DISEI - Università degli Studi di Firenze

Working Papers - Economics

Monopolistic Competition, As You Like It

Paolo Bertoletti and Federico Etro

Working Paper N. 31/2018

DISEI, Università degli Studi di Firenze
Via delle Pandette 9, 50127 Firenze, Italia
www.disei.unifi.it

Monopolistic Competition, As You Like It

Paolo Bertoletti and Federico Etro[1]

University of Pavia and University of Florence

December 2018

Keywords: Monopolistic competition, Asymmetric preferences, Heterogeneous firms, Generalized separability, Variable markups
JEL Codes: D11, D43, L11

Abstract

We study monopolistic competition with asymmetric preferences over a variety of goods provided by heterogeneous firms, and show how to compute equilibria through the Morishima measures of substitution. Further results concerning pricing and entry emerge under homotheticity and when demands depend on common aggregators, as for Generalized Additively Separable preferences (encompassing additive, Gorman-Pollak and implicit CES preferences). We discuss selection effects of changes in aggregate productivity, expenditure and market size, and present applications to trade, with markups variable across goods, and macroeconomics, with markups depending on aggregate variables.

[1] We thank Giacomo Calzolari, Lilia Cavallari, Mordecai Kurz, James Heckman, Alessandro Lizzeri, Florencio Lopez de Silanes, Mario Maggi, Peter Neary and seminar participants at Oxford University, EIEF (Rome), University of Pavia and the National University of Singapore.
Correspondence. Paolo Bertoletti: Dept. of Economics and Management, University of Pavia, Via San Felice, 5, I-27100 Pavia, Italy. Tel: +390382986202, email: *paolo.bertoletti@unipv.it.* Federico Etro: Florence School of Economics and Management, Via delle Pandette 32, Florence, 50127. Phone: 055-2759603. Email: *federico.etro@unifi.it.*

Which products and at which prices will be provided by markets where heterogeneous firms sell differentiated goods? This basic question should be at the core of modern economic theories that depart from the perfectly competitive paradigm by adopting the monopolistic competition set up pioneered by Chamberlin (1933). Most of these theories rely on a simplified model with symmetric and Constant Elasticity of Substitution (CES) preferences based on Dixit and Stiglitz (1977, Section I), which delivers constant markups, either across countries and among firms in trade models (Krugman, 1980; Melitz, 2003) or over time in macroeconomic models with flexible prices (Romer, 1990; Barro and Sala-i-Martin, 2004). Few applications use more general but still symmetric preferences (Dixit and Stiglitz, 1977, Section II; Bertoletti and Etro, 2016), even when considering variable productivity across firms (as in Melitz and Ottaviano, 2008, Parenti *et al.*, 2017, Arkolakis *et al.*, 2019, Dhingra and Morrow, 2019) and over time (as in Kimball, 1995, or Bilbiie *et al.*, 2012). In an attempt to capture the features of monopolistic competition in the spirit of Chamberlin,[2] we study a large industry with heterogeneous firms supplying genuinely differentiated commodities, and develop a methodology to characterize monopolistic competition in such a setting.[3] This suggests a richer approach to the differences between goods, firms and markets as well as over time, where such a variability matters also because it introduces new sources of inefficiency, both in the choice of the production mix and in the selection of firms, which are absent in the case of symmetric preferences and thus tend to be overlooked. We propose applications to trade, with markups variable across goods of different and endogenous quality, and to macroeconomics, with markups depending on aggregate variables that can magnify business cycle propagation.

Consider demand systems derived from preferences over a variety of different commodities that can be represented by well-behaved utility functions. Each commodity is produced with idiosyncratic marginal and fixed costs. Our basic question is simply which strategies are adopted by firms in such a market. The starting point is the analysis of Cournot and Bertrand equilibria in which firms choose either their quantities or their prices taking as given the strategies of the competitors and the demand systems. We generalize the familiar pricing conditions by expressing the equilibrium markup of firms in terms of their market shares and of the substitutability of their own products with those sold by competitors. Substitutability is measured by (the average of) the Morishima Elasticities of Substitution, as rediscovered and formalized by Blackorby and Russell (1981).[4] On this basis, we discuss how to solve for Cournot and

[2] Chamberlin (1933) defined monopolistic competition with reference to factors affecting the shape of the demand curve, and certainly did not intend to limit his analysis to the case of symmetric goods. He saw no discontinuity between its own market theory and the theory of monopoly as familiarly conceived, claiming *inter alia* that "monopolistic competition embraces the whole theory of monopoly. But it also looks beyond, and considers the interrelations, wherever they exist, between monopolists who are in some appreciable degree of competition with each other." (Chamberlin, 1937, p. 571-2).

[3] See also the seminal work of Spence (1976), who explicitly deals with the problem of product selection, focusing on quasi-linear preferences.

[4] The Morishima Elasticity of Substitution was originally proposed by Morishima (1967) in

Bertrand equilibria by computing the Morishima measures.

We then move to competition among a large number of firms where, in line with Spence (1976) and Dixit and Stiglitz (1977, 1993), market shares are negligible. In particular, we define monopolistic competition as the case in which firms "perceive" demand elasticity as given by the average Morishima elasticity (which approximately coincides with the actual one when market shares are indeed small enough). And we introduce free entry to ask a few fundamental questions, such as which products are provided by the market, what kind of selection is associated with changes in market size (i.e., opening up to free trade), expenditure (i.e., a demand shock) and aggregate productivity (i.e., technological growth) and what is the relation with an optimal provision of goods chosen by a social planner. The answers are simple under (asymmetric) CES preferences, because the set of goods provided by the market is not affected by changes of aggregate productivity, an increase of expenditure or market size delivers new goods but without affecting the entry sequence, and the latter corresponds to the optimal one. This is not the case in general, but we will show that the irrelevance of (common) productivity shocks is preserved under homothetic preferences, the neutrality of expenditure under directly additive preferences, the neutrality of the market size under indirectly additive preferences and the optimality of market entry under the unexplored class of implicit CES preferences.

Since typical demand systems depend on simple aggregators of firm strategies, we study in further depth monopolistic competition for the Generalized Additively Separable (GAS) preferences introduced by Pollak (1972) and Gorman (1970a, 1987), which deliver demand systems depending on one aggregator. Analyzing monopolistic competition with GAS preferences, intuition suggests that to take the common aggregator as given while computing the elasticity of demand should be approximately correct (i.e., profit maximizing) when market shares are negligible. We show that this is indeed the case, in the sense that these perceived demand elasticities are approximately equal to the average Morishima measures (which in turn are close to the actual ones) when shares are negligible. In addition, the equilibrium strategies do not depend on whether prices or quantities are chosen by the firms, implying that imperfectly competitive choices do actually "converge" to those of monopolistic competition.

This approach provides a simple way to solve for asymmetric equilibria, and it allows the computation of prices in closed-form solution for a variety of examples.[5] Under additivity of preferences we can show uniqueness of the free entry equilibrium in spite of asymmetries between goods, and make some comparison among firms that are active in equilibrium and in the social optimum.

a book review written in Japanese.

[5]We also extend the same approach to separable preferences that generate demand functions depending on multiple aggregators. In particular, we discuss preferences that feature separability of marginal utilities (including a generalization of the preferences employed by Melitz and Ottaviano, 2008), and the case of the so-called implicitly additive preferences (Hanoch, 1975), as well as an example of restricted AIDS preferences (Deaton and Muellbauer, 1980).

In addition, for the class of directly additive preferences we show that an increase in the market size (i.e. opening up to costless trade) favors the entry of goods with a less elastic demand, while changes in expenditure are neutral on the entry sequence. For the class of indirectly additive preferences, equilibrium pricing is independent across firms and the price of each firm only depends on its marginal cost, product substitutability and consumers' expenditure. Moreover, an increase in the market size is always neutral on prices as well as on the entry sequence, while an increase of expenditure (i.e. a positive demand shock) favors the entry of firms facing demands perceived as most elastic.

We then introduce the more general Gorman-Pollak preferences to the analysis of monopolistic competition, which deliver a markup depending both on the quantity or price of the firm and on the common aggregator, and can be exploited in trade applications where markups change between goods and across destinations (nesting results in Bertoletti *et al.*, 2018, and Arkolakis *et al.*, 2019). The special case of constant markups that differ across goods emerges in case of "power" additive subutilities and with the unexplored "self-dual addilog" preferences (see Houthakker, 1965). Such examples can be exploited in trade applications where firms sell goods of different qualities at different markups in different markets, whose empirical relevance has been pointed out in Manova and Zhang (2012) and others.

We finally discuss the class of "implicit CES" preferences (Gorman, 1970a,b, and Blackorby and Russell, 1981), which also belong to the GAS type and are suitable for trade and macroeconomic applications. These preferences deliver markups common across goods that vary directly with the utility level (which is the relevant aggregator). We employ them to present two general equilibrium applications. In a trade model *à la* Melitz with free entry of heterogeneous firms producing goods of different qualities these preferences generate novel selection effects: in particular, we show that opening up to costless trade can reduce markups, change the endogenous distribution of qualities across active firms and select more efficient firms. While none of these effects of "globalization" is produced under symmetric CES preferences, they emerge naturally in a setting with these and other asymmetric preferences. In a dynamic macroeconomic model, we show that these preferences provide a channel of propagation of aggregate shocks through endogenous markup variability: in particular, countercyclical markups can magnify positive temporary shocks by reducing the relative price of the final goods and promoting aggregate consumption. These insights extend to other non-homothetic preferences and deliver a new role for demand in business cycle models with flexible prices.

Our work is related to different literatures. We generalize the analysis of imperfect competition with differentiated products (usually studied under quasilinear preferences: see Vives, 1999) by reframing it in terms of the Morishima elasticities. After the seminal contribution of Spence (1976), only few papers have analyzed monopolistic competition with asymmetric preferences. The work of Dixit and Stiglitz (1977: Section III) only dealt with a specific example with intersectoral perfect substitutability. The earliest treatement we are aware of is in a work of Pascoa (1997), mainly focused on an example with Stone-Geary preferences and a continuum of goods. More recently, D'Aspremont and Dos Santos Ferreira (2016, 2017) have provided a related analysis of asymmetric preferences with an outside good adopting an alternative equilibrium concept, but their monopolistic competition limit is consistent with ours when market shares are negligible. We are not aware of other studies on equilibrium and optimal entry under asymmetric preferences. The trade literature with heterogeneous firms, started by Melitz (2003) and Melitz and Ottaviano (2008), has usually considered monopolistic competition with symmetric preferences; only a few works have added asymmetries to model quality differentials among goods (for instance Baldwin and Harrigan, 2012, Crozet *et al.*, 2012, and Feenstra and Romalis, 2014), but retaining a CES structure. We follow the spirit of this literature generalizing it to genuinely asymmetric preferences that deliver possibly different and variable markups.[6]

The work is organized as follows. Section 1 presents alternative equilibria of imperfect competition for the same demand microfoundation. Section 2 and 3 study monopolistic competition under homothetic and GAS preferences. Section 4 extends our approach to the case of other separable preferences. Section 5 sketches applications to trade and macroeconomics. Section 6 concludes. Propositions concerning particular classes of preferences are proved in the Appendix.

1 The Model

We consider L identical consumers with preferences over a finite number n of commodities represented by well-behaved direct and indirect utility functions:

$$U = U(\mathbf{x}) \quad \text{and} \quad V = V(\mathbf{s}), \tag{1}$$

where $\mathbf{x}$ is the n-dimensional vector of quantities and $\mathbf{s} = \mathbf{p}/E$ is the corresponding vector of prices normalized by expenditure E. We assume that the utility maximizing choices are unique, interior ($\mathbf{x}, \mathbf{p} > \mathbf{0}$) and characterized by the first-order conditions for utility maximization. Therefore, the inverse and direct (Marshallian) demand systems are delivered by Hotelling-Wold's and Roy's identities:

$$s_i(\mathbf{x}) = \frac{U_i(\mathbf{x})}{\widetilde{\mu}(\mathbf{x})}, \qquad x_i(\mathbf{s}) = \frac{V_i(\mathbf{s})}{\mu(\mathbf{s})}, \tag{2}$$

where

$$\widetilde{\mu}(\mathbf{x}) = \sum_{j=1}^{n} U_j(\mathbf{x}) x_j, \qquad \mu(\mathbf{s}) = \sum_{j=1}^{n} V_j(\mathbf{s}) s_j \tag{3}$$

and U_i and V_i denote marginal utilities, $i = 1, .., n$. Here $\widetilde{\mu}$ is the marginal utility of income *times* the expenditure level, and it holds that $|\mu(\mathbf{s})| = \widetilde{\mu}(\mathbf{x}(\mathbf{s}))$, as can be verified by adding up the market shares $b_j = s_j x_j$. As a simple

[6] See also Mrázová and Neary (2018) on selection effects with heterogeneous firms and Hottman *et al.* (2016) for an empirical approach based on a nested-CES utility system.

example we will occasionally refer to the asymmetric CES preferences, that can be represented by

$$U = \sum_{j=1}^{n} \widetilde{q}_j x_j^{1-\epsilon} \quad \text{and} \quad V = \sum_{j=1}^{n} q_j s_j^{1-\varepsilon}, \tag{4}$$

where $q_j = \widetilde{q}_j^{\epsilon} > 0$ can be interpreted as an idiosyncratic quality index for good j, and $\epsilon = 1/\varepsilon \in [0, 1)$ governs substitutability between goods.

Firm i produces good i at the marginal cost $c_i = \widetilde{c}_i/A > 0$, where the common parameter $A > 0$ represents aggregate productivity: the variable profits of firm i are then given by:

$$\pi_i = (p_i - c_i)x_i L. \tag{5}$$

We begin by studying market equilibria in which firms correctly perceive the demand system and choose their profit-maximizing strategies. In the tradition of industrial organization we have to consider two cases, with each firm choosing either its production level (Cournot competition) or its price (Bertrand competition). We then use these equilibria to define a generalized form of monopolistic competition, and to discuss entry.

1.1 Competition in quantities

Let us consider firms that choose their quantities on the basis of the inverse demand functions $s_i(\mathbf{x})$ in (2). Correctly anticipating the quantities produced by the competitors, each firm i chooses x_i to equate its marginal revenue to its marginal cost c_i. The relevant (per-consumer) marginal revenue of firm i is $MR_i = \partial (p_i x_i) / \partial x_i$, where $p_i(\mathbf{x}) = s_i(\mathbf{x})E$. It can be written as:

$$\begin{aligned} MR_i &= \frac{[U_i(\mathbf{x}) + U_{ii}(\mathbf{x})x_i]\, \widetilde{\mu}(\mathbf{x}) - U_i(\mathbf{x})x_i \left[U_i(\mathbf{x}) + \sum_{j=1}^{n} U_{ji}(\mathbf{x})x_j\right]}{\widetilde{\mu}(\mathbf{x})^2} E \\ &= p_i(\mathbf{x}) \left[1 - s_i(\mathbf{x})x_i - \sum_{j=1}^{n} \epsilon_{ij}(\mathbf{x}) s_j(\mathbf{x}) x_j\right], \end{aligned}$$

where the (gross) Morishima Elasticity of Complementarity, henceforth MEC, between varieties i and j is defined as:[7]

$$\epsilon_{ij}(\mathbf{x}) = -\frac{\partial \ln \{s_i(\mathbf{x})/s_j(\mathbf{x})\}}{\partial \ln x_i} = \frac{U_{ji}(\mathbf{x})x_i}{U_j(\mathbf{x})} - \frac{U_{ii}(\mathbf{x})x_i}{U_i(\mathbf{x})}. \tag{6}$$

This inverse measure of substitutability depends on preferences and not on the specific utility function which is chosen to represent them. Since substitutability

[7] See Blackorby and Russell (1981) on the corresponding *net* measure which applies to compensated demands. The larger is ϵ_{ij} the smaller is the possibility of good j to substitute for good i. Notice that $\epsilon_{ii} = 0$ and that in general $\epsilon_{ij} \neq \epsilon_{ji}$ for $i \neq j$.

can differ among goods, let us compute the weighted average of the MECs for good i with respect to all the other goods j, with weights based on the expenditure shares $b_j(\mathbf{x}) = s_j(\mathbf{x})x_j$, namely:

$$\bar{\epsilon}_i(\mathbf{x}) = \sum_{j \neq i}^{n} \epsilon_{ij}(\mathbf{x}) \frac{b_j(\mathbf{x})}{1 - b_i(\mathbf{x})}. \tag{7}$$

It is immediate to verify that the marginal revenue can be rewritten as $MR_i = p_i(\mathbf{x})[1 - b_i(\mathbf{x})]\,[1 - \bar{\epsilon}_i(\mathbf{x})]$, and that the Cournot equilibrium quantities satisfy the system:[8]

$$p_i(\mathbf{x}) = \frac{c_i}{1 - \epsilon_i^C(\mathbf{x})} \quad \text{for } i = 1, 2, ..., n, \tag{8}$$

where the left hand side comes from the inverse demand given in (2) and the right hand side depends on:

$$\epsilon_i^C(\mathbf{x}) = b_i(\mathbf{x}) + [1 - b_i(\mathbf{x})]\bar{\epsilon}_i(\mathbf{x}). \tag{9}$$

Here ϵ_i^C is an increasing function of the market share of firm i and of its average Morishima elasticity $\bar{\epsilon}_i$ (which we assume to be smaller than unity).[9] Intuitively, a firm's markup is higher when it supplies a good that is on average less substitutable with the other goods (high $\bar{\epsilon}_i$), and its market share is larger (high b_i). In the CES example (4) ϵ corresponds to the common and constant MEC, and one can obtain closed form solutions in simple cases.

