{1-\phi^k}{1-\phi}\beta\sigma_{uz}$

and

(A9)
$$\begin{aligned} &Cov(\Delta_2 y, \Delta_2 x) = Cov(\Delta x_t, \Delta y_t) + Cov(\Delta y_t \Delta x_{t-1}) + Cov(\Delta y_{t-1} \Delta x_t) + Cov(\Delta y_{t-1} \Delta x_{t-1}) \\ &= (\sigma_{uz} + \beta\sigma_z^2)(\phi-1)\sigma_{uz} + 0 + (\sigma_{uz} + \beta\sigma_z^2) = (1+\phi)\sigma_{uz} + 2\beta\sigma_z^2 \end{aligned}$$

Similarly we get:

(A10)
$$\begin{aligned} &Cov(\Delta_3 y, \Delta_3 x) = Cov(\Delta x_t, \Delta y_t) + Cov(\Delta y_t \Delta x_{t-1}) + Cov(\Delta y_t \Delta x_{t-2}) + Cov(\Delta y_{t-1} \Delta x_t) \\ &+ Cov(\Delta y_{t-1} \Delta x_{t-1}) + Cov(\Delta y_{t-1} \Delta x_{t-2}) + Cov(\Delta y_{t-2} \Delta x_t) + Cov(\Delta y_{t-2} \Delta x_{t-1}) + Cov(\Delta y_{t-2} \Delta x_{t-2}) \\ &= (\sigma_{uz} + \beta\sigma_z^2)(\phi-1)\sigma_{uz} + \phi(\phi-1)\sigma_{uz} + 0 + (\sigma_{uz} + \beta\sigma_z^2)(\phi-1)\sigma_{uz} + 0 + 0 + (\sigma_{uz} + \beta\sigma_z^2) \\ &= \sigma_{uz}\left[\sum_{i=1}^{3}(3-i)\phi^{i-1}(\phi-1)\right] + 3(\sigma_{uz} + \beta\sigma_z^2) \end{aligned}$$

which by induction leads to the general case of a k multiperiod change

(A11)
$$\begin{aligned} &Cov(\Delta_k y, \Delta_k x) = \sigma_{uz}\left[k + \sum_{i=1}^{k}(k-i)\phi^{i-1}(\phi-1)\right] + k(\beta\sigma_z^2) \\ &= k\sigma_{uz} + \sigma_{uz}\left[\sum_{i=1}^{k}(k-i)\phi^{i-1}(\phi-1)\right] + k(\beta\sigma_z^2) = k(\sigma_{uz} + \beta\sigma_z^2) + \sigma_{uz}[A] \end{aligned}$$

Where:

(A12)
$$A=\sum_{i=1}^{k}(k-i)\phi^{i-1}(\phi-1)=k(\phi-1)\sum_{i=1}^{k}\left(1-\frac{i}{k}\right)\phi^{i-1}=k(\phi-1)\left[\frac{1}{\phi}\sum_{i=1}^{k}\phi^{i}-\frac{1}{k}\sum_{i=1}^{K}i\phi^{i-1}\right]$$
$$=k(\phi-1)\left[\frac{1}{\phi}\frac{\left(\phi-\phi^{k+1}\right)}{1-\phi}-\frac{1}{k}\frac{\left[1-(k+1)\phi^{k}\right](1-\phi)-\left(\phi-\phi^{k+1}\right)}{(1-\phi)^{2}}\right]$$

Which, after simple algebra, simplifies to

(A13)
$$A=-\left[\left(k-i+\phi^{k}\right)-\frac{\phi\left(1-\phi^{k}\right)}{(1-\phi)}\right]$$

Now we can write the expression for the hedge ratio:

(A14)
$$MVHR(k)=\frac{Cov(\Delta_k y,\Delta_k x)}{Var(\Delta_k x)}=\frac{k\left(\sigma_{uz}+\beta\sigma_z^2\right)-\sigma_{uz}k\left[\left(k-1+\phi^{k}\right)-\frac{\phi\left(1-\phi^{k}\right)}{(1-\phi)}\right]}{k\sigma_z^2}=$$
$$=\frac{Cov(\Delta y,\Delta x)}{Var(\Delta x)}-\frac{\sigma_{uz}\left[\left(k-1+\phi^{k}\right)-\frac{\phi\left(1-\phi^{k}\right)}{(1-\phi)}\right]}{k\sigma_z^2}$$
$$=\frac{k\left(\sigma_{uz}+\beta\sigma_z^2\right)}{k\sigma_z^2}-\frac{\sigma_{uz}\left[\left(k-1+\phi^{k}\right)-\frac{\phi\left(1-\phi^{k}\right)}{(1-\phi)}\right]}{k\sigma_z^2}$$
$$=\frac{\sigma_{uz}}{\sigma_z^2}+\beta-\frac{\sigma_{uz}\left[\left(k-1+\phi^{k}\right)-\frac{\phi\left(1-\phi^{k}\right)}{(1-\phi)}\right]}{k\sigma_z^2}$$

The Ederington hedging effectiveness measure is given by:

(A15)
$$HE=\frac{\left[Cov(\Delta_k y\Delta_k x)\right]^2}{Var(\Delta_k y)Var(\Delta_k x)}$$

That is

(A16)
$$HE=\frac{\left[Cov(\Delta_k y\Delta_k x)\right]^2}{Var(\Delta_k y)}MVHR(k)$$

Where the first r.h.s. term is given by:

(A17)
$$\frac{Cov(\Delta_k y,\Delta_k x)^2}{Var(\Delta_k x)}=\frac{k\left(\sigma_{uz}+\beta\sigma_z^2\right)-\sigma_{uz}\left[\left(k-1+\phi^{k}\right)-\frac{\phi\left(1-\phi^{k}\right)}{(1-\phi)}\right]}{2\left(1-\phi^{k}\right)\sigma_u^2+k\beta^2\sigma_z^2+2\frac{1-\phi^{k}}{1-\phi}\beta\sigma_{uz}}=$$
$$=\frac{k\beta\sigma_z^2+\sigma_{uz}\left[\frac{\left(1-\phi^{k}\right)}{(1-\phi)}(1-2\phi)\right]}{2\left(1-\phi^{k}\right)\sigma_u^2+k\beta^2\sigma_z^2+2\frac{1-\phi^{k}}{1-\phi}\beta\sigma_{uz}}$$