1.2 Competition in prices

Consider now firms that choose their prices on the basis of the direct demand $x_i(\mathbf{s})$ in (2), while correctly anticipating the prices of the competitors. The elasticity of the Marshallian direct demand of firm i can be computed as:

$$\left|\frac{\partial \ln x_i}{\partial \ln p_i}\right| = -\frac{s_i}{x_i(\mathbf{s})} \frac{V_{ii}(\mathbf{s})\mu(\mathbf{s}) - V_i(\mathbf{s})\left[V_i(\mathbf{s}) + \sum_{j=1}^{n} V_{ji}(\mathbf{s})s_j\right]}{\mu(\mathbf{s})^2}.$$

Consider the (gross) Morishima Elasticity of Substitution, or MES, between goods i and j:[10]

$$\varepsilon_{ij}(\mathbf{s}) = -\frac{\partial \ln\{x_i(\mathbf{s})/x_j(\mathbf{s})\}}{\partial \ln s_i} = \frac{s_i V_{ji}(\mathbf{s})}{V_j(\mathbf{s})} - \frac{s_i V_{ii}(\mathbf{s})}{V_i(\mathbf{s})}, \tag{10}$$

[8] Throughout this work we assume that the first-order condition for profit maximization characterizes firm behaviour. Of course, existence and unicity of market equilibrium also require that the demand system satisfies a number of regularity conditions (for a related discussion see Vives, 1999, Ch. 6).

[9] Note that $\epsilon_{ij} < 1$ is equivalent to the condition that the relative expenditure $s_i x_i / s_j x_j$ is (locally) increasing with respect to x_i, but it would be compatible with i and j being either q-substitutes ($\partial \ln s_j / \partial \ln x_i < 0$) or q-complements ($\partial \ln s_j / \partial \ln x_i > 0$). Of course, $\bar{\epsilon}_i(\mathbf{x}) < 1$ is a weaker condition.

[10] See Blackorby and Russell (1981) and Blackorby *et al.* (2007). The higher is ε_{ij} the greater is the possibility of good j to substitute for good i. Notice that $\varepsilon_{ii} = 0$ and that in general $\varepsilon_{ij} \neq \varepsilon_{ji}$ for $i \neq j$.

which again depends on preferences and not on their specific representations, and compute the weighted average:

$$\bar{\varepsilon}_i(\mathbf{s}) \equiv \sum_{j \neq i}^{n} \varepsilon_{ij}(\mathbf{s}) \frac{b_j(\mathbf{s})}{(1 - b_i(\mathbf{s}))} \tag{11}$$

(assumed larger than unity),[11] where, with a little abuse of notation, $b_j(\mathbf{s}) = s_j x_j(\mathbf{s})$ is now the expenditure share of firm i as a function of normalized prices.

We can now rewrite the demand elasticity $|\partial \ln x_i / \partial \ln p_i|$ as:

$$\varepsilon_i^B(\mathbf{s}) = b_i(\mathbf{s}) + [1 - b_i(\mathbf{s})]\bar{\varepsilon}_i(\mathbf{s}), \tag{12}$$

to define the Bertrand equilibrium through the following system:

$$p_i = \frac{\varepsilon_i^B(\mathbf{s})\, c_i}{\varepsilon_i^B(\mathbf{s}) - 1} \quad \text{for } i = 1, 2, .., n. \tag{13}$$

Firms set higher markups if their goods are on average less substitutable than those of competitors (low $\bar{\varepsilon}_i$) and their market shares larger (high b_i). In the CES example (4) the parameter ε is the constant and symmetric MES and is the reciprocal of the common MEC.

1.3 Generalized monopolistic competition

The remainder of this work is dedicated to analyze monopolistic competition under asymmetric preferences. There are alternative ways to make sense of this concept but, in the spirit of Dixit and Stiglitz's (1993) reply to Yang and Heidra (1993), we interpret monopolistic competition as the result of having firms that correctly perceive market shares as negligible. In fact, what Dixit and Stiglitz (1977) did in their symmetric setting amounts to neglect any term of order $1/n$ in the demand elasticities, where n was a number of firms assumed sufficiently large to make the omitted terms small. Similarly, in our setting, when there are many goods we expect consumers to spread their expenditure if preferences are well-behaved and not too asymmetric, so that the market shares should be small for all goods.[12] On this basis, our previous results suggest to approximate the relevant demand elasticities with the corresponding averages of the Morishima measures.

Accordingly, we define as monopolistically competitive an environment where market shares are negligible, that is $b_i \approx 0$ for any $i = 1, .., n$, and where firms, correctly anticipating the value of actual demands, "perceive" the relevant elasticities as given by the average Morishima elasticities. This approach actually leads to two approximations according to whether we refer either to quantity

[11] Here $\varepsilon_{ij} > 1$ is equivalent to the condition that the relative expenditure $s_i x_i / s_j x_j$ is (locally) decreasing with respect to s_i, and it would be compatible with i and j being either so-called p-substitutes ($\partial \ln x_j / \partial \ln s_i > 0$) or p-complements ($\partial \ln x_j / \partial \ln s_i < 0$).

[12] Sufficient conditions on preferences to deliver this result are studied in Vives (1987).

or to price competition. In the first case we approximate (8) by using the expression:

$$p_i(\mathbf{x}) = \frac{c_i}{1-\bar{\epsilon}_i(\mathbf{x})} \quad \text{for } i=1,2,..,n. \tag{14}$$

In the second case we approximate (13) by:

$$p_i = \frac{\bar{\varepsilon}_i\left(\mathbf{p}/E\right)c_i}{\bar{\varepsilon}_i\left(\mathbf{p}/E\right)-1} \quad \text{for } i=1,2,..,n. \tag{15}$$

These simplified systems need to be solved to derive the prices and quantities which arise in a monopolistic competition equilibrium (that ought to imply negligible market shares). Once we depart from symmetry this may still be a formidable task, but in next sections we will consider a methodology that allows one to obtain explicit solutions for several classes of asymmetric preferences.

We can learn something more about this approach to monopolistic competition by considering the relevant cross demand elasticities. They can be computed as:

$$\begin{aligned}\frac{\partial \ln p_j\left(\mathbf{x}\right)}{\partial \ln x_i} &= \frac{U_{ji}\left(\mathbf{x}\right)x_i}{U_j\left(\mathbf{x}\right)} - \sum_{h=1}^{n}\frac{U_{hi}(\mathbf{x})x_i}{U_h(\mathbf{x})}b_h(\mathbf{x}) \\ &= \epsilon_{ij}\left(\mathbf{x}\right) - \bar{\epsilon}_i(\mathbf{x}) + b_i(\mathbf{x})\bar{\epsilon}_i(\mathbf{x}),\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\partial \ln x_j\left(\mathbf{s}\right)}{\partial \ln s_i} &= \varepsilon_{ij}\left(\mathbf{s}\right) - \left|\frac{\partial \ln x_i\left(\mathbf{s}\right)}{\partial \ln p_i}\right| \\ &= \varepsilon_{ij}\left(\mathbf{s}\right) - \bar{\varepsilon}_i\left(\mathbf{s}\right) - b_i\left(\mathbf{s}\right)\left(1-\bar{\varepsilon}_i\left(\mathbf{s}\right)\right).\end{aligned} \tag{17}$$

When shares are indeed negligible the cross effects should be perceived as negligible too whenever the differences $\epsilon_{ij}-\bar{\epsilon}_i$ and $\varepsilon_{ij}-\bar{\varepsilon}_i$ are small *and* the perceived own elasticities are not very large. Apparently, this is the case that Dixit and Stiglitz (1993) had in mind, and we expect it to apply to the typical monopolistic competition equilibrium with positive markups. Notice that the former condition is satisfied in any equilibrium of a symmetric environment. However, both conditions might be violated in our asymmetric setting: in similar cases the perceived cross demand elasticities can be large, and associated to a large own demand elasticity and therefore to small equilibrium markups. In other words, it can happen that goods are perceived as highly substitutable and that monopolistic competition pricing approximates marginal cost pricing as in a perfectly competitive setting.[13] We will exemplify this possibility in Section 2.1, namely for the case of translog preferences, and in Appendix F in the case of restricted AIDS preferences (Deaton and Muellbauer, 1980).

Note that in the CES case (4) the conditions (14) and (15) do characterize the same monopolistic competition solution:

$$\widehat{p}_i = \frac{c_i}{1-\epsilon} = \frac{\varepsilon c_i}{\varepsilon - 1}, \tag{18}$$

[13] Notice that, in general, the value of these cross demand elasticities need not be negligible in a strategic setting. In fact, if they were null there would be no reason for strategic interaction and we could think of those producers as "isolated monopolists".

and that in such a case the cross effects (16) and (17) actually vanish when market shares become negligible.

1.4 Entry

Which goods will be provided in a monopolistic competition equilibrium and which goods should be provided by a social planner? In this section we discuss these questions by considering free entry equilibria when the production of each good requires a positive fixed cost.[14] This analysis is of course important for general equilibrium applications of monopolistic competition.[15] Let us assume that preferences are defined over a large but finite set Ω of N different commodities, and that each good $i \in \Omega$ can be produced by a single firm only after paying a fixed entry cost $F_i > 0$. In the spirit of Chamberlin (1933), one can think of firms entering the market as long as they can price above the average cost.[16] Under monopolistic competition there are $n \leq N$ active firms: the price of the other $N - n$ goods should be set above their choke levels, or equivalently at ∞. The variable profits of an active firm $i = 1, ..., n$ can be written as $\pi_i = \frac{p_i - c_i}{p_i} b_i EL$. By using equilibrium pricing condition (15) and defining $\phi_i \equiv -\partial \ln V / \partial \ln s_i$ as the price elasticity of utility of commodity i, with average $\phi \equiv \frac{1}{n} \sum_{j=1}^{n} \phi_j$, we can express equilibrium profits as:

$$\widehat{\pi}_i = \frac{\phi_i(\widehat{\mathbf{s}})EL}{\phi(\widehat{\mathbf{s}})\bar{\varepsilon}_i(\widehat{\mathbf{s}})n} \tag{19}$$

(a corresponding formula can be obtained from the dual representation of preferences through the average MEC). Since EL/n are common to all firms, this implies that active firms with a lower average MES and a higher ratio ϕ_i/ϕ have higher variable profits because they can set higher markups and conquer larger market shares (these elasticities determine the intensive and extensive profit margins). In a free entry equilibrium only firms covering fixed costs with their variable profits can be active.

Let us briefly consider also an optimal allocation of resources. A social planner maximizing utility under a resource constraint $EL = \sum_{j=1}^{n}(c_j x_j L + F_j)$ would set a common markup on all goods: this already implies that a market

[14] It may be useful to remind the reader that without fixed costs a perfectly competitive market would optimally provide all the suitable goods by pricing them at marginal cost. The question of which goods are actually introduced becomes relevant under fixed costs and asymmetric preferences (since with symmetry it simplifies to the question of which *number* of goods should be provided).

[15] For these applications, one can also add to our basic setting a good representing the outside economy. This is particularly relevant for trade applications with a competitive sector and for macroeconomic applications with labor supply. Pricing within the monopolistically competitive sector carries on unchanged after imposing independent pricing for the outside good and taking this into account in the computation of the Morishima elasticities.

[16] One can also consider an entry process *à la* Melitz (2003) that exhausts *expected* profits: given an *ex ante* probability distribution over parameters indexing the goods to be produced, firms would enter the market until they expect profits to cover the entry cost. This would leave unchanged the competition stage whenever costs and market size attract a number of firms large enough to justify the assumption of small market shares (see Section 5.1).

equilibrium tends to provide too much of the goods with a low average MES. Without loss of generality, the optimal prices can be set at the marginal costs when the fixed costs are directly paid out of individual expenditure. Accordingly, the social planner chooses the set Γ of goods to be provided to solve:

$$\max_{\Gamma \subseteq \Omega} V \left(\underset{i \in \Gamma}{s_i} = \frac{c_i}{E - \sum_{i \in \Gamma} F_i / L}, \underset{i \notin \Gamma}{s_i} = \infty \right). \tag{20}$$

Of course, as long as unproduced goods become less costly, they can enter the set of optimally provided goods.

At this level of generality, we can neither exclude a multiplicity of market equilibria, nor we can compare them to a social optimum: however, progress can be made under further assumptions on the preferences. As a benchmark, let us reconsider the CES example (4), and let $\widehat{\Gamma} \subseteq \Omega$ be a set of goods provided in equilibrium at prices (18) and $\Gamma^* \subseteq \Omega$ an optimal commodity set.[17] We can directly compute profits (19) for a given expenditure as:

$$\widehat{\pi}_i = \frac{q_i \widetilde{c}_i^{1-\varepsilon} EL}{\varepsilon \sum_{j \in \widehat{\Gamma}} q_j \widetilde{c}_j^{1-\varepsilon}}, \tag{21}$$

which is independent from aggregate productivity A (an increase of productivity reduces prices while increasing proportionally demand so that profits, and thus $\widehat{\Gamma}$, remain unchanged), and linear with respect to the total market size EL (for a given set of firms). Thus, the condition of a non-negative profit for good i,

$$\frac{q_i \widetilde{c}_i^{1-\varepsilon}}{F_i} \geq \frac{\varepsilon \sum_{j \in \widehat{\Gamma}} q_j \widetilde{c}_j^{1-\varepsilon}}{EL}, \tag{22}$$

uniquely defines an order among firms based on the value of the left-hand side of (22): it is natural to think of it as establishing the sequence of market introduction (see Section 3.1). Moreover, the equilibrium order is identical to the welfare order implicitly defined by (20), which now reads as:

$$\max_{\Gamma \subseteq \Omega} V = \frac{\sum_{j \in \Gamma} q_j \widetilde{c}_j^{1-\varepsilon}}{\left[A \left(E - \sum_{j \in \Gamma} F_j / L \right) \right]^{1-\varepsilon}}.$$

Accordingly, a good i should be optimally provided only if:

$$\frac{q_i \widetilde{c}_i^{1-\varepsilon}}{F_i} \geq \lambda^* \left(A \frac{EL - \sum_{j \in \Gamma^*} F_j}{L} \right)^{1-\varepsilon}, \tag{23}$$

where λ^* is the relevant resource opportunity cost. Thus, as we will prove formally in Section 3, the asymmetric CES preferences generate a free entry equilibrium and a social optimum such that the identity of the goods introduced

[17] The sets $\widehat{\Gamma}$ and $\Gamma^*$are actually unique under CES or additive preferences, as we will prove in Section 3.

is independent from aggregate productivity A and it is determined by the total market size EL, while the sequence of introduction is unaffected from either expenditure E or market size L. In fact, one can also verify that the free entry equilibrium is optimal if industry profits are redistributed as additional income to consumers.[18]

In this example, as in endogenous growth models based on CES specifications *à la* Romer (1990) or Barro and Sala-i-Martin (2004), the sequence of new goods introduced in the economy does not deviate from the sequence of varieties that would be introduced by a social planner, in spite of asymmetries or shocks to aggregate productivity, income and population. Under more general conditions there is no reason why the market should be expected to provide the optimal set of goods, or to introduce them in the optimal order independently from shocks, but we will see that some of the special properties of the CES example will extend to more general classes of preferences. In particular, the irrelevance of productivity shocks will be preserved under homothetic preferences, the neutrality of expenditure under directly additive preferences, the neutrality of the market size under indirectly additive preferences and the optimality of market entry under implicit CES preferences.

2 Homothetic preferences

Monopolistic competition with symmetric homothetic preferences has been studied by Benassy (1996) and others.[19] Here we are concerned with the more general case of asymmetric homothetic preferences, because they are crucial for representative agent models and provide an interesting application of our proposed equilibria. Let us normalize the indirect utility function to be:

$$V = \frac{E}{P(\mathbf{p})} = P(\mathbf{s})^{-1}, \tag{24}$$

where P is homogeneous of degree 1 and represents a fully-fledged price index. For instance $P = \left[\sum_{j=1}^{n} q_j p_j^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$ in the CES case (4). Roy's identity delivers direct demands $x_i = P_i(\mathbf{s})/P(\mathbf{s})$ and market shares $b_i = s_i P_i(\mathbf{s})/P(\mathbf{s})$, which are homogeneous respectively of degree -1 and 0. This allows us to compute

[18] This can be confirmed by using the marginal utility of income for λ^*, and computing:

$$\begin{aligned} \hat{E} &= E + \frac{1}{L}\sum_{j\in\widehat{\Gamma}}(\widehat{\pi}_j - F_j) = E + \frac{\hat{E}}{\varepsilon} - \sum_{j\in\widehat{\Gamma}} F_j/L \\ &= \frac{\varepsilon}{\varepsilon-1}(E - \sum_{j\in\widehat{\Gamma}} F_j/L), \end{aligned}$$

where $\hat{E}$ is the equilibrium expenditure level.

[19] See Feenstra (2003) on translog preferences, and Feenstra (2018) for its generalization to the case of the so-called "quadratic mean of order r" (QMOR) preferences (with in addition heterogeneous firms).

the MES as:

$$\varepsilon_{ij}(\mathbf{s}) = \frac{s_i P_{ji}(\mathbf{s})}{P_j(\mathbf{s})} - \frac{s_i P_{ii}(\mathbf{s})}{P_i(\mathbf{s})},$$

which is homogeneous of degree 0, being the difference of two functions that are both homogeneous of that degree. Therefore also the average MES $\bar{\varepsilon}_i(\mathbf{s})$ are homogeneous of degree zero, which implies immediately that pricing is independent from the expenditure level (for a given set of firms).[20] Similar results can be derived starting from the direct utility (which can be written as a consumption index) and using the inverse demand system and the average MEC to study quantity competition.

2.1 Examples

We now consider equilibrium pricing for two specifications of homothetic preferences.

Translog preferences As a first example, let us consider the homothetic translog preferences (Christensen *et al.*, 1975) represented by the following price index:

$$P(\mathbf{s}) = \exp\left[\ln\alpha_0 + \sum_i \alpha_i \ln s_i + \frac{1}{2}\sum_i\sum_j \beta_{ij}\ln s_i \ln s_j\right], \tag{25}$$

where we assume without loss of generality $\beta_{ij} = \beta_{ji}$, and we need $\sum_i \alpha_i = 1$ and $\sum_j \beta_{ij} = 0$ to satisfy the linear homogeneity of P (a symmetric version of these preferences is used by Feenstra, 2003). The direct demand for good i is:

$$x_i(\mathbf{s}) = \frac{\alpha_i + \sum_j \beta_{ij}\ln s_j}{s_i},$$

which delivers the market share $b_i = \alpha_i + \sum_j \beta_{ij}\ln s_j$. Maximization of profits provides the Bertrand equilibrium conditions:

$$p_i = c_i\left(1 + \frac{b_i}{\beta_i}\right), \tag{26}$$

where the positiveness of $\beta_i \equiv -\beta_{ii}$ is necessary to ensure $\varepsilon_i^B = 1 + \beta_i/b_i > 1$.

We can obtain the same result, as well as the monopolistic competition equilibrium, by deriving the Morishima elasticity between goods i and j as:

$$\varepsilon_{ij} = 1 + \frac{\beta_i}{b_i} + \frac{\beta_{ji}}{b_j},$$

[20] When preferences are homothetic and symmetric, this implies that Morishima elasticities and markups in a symmetric equilibrium can be at most a function of the number of goods. While this result has been used elsewhere (for instance in Bilbiie *et al.*, 2012), we are not aware of a formal proof (we are thankful to Mordecai Kurz for pointing this out).

so that the average MES is:

$$\bar{\varepsilon}_i = \sum_{j \neq i}^{n} \varepsilon_{ij} \frac{b_j}{1 - b_i} = 1 + \frac{\beta_i}{(1 - b_i)\, b_i}.$$

This allows one to get (26) from (13), and to obtain the monopolistic competition prices:

$$p_i = c_i \left[1 + \frac{(1 - b_i)\, b_i}{\beta_i} \right] \tag{27}$$

from (15). Notice that the prices of monopolistic competition are below the Bertrand prices (26) for given market shares, and that, when market shares are negligible ($b_i \approx 0$), the average MES is large, goods are highly substitutable and prices must be close to the marginal costs (i.e., $\widehat{p}_i \approx c_i$), approaching the case of perfect competition.

Generalized linear preferences Let us now consider an example of homothetic preferences due to Diewert (1971). Suppose that preferences can be represented by the following direct utility/consumption index:

$$U = \sqrt{\mathbf{x}}' \mathbf{A} \sqrt{\mathbf{x}} = \sum_i \sum_j \sqrt{x_i} a_{ij} \sqrt{x_j}, \tag{28}$$

where, without loss of generality, we can take the matrix $\mathbf{A}$ to be symmetric. To satisfy the standard regularity conditions we assume that $a_{ij} \geq 0$ for any i, j (notice that parameters a_{ii}, $i = 1, .., n$ have no impact on the Hessian D^2U). Here we obtain $U_i = \sum_j a_{ij} \sqrt{x_j} / \sqrt{x_i}$ and $\widetilde{\mu} = U$, with market shares $b_i = (\sqrt{x_i} \sum_j a_{ij} \sqrt{x_j})/U$. Since the MECs can be computed as:

$$\epsilon_{ij} = \frac{1}{2} \left[1 + \frac{a_{ij} \sqrt{x_i}}{\sum_h a_{jh} \sqrt{x_h}} - \frac{a_{ii} \sqrt{x_i}}{\sum_h a_{ih} \sqrt{x_h}} \right],$$

we obtain the average MEC:

$$\bar{\epsilon}_i = \frac{1}{2} \left\{ 1 - \frac{a_{ii} \sqrt{x_i}}{\sum_h a_{ih} \sqrt{x_h}} + \frac{b_i - a_{ii} x_i / U(\mathbf{x})}{1 - b_i} \right\},$$

which allows us to determine the equilibrium conditions.[21] Here $\bar{\epsilon}_i$ is strictly positive for every good, implying positive markups, unless $a_{ij} = 0$ for any $i \neq j$ (in which case commodities would be perfect substitutes).

A simple case emerges when $a_{ii} = 0$ for any i, which implies $\bar{\epsilon}_i = 1/\left[2\,(1 - b_i)\right]$. This allows us to express Cournot prices as:

$$p_i = \frac{2c_i}{1 - 2b_i}, \tag{29}$$

[21] Notice that in the special, fully symmetric case with $a_{ij} = a > 0$ and $x_i = x$ for $i, j = 1, .., n$, one gets $\epsilon_{ij} = 1/2$.

and monopolistic competition prices as:

$$p_i = \frac{2(1-b_i)c_i}{1-2b_i}. \tag{30}$$

With these preferences markups do not vanish when market shares are negligible, but rather approach to twice the marginal cost: indeed we get $\widehat{p}_i \approx 2c_i$ when $b_i \approx 0$.

2.2 Entry

As discussed in Section 1.4, in general changes in market size, individual expenditure and productivity affect the set of active firms. However, under homotheticity the equilibrium variable profit (19) can be computed as:

$$\widehat{\pi}_i = \frac{\widehat{p}_i P_i(\widehat{\mathbf{p}}) EL}{\overline{\varepsilon}(\widehat{\mathbf{p}}) P(\widehat{\mathbf{p}})}, \tag{31}$$

where $\widehat{p}_i = \frac{\overline{\varepsilon}(\widehat{\mathbf{p}})c_i}{\overline{\varepsilon}(\widehat{\mathbf{p}})-1}$ for $i \in \widehat{\Gamma}$ (with infinite prices for $i \notin \widehat{\Gamma}$), and, one can verify that is independent from the productivity component A, and linear with respect to EL for a given set $\widehat{\Gamma}$. Thus changes in aggregate productivity do not affect $\widehat{\Gamma}$, while increases in market size and individual expenditures exert the same expansionary effect on it.

Applying (24) to (20) the social planner problem reduces to

$$AEL \max_{\Gamma \subseteq \Omega} \frac{1 - \sum_{i \in \Gamma} \frac{F_i}{EL}}{P\left(\underset{i\in\Gamma}{p_i} = \widetilde{c}_i, \underset{i\notin\Gamma}{p_i} = \infty\right)}. \tag{32}$$

While there is no guarantee that an equilibrium set of goods $\widehat{\Gamma}$ would be optimal, also an optimal set of goods Γ^* would be neutral in common productivity shocks, and would increase in market size and individual expenditures through their product. We summarize these facts as follows (see the proof in Appendix A):

Proposition 1. *When preferences are homothetic, the identity of the goods provided in a free entry equilibrium or in a social optimum does not depend on aggregate productivity, and is symmetrically affected by expenditure and market size.*

Of course these results hold for CES preferences (4), but both vanish for non-homothetic preferences, as we will discuss next.

3 GAS preferences

Although well-behaved demands can depend on prices in a general way, typical demand systems are simpler and depend on price aggregators or indices (as in the CES case), which allows us to study an alternative approach to monopolistic competition and to verify its consistency with our previous discussion. In this section we explore preferences that generate direct demand functions that depend on the own price and *one* common aggregator of all prices or, equivalently, inverse demand functions that depend on the own quantity and *one* common aggregator of all quantities. Pollak (1972) termed these as *Generalized Additively Separable* (GAS) preferences and showed that they encompass the following main classes: the directly additive (DA) and indirectly additive (IA) preferences (Houthakker, 1960), their extension to an unexplored class of preferences analyzed by Gorman (1970a, 1987) that we will call "Gorman-Pollak preferences", and the class of implicit CES preferences whose discussion will be postponed to Section 4 as part of a wider type, the implicitly additive preferences (Hanoch, 1975). We will show that under GAS preferences an equilibrium of monopolistic competition can be identically defined starting from either price or quantity competition and having firms to perceive as given the value of the common aggregator of individual behaviors. This approach is entirely consistent with that adopted by Dixit and Stiglitz (1977) who suggested to neglect the impact of an individual firm on marginal utility of income (the relevant aggregator in their setting), provided that this is sufficiently small to make this behaviour approximately "correct" (i.e., profit maximizing).

Pollak (1972) defined GAS preferences as those exhibiting demand functions that can be written as:

$$s_i = s_i(x_i, \xi(\mathbf{x})) \quad \text{and} \qquad x_i = x_i(s_i, \rho(\mathbf{s})), \tag{33}$$

where $\partial s_i/\partial x_i$, $\partial x_i/\partial p_i < 0$ and $\xi(\mathbf{x})$ and $\rho(\mathbf{s})$ are common functions ("aggregators") of respectively quantities and prices. Notice that we can write $s_i = x_i^{-1}(x_i; \xi(\mathbf{x}))$, so that $s_i(\cdot)$ is the partial inverse of $x_i(\cdot)$ with respect to its first argument, and $\xi(\mathbf{x}) = \rho(\mathbf{s}(\mathbf{x}))$.

GAS preferences provide an ideal setting to study monopolistic competition, since we can naturally define it as the environment in which each firm correctly anticipates the value of the aggregators ρ and ξ, but takes ("perceives") them as given while choosing its strategy to maximize profits:[22]

$$\pi_i = (s_i E - c_i)x_i(s_i, \rho)\, L = (s_i(x_i, \xi)E - c_i)x_i L. \tag{34}$$

It is important to stress that in this case the price and quantity equilibria of monopolistic competition do coincide. Since the "perceived" inverse demand of a commodity is just the inverse of the "perceived" direct demand, the corresponding elasticities ϵ_i and ε_i are simply related by the condition $\varepsilon_i = 1/\epsilon_i$ (as in a monopoly). Moreover, in Appendix B we prove that, provided that the market shares are negligible, to take the aggregator as given is approximately profit-maximizing for firms, since the perceived demand elasticity is approximately equal to the average Morishima measure. Formally, we have:

[22] Notice that the GAS preferences provide also the most general microfoundation for aggregative games of quantity and price competition (see e.g. Nocke and Schutz, 2018).

Proposition 2. *When preferences are of the GAS type and market shares become negligible, the perceived demand elasticity approximates the average Morishima elasticity.*

Accordingly, a monopolistic competition equilibrium where firms take aggregators as given approximates the imperfect competition equilibria of Section 1, which in this sense do "converge", when market shares become negligible.

The conditions for profit maximization of (34) taking as given either ρ or ξ define a system of pricing or production rules as:

$$p_i = \underline{p}_i(c_i, \rho) \quad \text{and} \quad x_i = \underline{x}_i(c_i, \xi). \tag{35}$$

These rules, together with the budget constraint $\sum_j p_j x_j = E$ and the assumption that firms correctly anticipate the actual demands, can be used to derive the equilibrium value of the aggregators as a function of the marginal cost vector $\mathbf{c}$ and of expenditure E, and therefore the equilibrium prices $\widehat{p}_i(\mathbf{c}, E)$ and quantities $\widehat{x}_i(\mathbf{c}, E)$.

If we now assume that firms decide on entry in the same spirit of their pricing/production decisions, namely by taking as given the relevant aggregator, we can make substantial progress in the analysis of monopolistic competiton. In particular, with additive preferences we can characterize which goods are going to be provided under free entry, and the selection effects associated to changes in market size (i.e., opening up to free trade), expenditure (i.e., a demand shock) and aggregate productivity (i.e., technological growth).

3.1 Directly Additive preferences

DA preferences can be represented by a direct utility that is additive as in:

$$U = \sum_{j=1}^{n} u_j(x_j), \tag{36}$$

where the sub-utility functions u_j are potentially all different but always increasing and concave. The inverse demand system is given by

$$s_i(x_i, \xi(\mathbf{x}) = \frac{u_i'(x_i)}{\xi(\mathbf{x})},$$

where $\xi = \widetilde{\mu} = \sum_j x_j u_j'$ and $x_i(s_i, \rho) = u_i'^{-1}(s_i \xi)$. These preferences clearly belong to the GAS type, and were originally used by Dixit and Stiglitz (1977) in the symmetric version with $u_j(x) = u(x)$ for all j.[23] We can express the profits of firm i as:

$$\pi_i = \left[\frac{u_i'(x_i)E}{\xi} - c_i \right] x_i L. \tag{37}$$

[23] For a further analysis of symmetric DA preferences see Zhelobodko *et al.* (2012), as well as Bertoletti and Epifani (2014) and Arkolakis *et al.* (2019) for applications to trade, and Cavallari and Etro (2017) for applications to macroeconomics.

The profit-maximizing condition with respect to x_i, taking ξ as given, can be rearranged in the pricing conditions:

$$p_i\left(x_i\right) = \frac{c_i}{1-\epsilon_i(x_i)}, \quad i = 1, 2, ..., n, \tag{38}$$

where $p_i\left(x_i\right) = u_i'(x_i)E/\xi$ and we define the elasticity of the marginal subutility $\epsilon_i(x) \equiv -xu_i''(x)/u_i'(x)$, which corresponds to the elasticity of the inverse demand $s_i(x, \xi)$ for given ξ. In this case ϵ_i is also the MEC ϵ_{ij} between good i and any other good $j \neq i$, therefore it coincides also with the average MEC $\overline{\epsilon}_i$ discussed in Section 1. In general, the markups can either increase or decrease in the consumption, depending on whether $\epsilon_i(x)$ is increasing or decreasing.

Given a set of active firms, a monopolistic competition equilibrium is a vector $(\mathbf{x}, \xi)$ that satisfies the $n+1$ equations $u_i'(x_i)E = \xi c_i\left[1-\epsilon_i(x_i)\right]$ for each $i = 1, .., n$ and $\xi = \sum_j x_j u_j'\left(x_j\right)$. Asymmetries of preferences and costs complicate its derivation because the quantity of each good depends on the quantities of all the other goods through the inverse demand system. However, under the assumptions that the profit-maximization problem is well defined for all firms (essentially, that all marginal revenues are positive but decreasing), in Appendix C we show that it must be unique. Formally, we have:

Lemma 1. *Assume that preferences are DA, that $r_i'\left(x\right) > 0 > r_i''\left(x\right)$, where $r_i(x) \equiv xu_i'\left(x\right)$, and that a solution to the profit maximization problem exists for a given set of active firms $i = 1, 2, ..., n$. Then the equilibrium is unique.*

We can easily study the comparative statics of this equilibrium. In particular, an increase in the expenditure level E increases all quantities, and raises the markup of firm i if and only if $\epsilon_i'(x) > 0$: this allows one to obtain different forms of "pricing to market" for different goods depending on their MEC functions. A rise of the marginal cost c_i decreases the quantity x_i, inducing an "incomplete pass-through" on the price of firm i if and only if its MEC is increasing. Also the indirect effect on the markups of the other firms (taking place through the change of the aggregator) depends on whether their MECs are increasing or decreasing. Finally, when a new good is introduced in the market through entry of an additional firm, the production of all other commodities decreases and therefore the markup of a firm decreases if and only if its MEC is increasing.

Let us now move to the question of which goods will be actually introduced: in Appendix C we show that both the free entry equilibrium and the optimal allocation of resources are unique, and we compare them. In particular, extending the approach of Spence (1976) we (inversely) rank the firms both according to their survival coefficient, $S_i \equiv \underset{x_i}{Min}\left\{\frac{c_i x_i + F_i/L}{x_i u_i'(x_i)}\right\}$, capturing their ability to survive in a free market, and according to their welfare coefficient, $W_i \equiv \underset{x_i}{Min}\left\{\frac{c_i x_i + F_i/L}{u_i(x_i)}\right\}$, capturing their incremental contribution to welfare, and we identify the active firms under equilibrium as well as under optimality. Notice that the survival coefficient is proportional to the (minimum) of the ratio between average cost and average revenue, while the welfare coefficient is proportional to the ratio of a commodity social cost and its incremental contribution to social welfare. We can think of these rankings as determining the actual sequences of introduction (respectively in a market equilibrium and in a social optimum), and they allow us to study how entry is affected by a change of market size, expenditure or aggregate productivity. The main result concerning the equilibrium of monopolistic competition with free entry is the following:

PROPOSITION 3. *When preferences are DA, the identity of the goods provided in the free entry equilibrium is uniquely determined, and an increase of the market size or a fall of productivity favor firms with the largest MECs, while a change of expenditure is neutral on the survival ranking.*

With the expression "to favor" we refer to improvements of the survival ranking (which apply to the marginal firm selected by the market to be active at the equilibrium). The intuition for the effects of market size and aggregate productivity is that firms with the largest unit profitability (determined by the value of their MEC), i.e., facing more rigid perceived demands, can make the best of an increase of market size and are less harmed by a common marginal cost increase. Instead, the neutrality of expenditure relies on the fact that its has a proportional impact on the profitability of all firms: therefore an expansion of demand attracts new firms in the market, but without altering their relative profitability.[24].

The welfare ranking differs in general from the equilibrium one, and depends crucially on the elasticity of the subutility $\eta_i(x) = u_i'(x)x/u_i(x)$, which measures how much of the utility provided by commodity i is captured by the revenue it generates (in utility terms). In Appendix C we prove the following:

PROPOSITION 4. *When preferences are DA, in the social optimum an increase of the market size or a productivity fall favor goods with the smallest elasticities of the subutility, while a change of expenditure is neutral on the welfare ranking.*

An increase of expenditure and/or market size and a rise of productivity relaxe the resource constraint of the social planner, allowing for the introduction of more goods and in larger quantities. But while a change of expenditure does not alter the welfare ranking of commodities (as in equilibrium), an increase of the market size or a fall of productivity change the relative incremental benefit of producing more of each of them, favoring those with the smallest elasticities η_i, where these are evaluated at the quantity which defines the welfare coefficient. The intuition is the following. When a new commodity is optimally introduced,

[24] This provides a rationale for the results on the selection effects of globalization derived by Zhelobodko *et al.* (2012) and Bertoletti and Epifani (2014) in a setting with symmetric goods and firms with heterogeneous marginal costs. They show that a market size increase has an impact on efficiency which depends on whether the MEC is increasing or decreasing. In fact, in that setting the firms facing the largest MECs are either the most or the least efficient firms according to the MEC being either decreasing or increasing in consumption.

it is priced proportionally to its marginal cost (see Section 1.4). Thus η_i is proportional to $c_i x_i/u_i$, implying that when market size increases or productivity falls the welfare ranking is rebalanced favoring those commodities with the smallest ratio between their variable cost and their incremental contribution to utility.

What can be said in general about the comparison of the free entry equilibrium to the social optimum? Under DA preferences, a possible approach is to compare the commodity rankings discussed above. Comparing W_i to S_i one can notice immediately that their ratio will tend to be small when η_i is small, making relatively more difficult the introduction of such a commodity in a market equilibrium. The reason is that η_i is proportional to the ratio between the revenue generated by commodity i and its contribution to the welfare, $p_i x_i/u_i$. In practice, a social planner may want to offer goods that are not profitable enough to be provided by the market, and the process of increase of the market size associated with globalization may also lead the market to introduce new goods whose demand is relatively rigid, while it would be optimal to introduce other goods whose incremental surplus is relatively larger. The equilibrium and optimal paths of product creation may go hand in hand, as in our CES example, but they may also diverge. We will now illustrate this with other examples.

Power sub-utility A simple case of DA preferences is based on the sub-utility power function:

$$u_i(x_i) = \tilde{q}_i x_i^{1-\epsilon_i}, \tag{39}$$

where both the MEC $\epsilon_i \in [0, 1)$ and the shift parameters $\tilde{q}_i > 0$ can differ among goods. These preferences are a special instance of the "direct addilog" preferences discussed by Houthakker (1960). Of course, they are CES as in (4) only when the exponents are identical, otherwise they are not even homothetic. As a natural generalization of the CES case they have been used in applications with perfect competition.[25] Under monopolistic competition, since the MECs are constant, markups are also constant and different across firms, and the equilibrium prices are:

$$\widehat{p}_i = \frac{c_i}{1-\epsilon_i}, \tag{40}$$

which show a full pass-through of changes in the marginal cost and independence from the pricing behavior of competitors and the expenditure level. Instead the equilibrium quantities $\widehat{x}_i$ depend on the equilibrium value $\widehat{\xi}$, but explicit solutions are not generally available. Finally, $\eta_i = 1 - \epsilon_i$ is a constant in this case, and we can compute the survival and welfare coefficients as:

$$S_i = \frac{W_i}{1-\epsilon_i} \quad \text{with} \quad W_i = \frac{c_i^{1-\epsilon_i} (F_i/L)^{\epsilon_i}}{\tilde{q}_i \epsilon_i^{\epsilon_i} (1-\epsilon_i)^{1-\epsilon_i}},$$

suggesting that the commodities that fare poorly in the market equilibrium compared to the optimum tend to be those with the largest MEC (which tend

[25] Dhrymes and Kurz (1964) is an early example of these functional forms as production technologies. More recently, Fieler (2011) has used them as utility functions in a trade model.

to be associated with a larger profit, but also with a larger welfare contribution). Interestingly, in this case the equilibrium selection effects provided by changes in market size or aggregate productivity point to the same direction suggested by the alleged change in the welfare ranking (of course the survival ranking coincides with the welfare ranking in the special CES case, as already noticed in Section 1.4).

Stone-Geary sub-utility Consider a simple version of the well-known Stone-Geary preferences (see Geary, 1950-51 and Stone, 1954) where:

$$u_i(x_i) = \log(x_i + \bar{x}_i), \tag{41}$$

with every $\bar{x}_i$ positive but small enough to insure a positive demand.[26] Solving for the elasticity of the perceived inverse demand we get $\epsilon_i(x) = x/(x + \bar{x}_i)$ and then the pricing condition:

$$p_i\left(x_i\right) = c_i\left(1 + \frac{x_i}{\bar{x}_i}\right).$$

The right-hand side is decreasing in $\bar{x}_i$ because a higher value of it increases demand elasticity. However, the equilibrium price of each firm cannot be derived independently from the behavior of competitors: the interdependence between firms created by demand conditions requires a fully-fledged equilibrium analysis. By the Hotelling-Wold identity we have:

$$s_i(x_i, \xi) = \frac{1}{(x_i + \bar{x}_i)\xi},$$

where $\xi = \sum_j x_j/\left(x_j + \bar{x}_j\right)$. Combining this with the pricing condition we can compute the quantity $x_i = \sqrt{\bar{x}_i E/(c_i\xi)} - \bar{x}_i$ and the (normalized) price rules $s_i = \sqrt{c_i/(\bar{x}_i E\xi)}$ for firm i. Defining $\Psi = \sum_{j=1}^n \sqrt{\bar{x}_j c_j}$ and using the adding up constraint we obtain the condition $\frac{n}{\xi} - \Psi/\sqrt{E\xi} = 1$, which can be solved for the equilibrium value:

$$\widehat{\xi} = \frac{\left[\sqrt{\Psi^2 + 4nE} - \Psi\right]^2}{4E}.$$

Replacing $\widehat{\xi}$ in the price rule we get the final closed-form solution for the monopolistic competition price of firm i:

$$\widehat{p}_i = \frac{2E\sqrt{\frac{c_i}{\bar{x}_i}}}{\sqrt{\Psi^2 + 4nE} - \Psi}. \tag{42}$$

In this example the price of each firm i increases less than proportionally in its marginal cost c_i and decreases in the preference parameter $\bar{x}_i$ (which reduces ϵ_i).

[26] Simonovska (2015) has recently used a symmetric version of these preferences to study monopolistic competition among heterogeneous firms.

Moreover, an increase in expenditure raises the markup of each good less than proportionally. Note that each price is increasing in Ψ, therefore an increase in the marginal cost c_j of a competitor or in the preference parameter $\bar{x}_j$ (which reduces the associated marginal utility) induce, albeit indirectly, a small increase in the markup of firm i. Finally, entry of new firms reduces all prices, and when it is due to an increase in the market size tends to favor the introduction of goods with a low $\bar{x}_j$, i.e., a more rigid demand.

Quadratic sub-utility Consider quadratic sub-utilities as in:

$$u_i(x_i) = \alpha_i x_i - \frac{\gamma_i}{2} x_i^2, \tag{43}$$

with $\alpha_i, \gamma_i > 0$. The associated preferences are a special instance of the quasi-homothetic preferences analyzed in Pollak (1971). Assuming that $s_i > 0$ for all i,[27] the perceived inverse demand elasticity is given by $\epsilon_i(x) = \gamma_i x/(\alpha_i - \gamma_i x)$, so that monopolistic competition pricing can be computed as follows:

$$p_i\left(x_i\right) = c_i \frac{\alpha_i - \gamma_i x_i}{\alpha_i - 2\gamma_i x_i}.$$

Since $s_i(x_i, \xi) = \left(\alpha_i - \gamma_i x_i\right)/\xi$ we obtain the production and pricing rules $x_i = \left(\alpha_i - c_i \xi/E\right)/2\gamma_i$ and $s_i = \left(\alpha_i + c_i \xi/E\right)/2\xi$. Using the adding up constraint allows us to solve for the equilibrium value of the quantity aggregator as:

$$\widehat{\xi} = 2\frac{\sqrt{\frac{\Upsilon\Phi}{4E^2} + 1} - 1}{\Upsilon},$$

where $\Upsilon = \sum_{j=1}^n \frac{c_j^2}{\gamma_j}$ and $\Phi = \sum_{j=1}^n \frac{\alpha_j^2}{\gamma_j}$, and eventually to obtain the equilibrium price:

$$\widehat{p}_i = \frac{c_i}{2} + \frac{\alpha_i \Upsilon}{4\left(\sqrt{E^2 + \frac{\Upsilon\Phi}{4}} - E\right)}. \tag{44}$$

The price of each firm i increases less than proportionally in its marginal cost c_i as well as in the intensity of preferences for the good α_i and in the expenditure level E. An increase in market size tend to favor the introduction of goods with small α_i and high γ_i, since their demands are more rigid.

3.2 Indirectly Additive preferences

IA preferences can be represented by an additive indirect utility as:

$$V = \sum_{j=1}^n v_j(s_j), \tag{45}$$

[27] This requires $\alpha_i > 2\gamma_i x_i$.

with sub-utilities v_j decreasing and convex (Houthakker, 1960). Elsewhere we have used the symmetric version of these preferences for the analysis of monopolistic competition.[28] The direct demand function is given by:

$$x_i(s_i, \rho(\mathbf{s})) = \frac{-v_i'(s_i)}{\rho(\mathbf{s})},$$

with $\rho = |\mu| = -\sum_{j=1}^n s_j v_j'$ and $s_i(x_i, \xi) = v_i'^{-1}(-x_i \rho)$, which confirms that these preferences belong to the GAS type. Here we can express the profits of firm i as:

$$\pi_i = \frac{(p_i - c_i)(-v_i'(p_i/E))}{\rho} L. \tag{46}$$

For a given value of the price aggregator ρ the elasticity of perceived demand $x_i(s_i, \rho)$ is given by $\varepsilon_i(s) = -sv_i''(s)/v_i'(s)$, which is also the MES ε_{ij} between goods i and j $(i \neq j)$ and thus coincides with the average MES $\bar{\varepsilon}_i$ of Section 1. The profit-maximizing price for each firm is then given by the solution to the price condition:

$$p_i = \frac{\varepsilon_i(p_i/E)c_i}{\varepsilon_i(p_i/E) - 1}, \quad i = 1, 2, ..., n \tag{47}$$

(which requires $\varepsilon_i > 1$). Under weak conditions a solution to (47) exists, and it is unique under the assumption that $2\varepsilon_i > -sv_i'''/v_i''$ for any i, which ensures that the second-order condition for profit maximization is satisfied.

Remarkably, each condition (47) is now sufficient to determine the monopolistic competition price of each firm in function of its own marginal cost and consumers' expenditure. This means that for the entire class of IA preferences each firm i can choose its price $\widehat{p}_i(c_i, E)$ independently from the behavior of competitors, as well as from their cost conditions or from parameters concerning their goods (e.g., from their "qualities"). An increase of expenditure increases the price of a good, and changes in its marginal cost are undershifted on the price if and only if the MES is increasing, which means that the demand is perceived as less elastic when expenditure is higher. Contrary to the DA case, the entry of a new firm does not change the prices of the pre-existing goods, but just reduces their production (through the increase of ρ).

All the equilibrium quantities (and the other firm-level variables, such as sales and profits) as well as welfare measures can then be recovered from the direct demand functions. For this reason, this class of preferences can be naturally employed in multicountry trade models, whereas the effects of differential trade costs, qualities and demand elasticities can be empirically assessed. A natural outcome of this model is that goods of higher quality or lower substitutability generate higher revenues in a given market and therefore are more likely to be exported to more distant countries. Similar *Alchian-Allen effects* ("shipping the good apples out") have been explored in recent works by Baldwin and Harrigan (2011), Crozet *et al.* (2012), Feenstra and Romalis (2014)

[28] See Nocke and Schutz (2018) for the extension to multiproduct firms under Bertrand competition, Bertoletti *et al.* (2018) and Macedoni and Weinberger (2018) for applications to trade and Boucekkine *et al.* (2017) and Anderson *et al.* (2018) for applications to macroeconomics.

and others, but always retaining a CES structure (4) that generates identical markups across goods. The IA specification allows us to move easily beyond the case of common markups, and to endogenize quality differentiation across firms and across destination within firms, whose empirical relevance has been pointed out in Manova and Zhang (2012).

In addition, under IA we can characterize the unique free entry equilibrium in terms of a survival ranking, which is simply given by the relative profitability, i.e., $\widetilde{S}_i = \underset{p_i}{Min}\left\{\frac{F_i}{v_i'(p_i/E)(c_i-p_i)}\right\}$. In Appendix D we prove:

PROPOSITION 5. *When preferences are IA, the identity of the goods provided in the free entry equilibrium is uniquely determined, and an increase of the expenditure level or a rise of productivity favors firms with the largest MESs, while an increase of market size is neutral on the survival ranking.*

The intuition is that an increase of expenditure (which exerts its impact through the demand size) or of aggregate productivity (which has a direct impact on unit profitability) affects proportionally more firms facing the most elastic demands, which make the best of them in terms of their survival ability, while an increase of market size has a proportional impact on all firms.[29]

The social planner problem (20) simplifies to $\max_{\Gamma\subseteq\Omega}\sum_{j\in\Gamma} v_j\left(\frac{c_j}{E-\sum_{i\in\Gamma}F_i/L}\right)$, but in this case it is not possible to establish a welfare ranking among commodities (essentially because the incremental welfare depends on the optimal value of the aggregator). Once again, there is no presumption that the market should provide the same goods provided by a social planner (except for the particular case of CES preferences).

Power sub-utility Consider a power sub-utility as:

$$v_i(s_i) = q_i s_i^{1-\varepsilon_i}, \tag{48}$$

where heterogeneity derives from the shift parameter $q_i > 0$ and the constant MES parameter $\varepsilon_i > 1$, implying that preferences are neither CES nor homothetic (unless $\varepsilon_j = \varepsilon$ for any j).[30] The pricing of firm i under monopolistic competition is immediately derived as:

$$\widehat{p}_i = \frac{\varepsilon_i c_i}{\varepsilon_i - 1}, \tag{49}$$

[29] This provides a rationale for results derived by Bertoletti and Etro (2017) in a setting with symmetric goods and firms with heterogeneous marginal costs. They show that an increase of expenditure has an impact on efficiency which depends on whether the MES is increasing or decreasing. In fact, the firms facing the largest MESs are in that setting either the least or the most efficient firms according to the MES being increasing or decreasing.

[30] This generalization of the CES case is a special instance of the "indirect addilog" preferences of Houthakker (1960) and differs from the one based on DA power sub-utilities presented above.

which implies again full pass-through of changes of the marginal cost. It is straightforward to derive the equilibrium quantity (for a given set of active firms):

$$\widehat{x}_i = \frac{q_i(\varepsilon_i - 1)^{\varepsilon_i+1}\left[\frac{E}{c_i\varepsilon_i}\right]^{\varepsilon_i}}{\sum_{j=1}^{n} q_j\left(\varepsilon_j - 1\right)^{\varepsilon_j}\left[\frac{E}{\varepsilon_j c_j}\right]^{\varepsilon_j - 1}},$$

and consequently sales and profits. Clearly, q_i is a shift parameter capturing the quality of good i, that leaves unchanged the price but increases profit by increasing sales. The relative productions, sales and profits of firms depend on the relative quality of their goods, on their cost efficiency and demand elasticity, and on the level of expenditure in simple ways that can be exploited in empirical work.

Translated power sub-utility Consider the following sub-utility:

$$v_i(s) = \frac{(a_i - s)^{1+\gamma_i}}{1+\gamma_i}, \tag{50}$$

with quality parameter $a_i > 0$ (such that $v_i(s) = 0$ if $s > a_i$) and $\gamma_i > 0$. It delivers simple perceived demand functions, including the case of a linear demand (for $\gamma_i = 1$) and the limit cases of a perfectly rigid demand ($\gamma_i \to 0$) and a perfectly elastic demand ($\gamma_i \to \infty$). These preferences have been recently applied by Bertoletti *et al.* (2018) and Macedoni and Weinberger (2018) to study the welfare impact of trade liberalization and quality regulation in multicountry models with heterogeneous firms. Since the MES for good i is $\varepsilon_i\left(s\right) = \gamma_i s/(a_i - s)$, the price of firm i can be computed as:

$$\widehat{p}_i = \frac{a_i E + \gamma_i c_i}{1+\gamma_i}, \tag{51}$$

which shows incomplete pass-through of marginal cost changes (parametrized by the firm-specific parameter γ_i) and markups increasing in the intensity of preference for each good (as captured by willingness-to pay parameter a_i) and in the expenditure level. A positive shock to the expenditure level tends to favor firms producing commodities whose a_i-parameters are relatively small while the γ_i- parameters are relative large (thus facing relatively more elastic demands).

3.3 Gorman-Pollak preferences

Building on Gorman (1970a) and Pollak (1972),[31] Gorman (1987) has characterized the main class of GAS preferences through the following extension of additivity. Suppose that preferences can be represented by the utility functions:

$$U\left(\mathbf{x}\right) = \sum_j u_j\left(\xi\left(\mathbf{x}\right)x_j\right) - \phi\left(\xi\left(\mathbf{x}\right)\right) \text{ and } V(\mathbf{s}) = \sum_j v_j(\rho\left(\mathbf{s}\right)s_j) - \theta\left(\rho\left(\mathbf{s}\right)\right), \tag{52}$$

[31] See Terence Gorman's collected works published in Blackorby and Shorrocks (1995).

where ξ can be seen as generating the benefit of increasing the *effective* quantity of good i to ξx_i at the utility cost $\phi(\xi)$, which is equivalent (in the dual representation of preferences) to the possibility of reducing the inconvenience of consumption $\theta(\rho)$ at the cost of increasing the *effective* price of good i to ρs_i. We assume that ξ and ρ satisfy the conditions:

$$\phi'(\xi) \equiv \sum_{j=1}^{n} u_j'(\xi x_j) x_j \quad \text{and} \quad \theta'(\rho) \equiv \sum_{j=1}^{n} v_j'(\rho s_j) s_j. \tag{53}$$

Their role is to cancel out any cross effect on utility, as in the case of additive preferences. The demand system can then be easily computed as:

$$s_i(\mathbf{x}) = \frac{u_i'(\xi x_i)}{\sum_{j=1}^{n} u_j'(\xi x_j) x_j} \quad \text{and} \quad x_i(\mathbf{s}) = \frac{v_i'(\rho s_i)}{\sum_{j=1}^{n} v_j'(\rho s_j) s_j}. \tag{54}$$

It can be shown that $\rho = \phi'(\xi)$ and $\xi = -\theta'(\rho)$, so that the utility-maximizing choices satisfy $\rho s_i = u_i'(\xi x_i)$ and $\xi x_i = -v_i'(\rho s_i)$, and that $\widetilde{\mu} = \rho\xi$. We label these as "Gorman-Pollak" preferences and propose them for an application to monopolistic competition with asymmetric goods.[32]

It follows from (54) that, for given value of the aggregators, the properties of the perceived demand functions depend on the functional forms of u_i and v_i. Preferences are indeed CES when the subutilities u_i and v_i have a common power expression. They are *homothetic* when $\phi(\xi) = \ln \xi$ and $\theta(\rho) = -\ln \rho$,[33] a case which covers the GAS demand systems investigated in Matsuyama and Ushchev (2017). They are *directly additive* when $\theta(\rho) = -\rho$ (so that $\xi = 1$). Finally, they are *indirectly additive* when $\phi(\xi) = \xi$ (so that $\rho = 1$). Obviously, the functional forms in (52) have to satisfy the usual regularity conditions (explored in Fally, 2018).

To study monopolistic competition, let us define the elasticities $\epsilon_i(z) \equiv -\frac{u_i''(z)z}{u_i'(z)}$ and $\varepsilon_i(z) \equiv -\frac{v_i''(z)z}{v_i'(z)}$. When firms maximize their profits:[34]

$$\pi_i = \left[\frac{u_i'(\xi x_i) E}{\phi'(\xi)} - c_i\right] x_i L = \frac{(s_i E - c_i) v_i'(\rho s_i)}{\theta'(\rho)} L$$

taking as given the aggregators, it is immediate to verify that the perceived demand elasticities of monopolistic competition are given by $\epsilon_i(\xi x_i)$ and $\varepsilon_i(\rho s_i)$, which imply the equilibrium pricing conditions:

$$p_i = \frac{c_i}{1 - \epsilon_i(\xi(\mathbf{x}) x_i)} = \frac{\varepsilon_i(\rho(\mathbf{s}) s_i) c_i}{\varepsilon_i(\rho(\mathbf{s}) s_i) - 1}. \tag{55}$$

[32] Gorman (1987) writes: "I have not seen this system tried, which is a pity, since it is easily understood, is related to a leading theoretical model, and would be very useful should it fit."

[33] In this case the conditions (53) define aggregators that are homogeneous of degree -1, therefore the demand ratios are homogeneous of degree 0 and preferences are homothetic.

[34] The second-order conditions for profit maximization can be easily derived. They are satisfied in the example below, and other examples can be easily built starting from the sub-utilities used for the additive classes.

In contrast to the case of additive preferences, within the Gorman-Pollak class the relevant demand elasticities *do* depend in general on the values of the aggregators, and *do not* directly correspond to the Morishima measures. Nevertheless Proposition 2 applies and the equilibrium (55) approximates the imperfect competition equilibria of Section 1 when market shares are negligible. We have employed symmetric Gorman-Pollak preferences to study monopolistic competition with free entry of heterogeneous firms in Bertoletti and Etro (2018): beside the particular cases where they are homothetic and/or additive, the Gorman-Pollak preferences do not exhibit neutralities of productivity, expenditure or market size on pricing and product selection and deliver inefficient entry. Moreover, even asymmetric Gorman-Pollak preferences retain a tractability that is well illustrated by next example.

Self-dual addilog preferences The family of "self-dual addilog" preferences introduced by Houthakker (1965) and investigated by Pollak (1972) belongs to the Gorman-Pollak class. For this family of preferences the direct demand system is given by:

$$x_i(\mathbf{s}) = q_i \frac{s_i^{-\varepsilon_i}}{\rho\left(\mathbf{s}\right)^{\varepsilon_i + \frac{\delta-1}{\delta}}}, \tag{56}$$

where $q_i > 0$ is a shift parameter reflecting the quality of good i, $\varepsilon_i > 1$ governs the perceived elasticity of demand and $\rho\left(\mathbf{s}\right)$ is implicitly defined by the condition $\sum_{i=1}^{n} q_i s_i^{1-\varepsilon_i} \rho^{\frac{1-\delta}{\delta}-\varepsilon_i} = 1$. We assume $\delta \in (0,1)$, and $\varepsilon_i \neq \varepsilon_j$ for some i and j (otherwise preferences are CES). Moreover, the inverse demand system is given by:

$$s_i(\mathbf{x}) = \widetilde{q}_i \frac{x_i^{-\epsilon_i}}{\xi\left(\mathbf{x}\right)^{\epsilon_i + \frac{\widetilde{\delta}-1}{\widetilde{\delta}}}}, \tag{57}$$

where $\xi\left(\mathbf{x}\right)$ is implicitly defined by the condition $\sum_{i=1}^{n} \widetilde{q}_i x_i^{1-\epsilon_i} \xi^{\frac{1-\widetilde{\delta}}{\widetilde{\delta}}-\epsilon_i} = 1$, with $\varepsilon_i = \frac{1}{\epsilon_i} > 0$, $q_i = \widetilde{q}_i^{\,\epsilon_i}$ and $\delta = 1 - \widetilde{\delta}$.

Pollak (1972) showed that the underlying preferences can be represented for $\delta \neq 1/2$ by:

$$U\left(\mathbf{x}\right) = \sum_{j=1}^{n} \frac{\widetilde{q}_j \left(x_j \xi\right)^{1-\epsilon_j}}{1-\epsilon_j} - \frac{\widetilde{\delta}\xi^{\frac{2\widetilde{\delta}-1}{\widetilde{\delta}}}}{2\widetilde{\delta}-1} \quad \text{and} \quad V(\mathbf{s}) = \sum_{j=1}^{n} \frac{q_j \left(s_j \rho\right)^{1-\varepsilon_j}}{\varepsilon_j - 1} + \frac{\delta \rho^{\frac{2\delta-1}{\delta}}}{2\delta - 1}.$$

with the aggregators defined above. In the special case with $\delta = 1/2$ preferences are homothetic and ϕ and θ take a logarithmic form with respect to the corresponding aggregators.

Given the inverse and direct demand systems, when firms maximize profits taking as given the aggregators, we immediately obtain the following prices under monopolistic competition:

$$\widehat{p}_i = \frac{c_i}{1-\epsilon_i} = \frac{\varepsilon_i c_i}{\varepsilon_i - 1}, \tag{58}$$

where the idiosyncratic markups are constant as in our additive, power subutility examples. In fact, we can also derive the equilibrium quantities as:

$$\widehat{x}_i = \frac{q_i(\varepsilon_i - 1)^{\varepsilon_i} E^{\varepsilon_i}}{c_i^{\varepsilon_i} \varepsilon_i^{\varepsilon_i} \rho\left(\widehat{\mathbf{s}}\right)^{\frac{\delta-1}{\delta}+\varepsilon_i}}.$$

These results make this family the natural extension of the power additive preferences. The availability of a homothetic version, with the associated well-defined price and consumption indexes, and the flexibility of the general specification provide interesting advantages for applications that depart from the CES paradigm.

4 Separable preferences and many aggregators

In this section we finally deal with the extension of the approach of Section 3 to preferences generating demand systems that depend on multiple common aggregators. We begin with the case where each demand depends on *two* common aggregators, and can be written as:

$$s_i = s_i\left(x_i, \xi\left(\mathbf{x}\right), \psi\left(\mathbf{x}\right)\right) \quad \text{and} \quad x_i = x_i\left(s_i, \rho\left(\mathbf{s}\right), \omega\left(\mathbf{s}\right)\right). \tag{59}$$

Assuming $\partial s_i/\partial x_i$ and $\partial x_i/\partial p_i < 0$, again we obtain $s_i = x_i^{-1}(x_i, \xi(\mathbf{x}), \psi\left(\mathbf{x}\right))$ where $\xi(\mathbf{x}) = \rho\left(\mathbf{s}\left(\mathbf{x}\right)\right)$ and $\psi\left(\mathbf{x}\right) = \omega\left(\mathbf{s}\left(\mathbf{x}\right)\right)$. We can then keep defining unambiguously monopolistic competition as the environment in which firms adopt their strategies anticipating the correct value of the aggregators but taking them as given. We consider two types of preferences which satisfy property (59), and we conclude by discussing the case of more than two aggregators.

4.1 Separable marginal utility preferences

Property (59) holds for any preferences for which the marginal utility of each good can be written in a separable fashion as:

$$U_i\left(\mathbf{x}\right) = f_i\left(x_i, \xi\left(\mathbf{x}\right)\right), \tag{60}$$

with $\partial f_i/\partial x_i < 0$. Preferences with such a separable marginal utility include the three classes of GAS preferences studied in Section 3 and others. The inverse demand is $s_i\left(\mathbf{x}\right) = \frac{f_i(x_i, \xi(\mathbf{x}))}{\psi(\mathbf{x})}$, where $\psi = \widetilde{\mu} = \sum_j x_j f_j$ is the second aggregator. The perceived inverse demand elasticity when both aggregators are taken as given is provided by the function:

$$\epsilon_i\left(x, \xi\right) = -\frac{x_i f_{ii}(x, \xi)}{f_i(x, \xi)} \tag{61}$$

which depends crucially on the value of the aggregator ξ that affects marginal utility.

To show how one can adapt our approach in this environment, let us consider the direct utility:

$$U(\mathbf{x}) = \sum_{j=1}^{n} u_j(x_j) - \frac{\eta}{2}\xi(\mathbf{x})^2, \tag{62}$$

where $\eta > 0$ and $\xi = \sum_{j=1}^{n} x_j$. One can recognize here the non-linear component of the utility specification used by Melitz and Ottaviano (2008) under a common and quadratic subutility for any good. This class of preferences satisfies the separability (60) with $f_i(x_i, \xi(\mathbf{x})) = u_i'(x_i) - \eta\xi(\mathbf{x})$. Within this class we have market shares $b_i = (u_i' - \eta\xi) x_i/\psi$, and the corresponding perceived, inverse demand elasticity is given by:

$$\epsilon_i(x, \xi) = \frac{-u_i''(x)\, x}{u_i'(x) - \eta\xi}, \tag{63}$$

which allows one to compute the pricing conditions. It is useful to sketch how in principle one can apply the solution procedure of Section 3 in the presence of two aggregators. Consider the quadratic subutility $u_i(x) = \alpha_i x - \frac{\gamma_i}{2}x^2$ where the parameters α_i and γ_i change across goods. The pricing condition becomes:

$$p_i(x_i) = c_i\left(\frac{\alpha_i - \gamma_i x_i - \eta\xi}{\alpha_i - 2\gamma_i x_i - \eta\xi}\right),$$

and by the Hotelling-Wold identity we have:

$$s_i(x_i, \xi, \psi) = \frac{\alpha_i - \gamma_i x_i - \eta\xi}{\psi},$$

where $\psi = \sum_j x_j\left(\alpha_j - \gamma_j x_j - \eta\xi\right)$. Thus we can compute the quantity rules $x_i = (\alpha_i - \eta\xi - \psi c_i/E)/2\gamma_i$ and the pricing rules $s_i = (\alpha_i - \eta\xi + c_i\psi/E)/2\psi$. Using the budget constraint one can solve for the marginal utility of income $\psi(\xi)$ in function of the other aggregator and for the quantity $x_i(\xi, \psi(\xi))$. Using the definition $\xi = \sum_{j=1}^{n} x_j(\xi, \psi(\xi))$ we can solve for $\widehat{\xi}(\mathbf{c}, E)$ and then $\psi(\widehat{\xi}(\mathbf{c}, E))$, and derive the prices:

$$\hat{p}_i = \frac{c_i}{2} + \frac{\left(\alpha_i - \eta\widehat{\xi}(\mathbf{c}, E)\right)E}{2\psi(\widehat{\xi}(\mathbf{c}, E))}, \tag{64}$$

which generalize those in (44).

In a similar way one can analyze preferences whose indirect utility function features a marginal disutility that can be written in the separable fashion $V_i(\mathbf{s}) = g_i(s_i, \rho(\mathbf{s}))$, with $\partial g_i/\partial s_i > 0$. The corresponding demand satisfies (59) with $\omega = |\mu| = -\sum_i g_i s_i$. Again, preferences of this kind do nest the three classes of GAS preferences of Section 3, but they also include others. Examples have been occasionally explored in the literature, though under symmetric conditions - as for the homothetic preferences proposed by Datta and Dixon (2001) and the generalized quadratic indirect utility we studied elsewhere.

4.2 Implicitly additive preferences

The other demand system satisfying (59) is delivered by the implicitly additive preferences of Hanoch (1975). Particular cases have been used by Kimball (1995) under homotheticity to study nominal price rigidities in macroeconomics, and by Feenstra and Romalis (2014) to study endogenous qualities in trade.[35]

Let us assume that preferences can be represented by a direct utility $U(\mathbf{x})$ which is implicitly defined by:

$$F(\mathbf{x}, U) = \sum_{j=1}^{n} F^j(x_j, U) \equiv 1, \tag{65}$$

where the "transformation function" F satisfies the relevant regularity conditions (F must be monotonic). Then preferences are *directly* implicitly additive, and of course direct additivity is just a special case of it. They are also homothetic if $F^i(x_i, U) = F^i(x_i/U)$ for any i. The marginal utility of commodity i is given by:

$$U_i(\mathbf{x}) = \frac{-F_i^i(x_i, U(\mathbf{x}))}{\varphi(\mathbf{x})},$$

where $\varphi = \sum_{j=1}^{n} F_U^j$, so that the marginal utility depends now on two aggregators, namely φ and the same utility function U.

The Hotelling-Wold identity for utility maximization allows us to solve for $s_i(\mathbf{x}) = \frac{F_i^i(x_i, \xi(\mathbf{x}))}{\psi(\mathbf{x})}$, $i = 1, .., n$, where $\xi = U$ and $\psi = \sum_j F_j^j x_j$ (notice that no aggregator corresponds to the marginal utility of income, and that φ does not appear in the expression of demand). The perceived inverse demand elasticity when both aggregators are taken as given is provided by the function:

$$\epsilon_i(x, U) = -\frac{x F_{ii}^i(x, U)}{F_i^i(x, U)}, \tag{66}$$

which depends on the consumption of the specific good and on the utility level. As Hanoch (1975) noted, the properties of the "substitution function" $x_i F_{ii}^i / F_i^i$ completely determine the substitutability of good i along an indifference curve, and in our monopolistic competition setting this function determines markups.

In Appendix E we show that the result obtained for GAS preferences in Proposition 2 extends to preferences satisfying implicit additivity:

PROPOSITION 6. *When preferences are implicitly additive and the market shares become negligible, the perceived demand elasticity approximates the average Morishima elasticity.*

[35] Feenstra and Romalis (2014) actually use an expenditure function with an "implicitly additive" functional form. However, by a result of Blackorby *et al.* (1978: Theorem 4.10, p. 149) this is equivalent to direct implicit additivity of preferences under some regularity conditions. Also see the application by Matsuyama (2017) to study the impact of non-homotheticity on structural change.

Accordingly, also in this case the monopolistic competition equilibrium where firms take aggregators as given approximates the imperfect competition equilibria of Section 1, which thus do converge.

A similar analysis can be employed to study *indirectly* implicitly additive preferences. These can be represented by an indirect utility $V(\mathbf{s})$ that is implicitly defined by $G = \sum_{j=1}^{n} G^j(s_j, V) \equiv 1$, where the function G has to satisfy some regularity conditions. In this environment the marginal disutilities depend on two aggregators, one of which is the utility again, which affects the perceived direct demand elasticity. In general, preferences whose indirect utility is implicitly additive differ from those with a direct utility that is implicitly additive, but both properties are satisfied by the class of "implicit CES" preferences that we discuss below.

4.3 Implicit CES preferences

Consider the class of implicitly additive preferences with direct utility defined by (65) with:

$$F^i(x_i, U) = \tilde{q}_i(U)x_i^{1-\epsilon(U)}, \tag{67}$$

where $\epsilon(U)$ and $\tilde{q}_j(U)$ are constant for a given utility level but can change across indifference curves. Note that all F^i are homogeneous of the same degree $1-\epsilon$ with respect to x_j: then $x_i F_i^i = (1-\epsilon) F^i$ and $\psi = 1-\epsilon$. Thus, by the Hotelling-Wald identity, the inverse demand of commodity i is given by:

$$s_i(\mathbf{x}) = \tilde{q}_i(U(\mathbf{x}))x_i^{-\epsilon(U(\mathbf{x}))},$$

which depends on one aggregator only. This is also the last class of preferences that belongs to the GAS type (Gorman, 1970a,b). Equivalently one can represent the same preference class by an indirect implicitly additive utility function with $G^i(s_i, V) = q_i(V)s_i^{1-\varepsilon(V)}$.

It is easy to verify that monopolistic competition prices satisfy:

$$p_i = \frac{c_i}{1-\epsilon(U)} = \frac{\varepsilon(V)\, c_i}{\varepsilon(V) - 1}, \tag{68}$$

which shows that markups are identical across firms in spite of the differences among goods. Nevertheless, markups can vary with changes in expenditure or costs through their impact on the equilibrium utility.[36]

We have employed a symmetric version of these preferences to study monopolistic competition with free entry of heterogeneous firms in Bertoletti and Etro (2018), showing that they generate an optimal equilibrium. Intuitively,

[36] By using (67) the equilibrium utility must satisfy:

$$\sum_{j=1}^{n} \left\{ \frac{q_j(U)^{\frac{1}{\epsilon(U)-1}} c_j}{[1-\epsilon(U)]\, E} \right\}^{1-\frac{1}{\epsilon(U)}} = 1.$$

the property that markups are common across goods implies that monopolistic competition does not bias the equilibrium consumption distribution with respect to the optimal one. In fact, also with asymmetric implicit CES preferences the social planner allocation turns out to be an equilibrium allocation when the industry profits are redistributed to the consumers, which extends one of the properties of the explicit CES preferences. All these peculiarities make this class of preferences particularly attractive for applications.

4.4 Other separable preferences

There are other preferences generating demand systems that satisfy (59) without fitting in the earlier types. To exemplify this possibility, in Appendix D we build an example, based on the so-called *Almost Ideal Demand System* (AIDS) introduced by Deaton and Muellbauer (1980), which nests a translog specification of Matsuyama and Ushchev (2017). Monopolistic competition under the restricted AIDS preferences delivers demand elasticities which grow unboundedly when market share becomes negligible, providing marginal cost pricing in the limit.

We conclude by noting that in principle the approach to monopolistic competition that we have explored when preferences are separable can be extended to other cases in which each demand function depends on *more than two* aggregators. In fact, the associated procedure to determine the equilibrium can be applied to any system of well defined "perceived" demands as soon as the alleged behavioral rules (based on the perceived demand elasticities) are consistent with the demand system, so that firms can be seen as correctly anticipating the actual demands. However, to argue that taking all aggregators as given is approximately profit-maximizing for firms, one has to verify that when the market shares become negligible the perceived demand elasticity does converge to the relevant Morishima measure (this basically requires that the impact of each firm on the aggregators vanishes).

5 General equilibrium applications

Our analysis of monopolistic competition can shed some light on issues that cannot be approached on the basis of the classic Dixit-Stiglitz model with CES preferences, as endogenous quality differentiation in trade and the impact of markup variability on the business cycle. To exemplify, in this section we move to a continuum of goods in line with the applied literature and focus on the case of implicit CES preferences, which provide a new and rich generalization of the explicit CES preferences preserving their simplicity. We sketch two applications focused respectively on price differentiation across firms and over time, and we then discuss how our preliminary results should extend qualitatively to more general preferences.

5.1 Endogenous quality differentiation and trade

As a first application, we investigate the impact of globalization when goods have different and endogenous qualities. In the basic Melitz (2003) model with heterogeneous firms, opening up to costless trade generates gains from variety without any selection effects (which, as well known, would emerge if there was a change of trade costs). We show below that when active firms invest in quality differentiation a welfare-increasing globalization can directly reduce markups, affect the distribution of qualities across firms and generate selection effects.

Suppose that preferences are implicitly defined by:

$$V \equiv \int_{\Omega} q(\omega)s(\omega)^{1-\varepsilon(V)}d\omega, \tag{69}$$

where Ω is the set of consumed goods, $q(\omega)$ can be interpreted as the quality of commodity ω and $\varepsilon(V) > 1$ can vary with utility (unless preferences are explicit CES). To fix ideas we will endorse the assumption $\varepsilon'(V) \geq 0$, so that goods can become more substitutable when utility increases. By Roy' identity, individual demand for commodity ω is given by

$$x(\omega) = \frac{q(\omega)}{V}s(\omega)^{-\varepsilon(V)},$$

where utility V is the relevant aggregator.

As usual, labor is the only input and the wage is normalized at unity. Assume that, after paying an entry cost F_e, firms draw a cost parameter z from a distribution $G(z)$ on $[0, \infty)$ and pay a common fixed cost F to produce at a marginal cost $c(q, z)$, which is increasing and convex in quality q and increasing in z. A z-firm supplying quality q at price p faces variable profits:

$$\pi = \frac{[p - c(q, z)]q}{V}\left(\frac{p}{E}\right)^{-\varepsilon(V)}L,$$

where L is the population of identical consumers with expenditure/labour endowment E. Its optimal price $p(z)$ and quality $q(z)$ can be shown to satisfy:

$$p(z) = \frac{\varepsilon(V)c(q(z), z)}{\varepsilon(V) - 1} \quad \text{and} \quad \frac{\partial \ln c(q(z), z)}{\partial \ln q} = \frac{1}{\varepsilon(V) - 1}, \tag{70}$$

which implies a common markup but possibly heterogeneous qualities across firms. The model is consistent with either a positive or negative correlation between quality and cost efficiency, as well as with a common quality in the special case of a marginal cost "multiplicative" with respect to z. Moreover, $\varepsilon'(V) > 0$ implies that the markups decrease with the utility level, and that product quality increases (decreases) with the utility level if the elasticity of marginal cost to quality is decreasing (increasing) with respect to z.

The monotonicity of profits in z allows us to determine the cut-off $\hat{z}$ such that $\pi(\hat{z}) = F$ and the measure $n = NG(\hat{z})$ of active firms out of the mass N

of firms created. Free entry *ex ante* requires $\int_0^{\hat{z}}[\pi(z) - F]dG(z) = F_e$, and by using the budget constraint and the pricing rule we can compute that:

$$N = \frac{EL}{\varepsilon(V)[F_e + FG(\hat{z})]}.$$

The model is closed replacing price and quality rules in (69) to obtain an implicit value for the equilibrium utility V.

We can now investigate the effects of opening up to costless trade (namely an increase in L). It is standard to verify that with explicit CES preferences ($\varepsilon'(V) = 0$) the measure n of consumed varieties increases, but there are no selection effects: the associated increase in utility does not affect $\hat{z}$ and the quality distribution of goods. Consider now the case where $\varepsilon'(V) > 0$ and the marginal cost is multiplicative in z: then an increase in utility is associated with a markup reduction and a selection of the more efficient firms ($\hat{z}$ decreases), but all firms produce the same quality. Finally, with a more general cost function, opening up to trade can generate selection effects that change also the distribution of qualities across active firms. Similar qualitative results can be obtained with other classes of the separable preferences analyzed in this paper (but, interestingly, the selection effects of market size would vanish under indirect additivity, as shown in Bertoletti and Etro, 2017). Multicountry applications could reproduce the quality differentiation among firms and across destinations that seems to emerge empirically (Manova and Zhang, 2012) and could be used to investigate the alledged gains from trade.

5.2 Markup variability and business cycles propagation

As a second application, we introduce monopolistic competition with implicit CES preferences in a standard flexible price dynamic model (see for instance Woodford, 2003, or Barro and Sala-i-Martin, 2004). In line with the macroeconomic literature we assume an exogenous measure of goods normalized to unity and a single representative agent living forever with utility:

$$\mathbb{U} = \mathbb{E}\left[\sum_{t=1}^{\infty} \beta^{t-1} \log C_t\right] \quad \text{with } C_t \equiv \left[\int_0^1 x_t(\omega)^{1-\epsilon(C_t)} d\omega\right]^{\frac{1}{1-\epsilon(C_t)}}, \tag{71}$$

where $\beta \in (0, 1)$ is the discount factor, $x_t(\omega)$ is the consumption of good ω at time t and C_t is aggregate consumption (intratemporal utility) at t. A perfectly competitive sector employs capital K_t and exogenous labor L to produce intermediate goods according to a neoclassical production function $Y_t = F(K_t, L)$ subject to productivity shocks. The intermediate good is used to produce final goods with a one-to-one technology by monopolistically competitive firms and is the *numeraire*. In each period, the expenditure on each variety $x_t(\omega)$ and on the future stock of capital K_{t+1} have to maximize utility under a standard resource constraint:

$$K_{t+1} = K_t(1 - \delta) + Y_t + \Pi_t - \int_0^1 p_t(\omega)x_t(\omega)d\omega,$$

where $\delta \in [0, 1]$ is the depreciation rate, and total profits $\Pi_t = \int_0^1 [p_t(\omega) - 1] x_t(\omega) d\omega$ and all prices $p_t(\omega)$ are taken as given. Optimality implies $p_t(\omega) = x_t(\omega)^{-\epsilon(C_t)} / \lambda_t C_t^{1-\epsilon(C_t)}$ and the Euler condition $\lambda_t = \beta \mathbb{E}[R_{t+1} \lambda_{t+1}]$, where the Lagrange multiplier λ_t corresponds to the marginal utility of income at time t and the gross return to capital is as usual given by $R_{t+1} = 1 + F_K(K_{t+1}, L) - \delta$.

Under monopolistic competition, all firms set the same price so that $C_t = x_t(\omega)$ and:

$$p(C_t) = \frac{1}{1 - \epsilon(C_t)}. \tag{72}$$

The markup is common but counter- or pro-cyclical according to the sign of $\epsilon'(C)$. The Euler condition can be rewritten as:

$$\frac{1}{C_t} = \beta \mathbb{E} \left[\frac{R_{t+1}}{C_{t+1}} \frac{p(C_t)}{p(C_{t+1})} \right], \tag{73}$$

which implies a unitary intertemporal elasticity of substitution but corresponds to the canonical version only under explicit CES preferences. Otherwise markup variability affects consumption choices and the consequent propagation of the business cycle. In particular, under the assumption $\epsilon'(C) < 0$, a temporary increase of productivity generates a temporary reduction in markups which promotes current consumption and magnifies the shock propagation. Accordingly, through the impact on relative prices over time, the demand side plays a new role in the business cycle generated by flexible price models under monopolistic competition.[37] The same qualitative results emerge with other non-homothetic preferences as long as the relevant elasticity, namely the (average) MEC, is decreasing in consumption. A quantitative analysis of similar mechanisms in a fully-fledged Real Business Cycle model with endogenous labor supply (which would be directly related to the real wage and therefore inversely to the markup) can be found in Cavallari and Etro (2017): their estimate of preference parameters delivers countercyclical markups under monopolistic competition, and allows the model to outperform the standard version with perfect competition in matching moments of the aggregate variables. Further applications with nominal frictions would be natural.

6 Conclusion

We have analyzed imperfect and monopolistic competition when consumers have asymmetric preferences over many differentiated commodities and firms are heterogeneous in costs. Defining monopolistic competition as the market structure which arises when market shares are negligible, we have been able to obtain a well-defined and workable characterization of monopolistic competition pricing.

[37] Notice that the equilibrium is inefficient due to markup variability over periods (except for the explicit CES case). Endogenous entry can be added to enrich propagation mechanisms and distortions (see Etro, 2018).

Moreover, we have presented a simple and consistent approach to the functioning of a market with monopolistic competition when demand functions depend on common aggregators.

While asymmetric CES aggregators preserve the optimality of the entry process and the neutrality of productivity, expenditure and market size on it, under more general preferences the goods introduced in an equilibrium can differ from those provided in the social optimum, and productivity growth and changes in expenditure and market size affect the identity of goods selected by the market. As we have argued, our approach can be usefully employed in trade and macroeconomic applications. Most of the recent research on heterogeneous firms is actually based on symmetric preferences (Melitz, 2003; Melitz and Ottaviano, 2008; Dhingra and Morrow, 2019; Arkolakis *et al.*, 2019), which is hardly realistic, especially to analyze welfare. Also the macroeconomic applications of monopolistic competition have usually focused on symmetric homothetic aggregators (Kimball, 1995; Woodford, 2003; Bilbiie *et al.*, 2012). Departing from symmetry and homotheticity allows to examine markup variability among goods and over time and its influence on welfare along the business cycle and across countries in more realistic ways.

References

Anderson, Eric, Sergio Rebelo and Arlene Wong, 2018, Markups Across Space and Time, NBER WP No. 24434

Arkolakis, Costas, Arnaud Costinot, Dave Donaldson and Andrés Rodríguez-Clare, 2019, The Elusive Pro-Competitive Effects of Trade, *Review of Economic Studies*, in press

Baldwin, Richard and James Harrigan, 2011, Zeros, Quality, and Space: Trade theory and trade evidence, *American Economic Journal: Microeconomics*, 3, 2, 60-88

Barro, Robert and Xavier Sala-i-Martin, 2004, *Economic Growth*, MIT Press

Benassy, Jean Paul, 1996, Taste for Variety and Optimum Production patterns in Monopolistic Competition, *Economics Letters*, 52, 1, 41-7

Bertoletti, Paolo and Paolo Epifani, 2014, Monopolistic Competition: CES Redux?, *Journal of International Economics*, 93, 2, 227-38

Bertoletti, Paolo and Federico Etro, 2016, Preferences, Entry and Market Structure, *RAND Journal of Economics*, 47, 4, 792-821

Bertoletti, Paolo and Federico Etro, 2017, Monopolistic Competition when Income Matters, *The Economic Journal*, 127, 603, 1217-43

Bertoletti, Paolo and Federico Etro, 2018, Monopolistic Competition with Gas Preferences, DEM Working Paper Series # 165, Department of Economics and Management, Pavia University

Bertoletti, Paolo, Federico Etro and Ina Simonovska, 2018, International Trade with Indirect Additivity, *American Economic Journal: Microeconomics*, 10, 2, 1-57

Bilbiie, Florin, Fabio Ghironi and Marc Melitz, 2012, Endogenous Entry, Product Variety, and Business Cycles, *Journal of Political Economy*, 120, 2, 304-45

Blackorby, Charles and R. Robert Russell, 1981, The Morishima Elasticity of Substitution: Symmetry, Constancy, Separability, and its Relationship to the Hicks and Allen Elasticities, *Review of Economic Studies*, 48, 147-58

Blackorby, Charles and Anthony F. Shorrocks, 1995, *Collected Works of W.M. Gorman: Separability and Aggregation*, Oxford: Oxford University Press

Blackorby, Charles, Daniel Primont and R. Robert Russell, 1978, *Duality, Separability, and Functional Structure: Theory and Economic Applications*, New York: Elsevier North-Holland

Blackorby, Charles, Daniel Primont and R. Robert Russell, 2007, The Morishima Gross Elasticity of Substitution, *Journal of Productivity Analysis*, 28, 203-8

Boucekkine, Raouf, Hélène Latzer and Mathieu Parenti, 2017, Variable Markups in the Long-Run: A Generalization of Preferences in Growth Models, *Journal of Mathematical Economics*, 68C, 80-6

Cavallari, Lilia and Federico Etro, 2017, Demand, Markups and the Business Cycle, WP 30/18, University of Florence

Chamberlin, Edward, 1933, *The Theory of Monopolistic Competition: A Reorientation of the Theory of Value*, Cambridge: Harvard University Press

Chamberlin, Edward, 1937, Monopolistic or Imperfect Competition?, *Quarterly Journal of Economics*, 51, 557-80

Christensen, Laurits, Dale Jorgenson and Lawrence Lau, 1975, Transcendental Logarithmic Utility Functions, *The American Economic Review*, 65, 3 367-83

Crozet, Matthieu, Keith Head and Thierry Mayer, 2012, Quality Sorting and Trade: Firm-level evidence for French wine, *Review of Economic Studies*, 79, 2, 609-44

d'Aspremont, Claude and Rodolphe Dos Santos Ferreira, 2016, Oligopolistic vs. Monopolistic Competition: Do intersectoral effects matter?, *Economic Theory*, 62, 1, 299-324

d'Aspremont, Claude and Rodolphe Dos Santos Ferreira, 2017, The Dixit–Stiglitz Economy with a 'Small Group' of Firms: A simple and robust equilibrium markup formula, *Research in Economics*, 71, 4, 729-39

Datta, Bipasa and Huw Dixon, 2001, ω-Homothetic Preferences: Theory and Applications, *Scottish Journal of Political Economy*, 48, no. 2, 148–63

Deaton, Angus, 1974, A Reconsideration of the Empirical Implications of Additive Preferences, *The Economic Journal*, 84, 338–48

Deaton, Angus and John Muellbauer, 1980, An Almost Ideal Demand System, *The American Economic Review*, 70, 312-26

Dhingra, Swathi and John Morrow, 2019, The Impact of Integration on Productivity and Welfare Distortions Under Monopolistic Competition, *Journal of Political Economy*, in press

Dhrymes, Phoebus and Mordecai Kurz, 1964, Technology and Scale in Electricity Generation, *Econometrica*, 32, 3, 287-315

Diewert, Walter E., 1971, An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function, *Journal of Political Economy*, 79, 3, 481-507

Dixit, Avinash and Joseph Stiglitz, 1977, Monopolistic Competition and Optimum Product Diversity, *The American Economic Review*, 67, 297-308

Dixit, Avinash and Joseph Stiglitz, 1993, Monopolistic Competition and Optimum Product Diversity: Reply, *The American Economic Review*, 83, 1, 302-4

Etro, Federico, 2018, Macroeconomics with Endogenous Markups and Optimal Taxation, *Southern Economic Journal*, 85, 2, 378-406

Fally, Thibault, 2018, Integrability and Generalized Separability, mimeo, University of California at Berkeley

Feenstra, Robert, 2003, A Homothetic Utility Function for Monopolistic Competition Models without Constant Price Elasticity, *Economics Letters*, 78, 1, 79-86

Feenstra, Robert, 2018, Restoring the Product Variety and Pro-Competitive Gains from Trade with Heterogeneous Firms and Bounded Productivity, Journal of International Economics, 110 (C), 16-27

Feenstra, Robert, and John Romalis, 2014, International Prices and Endogenous Quality, *Quarterly Journal of Economics*, 129, 2, 477-527

Fieler, Ana Cecilia, 2011, Non-homotheticity and Bilateral Trade: Evidence and a Quantitative Explanation, *Econometrica*, 79, 4, 1069-101

Geary, Roy, 1950-51, A Note on 'A Constant Utility Index of the Cost of Living', *Review of Economic Studies*, 18, 1, 65-6

Gorman, William Moore (Terence), 1970a, Conditions for Generalized Additive Separability, in Charles Blackorby and Anthony F. Shorrocks Eds., 1995, Vol. 1, 186-207

Gorman, William Moore (Terence), 1970b, Quasi-Separable Preferences, Costs, and Technologies, in Charles Blackorby and Anthony F. Shorrocks Eds., 1995, Vol. 1, 104-14

Gorman, William Moore (Terence), 1987, Separability, *The New Palgrave: A Dictionary of Economics*, 4, London: Macmillan Press, 305-11

Hanoch, Giona, 1975, Production and Demand Models with Direct or Indirect Implicit Additivity, *Econometrica*, 43, 3, 395-419

Hottman, Colin J., Stephen J. Redding and David E. Weinstein, 2016, Quantifying the Source of Firm Heterogenity, *Quarterly Journal of Economics*, 131, 3, 1291-364

Houthakker, Hendrik, 1960, Additive Preferences, *Econometrica*, 28, 2, 244-57

Houthakker, Hendrik, 1965, A Note on Self-dual Preferences, *Econometrica*, 33, 4, 797-801

Kimball, Miles, 1995, The Quantitative Analytics of the Basic Neomonetarist Model, *Journal of Money, Credit and Banking*, 27, 1241-77

Krugman, Paul, 1980, Scale Economies, Product Differentiation, and the Pattern of Trade, *The American Economic Review*, 70, 950-9

Macedoni, Luca and Ariel Weinberger, 2018, The Welfare Benefits of Raising your Standards. Unpublished paper, University of Oklahoma

Manova, Kalina and Zhiwei Zhang, 2012, Export Prices across Firms and Destinations, *Quarterly Journal of Economics*, 127, 1, 379-436

Matsuyama, Kiminori, 2017, Engel's Law in the Global Economy: Demand-Induced Patterns of Structural Change and Trade across Countries, mimeo, Northwestern University

Matsuyama, Kiminori and Philip Ushchev, 2017, Beyond CES: Three Alternative Classes of Flexible Homothetic Demand Systems, CEPR DP12210

Melitz, Marc, 2003, The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity, *Econometrica*, 71, 6, 1695-725

Melitz, Marc and Gianmarco Ottaviano, 2008, Market Size, Trade, and Productivity, *Review of Economic Studies*, 75, 1, 295-316

Morishima, Michio, 1967, A Few Suggestions on the Theory of Elasticity, *Keizai Hyoron*, 16, 144-50

Mrázová, Monika and Peter Neary, 2018, Selection Effects with Heterogeneous Firms, *Journal of the European Economic Association*, forthcoming

Nocke, Volker and Nicolas Schutz, 2018, Multiproduct-Firm Oligopoly: An Aggregative Games Approach, *Econometrica*, 86, 2, 523-557

Parenti, Mathieu, Philip Ushchev and Jean-Francois Thisse, 2017, Toward a Theory of Monopolistic Competition, *Journal of Economic Theory*, 167 (C), 86-115

Pascoa, Mario Rui, 1997, Monopolistic Competition and Non-Neighboring-Goods, *Economic Theory*, 9, 129-42

Pollak, Robert, 1971, Additive Utility Functions and Linear Engel Curves, *Review of Economic Studies*, 38, 4, 401-14

Pollak, Robert, 1972, Generalized Separability, *Econometrica*, 40, 3, 431-53

Romer, Paul, 1990, Endogenous Technological Change, *Journal of Political Economy*, 98, 5, S71-102

Simonovska, Ina, 2015, Income Differences and Prices of Tradables, *Review of Economic Studies*, 82, 4, 1612-56

Spence, Michael, 1976, Production Selection, Fixed Costs, and Monopolistic Competition, *Review of Economic Studies*, 43, 2, 217-35

Stone, Richard, 1954, Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand, *The Economic Journal*, 64, 255, 511-27

Vives, Xavier, 1987, Small Income Effects: A Marshallian theory of consumer surplus and downward sloping demand, *Review of Economic Studies*, 54, 1, 87-103

Vives, Xavier, 1999, *Oligopoly Pricing. Old Ideas and New Tools*, London: MIT Press

Yang, Xiaokai and Ben J. Heijdra, 1993, Monopolistic Competition and Optimum Product Diversity: Comment, *The American Economic Review*, 83, 1, 295-301

Woodford, Michael, 2003, *Interest and Prices*, Princeton University Press

Zhelobodko, Evgeny, Sergey Kokovin, Mathieu Parenti and Jacques-François Thisse, 2012, Monopolistic Competition in General Equilibrium: Beyond the CES, *Econometrica*, 80, 6, 2765-84

Appendix

A: Monopolistic competition with Homothetic preferences.

Proof of Proposition 1. Changes in aggregate productivity A cannot affect the profit of active firms due to the homogenenity of degree zero of both the average MES $\bar{\varepsilon}(\mathbf{p})$ and the market share $p_i P_i(\mathbf{p}) EL/P(\mathbf{p})$: thus they do not affect the set $\widehat{\Gamma}$ of firms that are active in a free entry equilibrium. Expression (31) and (32) immediately show that the same happens to Γ^*, and that only the product EL matters for both variable profits and social welfare. □

B: Monopolistic competition with GAS preferences.

Proof of Proposition 2. Assume that preferences belong to the GAS type. Taking as given the relevant aggregator, in a monopolistic competition equilibrium firms compute the perceived (inverse) demand elasticity according to:

$$\epsilon_i = -\frac{\partial \ln s_i(x_i, \xi)}{\partial \ln x_i}.$$

We now show that, when market shares are negligible, to take the aggregator ξ as given approximately coincides with using the average Morishima measures as the relevant demand elasticities, and is thus approximately profit maximizing. Let us start by computing the MEC between commodities i and j ($i \neq j$):

$$\begin{aligned} \epsilon_{ij} &= -\frac{\partial \ln \{s_i(\mathbf{x})/s_j(\mathbf{x})\}}{\partial \ln x_i} \\ &= \left[\frac{\partial \ln s_j(x_j, \xi(\mathbf{x}))}{\partial \ln \xi} - \frac{\partial \ln s_i(x_i, \xi(\mathbf{x}))}{\partial \ln \xi}\right] \frac{\partial \ln \xi(\mathbf{x})}{\partial \ln x_i} - \frac{\partial \ln s_i(x_i, \xi(\mathbf{x}))}{\partial \ln x_i}. \end{aligned}$$

This implies that the average MEC is:

$$\begin{aligned} \bar{\epsilon}_i &= \left[\sum_{j \neq i} \frac{\partial \ln s_j(x_j, \xi(\mathbf{x}))}{\partial \ln \xi} \frac{b_j(\mathbf{x})}{1 - b_i(\mathbf{x})} - \frac{\partial \ln s_i(x_i, \xi(\mathbf{x}))}{\partial \ln \xi}\right] \frac{\partial \ln \xi(\mathbf{x})}{\partial \ln x_i} \\ &\quad - \frac{\partial \ln s_i(x_i, \xi(\mathbf{x}))}{\partial \ln x_i}. \end{aligned}$$

By differentiating the identity $\sum_j s_i(x_j, \xi) x_j = 1$ we can also compute:

$$\frac{\partial \ln \xi(\mathbf{x})}{\partial \ln x_i} = -\frac{\frac{\partial \ln s_i(x_i, \xi(\mathbf{x}))}{\partial \ln x_i} + 1}{\sum_{j=1}^n \frac{\partial \ln s_j(x_j, \xi(\mathbf{x}))}{\partial \ln x_j}} \frac{b_i(\mathbf{x})}{\xi(\mathbf{x})^2}.$$

Accordingly we have $\bar{\epsilon}_i \approx \epsilon_i \approx \epsilon_{ij}$ when $b_i \approx 0$.[38] Notice that $\bar{\epsilon}_i = \epsilon_i = \epsilon_{ij}$ even when shares are *not* negligible if both preferences and the consumption bundle

[38] This formally assumes that not all the demand own elasticities *and* the quantity aggregator are too small.

(and then the price vector) are symmetric (as in Bertoletti and Etro, 2016).[39] Analogously, one can derive the MES and show that with GAS preferences small market shares imply $\bar{\varepsilon}_i \approx \varepsilon_i \approx \varepsilon_{ij}$ and thus $\bar{\varepsilon}_i \approx \bar{\epsilon}_i^{-1}$. Thus to take the aggregator ρ as given while choosing the own price is approximately correct when market shares are indeed negligible. □

C: Monopolistic competition with DA preferences.

Proof of Lemma 1. Assume that preferences are DA, that solutions to the maximization of profits (37) exist[40] and that $r_i'(x_i) > 0 > r_i''(x_i)$, where $r_i = x_i u_i'(x_i)$, for $i = 1, 2, ..., n$. It follows immediately from the first-order condition for profit maximization:

$$r_i'(x_i)\frac{E}{\xi} = c_i$$

that the monopolistically competitive quantity of any firm is increasing with respect to E/ξ. Moreover, it follows from (38) that:

$$p_i x_i = \frac{c_i r_i(x_i)}{r_i'(x_i)}.$$

Thus, the profit-maximizing revenue of a monopolistically competitive firm is increasing with respect to its equilibrium quantity. Since total revenue must be equal to the expenditure level E, it follows that there is a single value of ξ which characterizes an equilibrium for a given set of firms (and a vector of marginal cost $\mathbf{c}$).□

Proof of Proposition 3. Consider free entry under DA of preferences. By using (37) we can write the condition of a non-negative profit as:

$$\frac{E}{\xi} \geq \frac{c_i x_i + F_i/L}{x_i u_i'(x_i)}.$$

Following Spence (1976), let us rank firms increasingly according to their *survival coefficient* ($S_N \geq S_{N-1} \geq ... \geq S_1$):

$$S_i = \underset{x_i}{Min}\left\{\frac{c_i x_i + F_i/L}{x_i u_i'(x_i)}\right\}.$$

The equilibrium can be described as follows: for a given E/ξ, any active firm i maximizes profit by setting its Lerner index equal to the MEC ϵ_i, independently from F_i/L. This determines the whole set of quantities for the active firms, and

[39] Since ($h \neq i \neq j$)

$$\epsilon_{ij} - \epsilon_{ih} = \left[\frac{\partial \ln s_j(x_j, \xi(\mathbf{x}))}{\partial \ln \xi} - \frac{\partial \ln s_h(x_i, \xi(\mathbf{x}))}{\partial \ln \xi}\right]\frac{\partial \ln \xi(\mathbf{x})}{\partial \ln x_i},$$

from (16) cross demand effects are approximately zero when market shares are negligible, unless the own demand elasticities are indeed large.

[40] Sufficient but not necessary conditions are $\lim_{x\to\infty} u_i'(x) = 0$ and $\lim_{x\to 0} u_i'(x) = \infty$.

then the aggregator ξ. After any entry, the value of aggregator ξ must increase to reduce the expenditure in the incumbent commodities, making room for the entrant and survival more difficult for all firms. In an equilibrium, all active firms get non-negative profits, and their quantities are consistent with the value of the aggregator ξ. All the other firms would expect a negative profit if entering the market (taking ξ as given). It is then natural to characterize the unique free entry equilibrium by $S_n \geq E/\xi$, where only firms $i = 1, .., n$ are active.[41] Differentiating S_i and using the envelope theorem, we have:

$$\frac{\partial \ln S_i}{\partial \ln L} = -\epsilon_i (x_i), \quad \frac{\partial \ln S_i}{\partial \ln E} = 0 \quad \text{and} \quad \frac{\partial \ln S_i}{\partial \ln A} = \epsilon_i (x_i) - 1,$$

where ϵ_i is evaluated at the quantity $x_i = \frac{F_i}{c_i L} \frac{1-\epsilon_i(x_i)}{\epsilon_i(x_i)}$ which defines S_i. Accordingly, an increase of market size (which has a positive impact on all firms) or in a common component of the marginal cost (a reduction of productivity) alters the survival ranking favoring firms producing varieties with the largest MECs (thus facing steeper perceived demand functions), while expenditure is neutral. □

Proof of Proposition 4. Consider the optimal allocation. It is easy to verify that only goods with the smallest *welfare coefficient*:

$$W_i = \underset{x_i}{Min} \left\{ \frac{c_i x_i + F_i/L}{u_i (x_i)} \right\},$$

will be introduced. Suppose not: then, the same sub-utility level could be realized and some resources saved by introducing another commodity with a smaller coefficient. We can thus rank commodities according to their welfare coefficients. By direct computation we have the following derivatives in elasticity terms:

$$\frac{\partial \ln W_i}{\partial \ln L} = \eta_i (x_i) - 1, \quad \frac{\partial \ln W_i}{\partial \ln E} = 0 \quad \text{and} \quad \frac{\partial \ln W_i}{\partial \ln A} = -\eta_i (x_i),$$

where $\eta_i = d \ln u_i / d \ln x_i$ is evaluated at the quantity $x_i = \frac{F_i}{c_i L} \frac{\eta_i(x_i)}{1-\eta_i(x_i)}$ which defines W_i. Accordingly, an increase of market size or a reduction of productivity requires to increase proportionally more the welfare coefficient of commodities with the largest η_i, while expenditure is neutral. □

D: Monopolistic competition with IA preferences.

Proof of Proposition 5. The free entry equilibrium under indirectly additive preferences can be characterized starting from the non-negative profit condition:

$$\frac{L}{\rho} \geq \frac{F_i}{(c_i - p_i)\, v_i'(p_i/E)},$$

[41] Notice that the ranking is independent from the values of expenditure and ξ. Basically, it simply says that a firm ranked m cannot be active if a firm ranked $l \leq m$ is not. The identity of the marginal active firm depends on the contrary also on the expenditure level.

Let us define the *survival coefficient*:

$$\widetilde{S}_i = \underset{p_i}{Min}\left\{\frac{F_i}{(c_i - p_i)\, v_i'(s_i)}\right\},$$

which is proportional to the ratio of fixed cost and variable profit, and it has to be evaluated at the profit-maximizing value of p. Firms can then be ranked in terms of their survival coefficient: after any entry, the value of aggregator ρ must increase, to reduce demand and expenditure on the incumbent firms, making survival more difficult for all firms. A free entry equilibrium is determined by the unique value of ρ that makes all active firms to get non negative profits (and an aggregate revenue equal to EL), while all other firms expect a negative profit if entering the market (taking ρ as given). The market equilibrium then satisfies $\widetilde{S}_n \geq \frac{L}{\rho}$ for the set of active firms, $i = 1, .., n$. Since

$$\frac{\partial \ln \widetilde{S}_i}{\partial \ln L} = 0, \quad \frac{\partial \ln \widetilde{S}_i}{\partial \ln E} = -\varepsilon_i \quad \text{and} \quad \frac{\partial \ln \widetilde{S}_i}{\partial \ln A} = 1 - \varepsilon_i$$

an increase of expenditure or aggregate productivity unambigously favours firms with larger MESs, while an increase in market population is neutral on the ranking. □

E: Monopolistic competition with Implicitly Additive preferences.

Proof of Proposition 6. Let us consider the directly implicitly additive preferences defined by (65). Since $\ln(s_i/s_j) = \ln F_i^i(x_i, \xi) - \ln F_j^j(x_{j,}\xi)$, it follows that:

$$\begin{aligned} \epsilon_{ij}(\mathbf{x}) &= \left[\frac{F_{jU}^j(x_j, \xi(\mathbf{x}))}{F_j^j(x_j, \xi(\mathbf{x}))} - \frac{F_{iU}^i(x_i, \xi(\mathbf{x}))}{F_i^i(x_i, \xi(\mathbf{x}))}\right] U_i(\mathbf{x})x_i - \frac{F_{ii}^i(x_i, \xi(\mathbf{x}))x_{i,}}{F_i^i(x_i, \xi(\mathbf{x}))} \\ &= \left[\frac{F_{iU}^i(x_i, \xi(\mathbf{x}))}{F_i^i(x_i, \xi(\mathbf{x}))} - \frac{F_{jU}^j(x_j, \xi(\mathbf{x}))}{F_j^j(x_j, \xi(\mathbf{x}))}\right] \frac{\psi(\mathbf{x})b_i(\mathbf{x})}{\sum_j F_U^j(x_j, \xi(\mathbf{x}))} + \epsilon_i(\mathbf{x}) \end{aligned}$$

where we used the fact that $b_i = F_i^i(x_i, \xi)x_i/\psi$. This allows us to compute the average MEC as:

$$\bar{\epsilon}_i(\mathbf{x}) = \left[\frac{F_{iU}^i(x_i, \xi(\mathbf{x}))\psi(\mathbf{x})}{F_i^i(x_i, \xi(\mathbf{x}))} - \frac{\sum_{j\neq i} F_{jU}^j(x_j, \xi(\mathbf{x}))x_j}{1 - b_i(\mathbf{x})}\right] \frac{b_i(\mathbf{x})}{\sum_j F_U^j(x_j, \xi(\mathbf{x}))} + \epsilon_i(\mathbf{x}).$$

Therefore, $b_i \approx 0$ implies $\bar{\epsilon}_i \approx \epsilon_i \approx \epsilon_{ij}$, and taking the aggregates as given is approximately correct when the market shares are negligible. Notice that $\bar{\epsilon}_i = \epsilon_i = \epsilon_{ij}$ even when this is *not* the case if both preferences and the consumption bundle (and then the price vector) are symmetric.[42] Similar results can be shown when preferences are indirectly implicitly additive. □

[42] Since ($h \neq i \neq j$):

$$\epsilon_{ih}(\mathbf{x}) - \epsilon_{ij}(\mathbf{x}) = \left[\frac{F_{hU}^h(x_h, \xi(\mathbf{x}))}{F_h^i(x_h, \xi(\mathbf{x}))} - \frac{F_{jU}^j(x_j, \xi(\mathbf{x}))}{F_j^j(x_j, \xi(\mathbf{x}))}\right] \frac{\psi(\mathbf{x})b_i(\mathbf{x})}{\sum_j F_U^j(x_j, \xi(\mathbf{x}))}.$$

F. Monopolistic Competition with restricted AIDS preferences.

We present here an example of preferences that do not belong to the types analyzed in Section 4, but do satisfy the property (59) for which the demand system depends on two aggregators. The specification is based on the so-called Almost Ideal Demand System (AIDS) introduced by Deaton and Muellbauer (1980). Consider preferences represented by the following indirect utility:

$$V(\mathbf{s}) = -\frac{\rho(\mathbf{s})}{\zeta(\mathbf{s})},$$

where the aggregators ρ and ζ are defined by $(i, j = 1, .., n)$

$$\rho \equiv \alpha_0 + \sum_j \alpha_j \ln s_j + \frac{1}{2} \sum_j \sum_i \gamma_{ij} \ln s_j \ln s_i \quad \text{and} \quad \zeta \equiv \beta_o \prod\nolimits_j s_j^{\beta_j},$$

and assume $\sum_j \gamma_{ij} = \sum_j \beta_j = 0$ and $\sum_{j=1}^n \alpha_j = 1$ to satisfy the regularity conditions, and $\gamma_{ij} = \gamma_{ji}$ without loss of generality. For any commodity i the marginal disutility is:

$$V_i(\mathbf{s}) = -\frac{\alpha_i + \sum_j \gamma_{ij} \ln s_j - \beta_i \rho(\mathbf{s})}{s_i \zeta(\mathbf{s})},$$

and the direct demand functions can be derived by Roy's identity as:

$$x_i(\mathbf{s}) = \frac{\alpha_i + \sum_j \gamma_{ij} \ln s_j - \beta_i \rho(\mathbf{s})}{s_i}.$$

This demand system does not depend in general just on one or two common aggregators, thus we cannot use it to study monopolistic competition as an environment where firms set prices taking aggregators as given as in Section 4. However, consider the following additional restrictions that introduce some symmetry between goods:

$$\gamma_{ij} = \gamma\gamma_i\gamma_j \text{ for } i \neq j, \ \sum_{j=1}^n \gamma_j = 1, \, \gamma_{ii} = \gamma\gamma_i(\gamma_i - 1)$$

with $\gamma > 0$. In this case, the marginal disutility becomes:

$$V_i(\mathbf{s}) = -\frac{\alpha_i + \gamma\gamma_i \left[\omega(\mathbf{s}) - \ln s_i\right] - \beta_i \rho(\mathbf{s})}{s_i \zeta(\mathbf{s})},$$

which is separable in three aggregators, namely ρ, ζ and:

$$\omega = \sum_j \gamma_j \ln s_j.$$

The direct demand functions read as:

$$x_i(s_i, \rho(\mathbf{s}), \omega(\mathbf{s})) = \frac{\alpha_i + \gamma\gamma_i \left[\omega(\mathbf{s}) - \ln s_i\right] - \beta_i \rho(\mathbf{s})}{s_i},$$

where only two aggregators remain. Accordingly, this restricted AIDS specification satisfies (59). Notice that the aggregator ρ also disappears when $\beta_i = 0$ for any i, delivering the homothetic translog demand considered in Matsuyama and Ushchev (2017), which has the GAS property. Otherwise, perceived demands and demand elasticities depend on two aggregators. Taking both of them as given, this elasticity can be computed as:

$$\varepsilon_i = 1 + \frac{\gamma\gamma_i}{b_i},$$

and it grows unboundedly when the market share becomes negligible, implying $\widehat{p}_i \approx c_i$ when $b_i \approx 0$. Of course, this outcome is consistent with what one could obtain using the average Morishima elasticities to study strategic interactions as in Section 1.