Vertical Integration and Strategic Delegation

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Abstract

With backward acquisitions, downstream firms profitably internalize the effects of their actions on their rivals’ sales. With passive such acquisitions, upstream competition is also relaxed. Comparing the effects of downstream firms’ acquisition of pure vs. controlling cash flow rights in an efficient supplier when all firms compete in prices, downstream prices increase with passive acquisition, but decrease with controlling acquisition. Passive acquisition is profitable when controlling acquisition is not. Downstream acquirers strategically abstain from vertical control, thus delegating commitment to high prices to the supplier. The results are sustained when suppliers charge two part tariffs.

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1 Introduction

Partial ownership across horizontally and vertically related firms is very common, but has been of welfare concern or of concern in competition policy only if associated with control.\(^1\) Whereas the anti-competitive effect of horizontal cross-shareholding on prices is hardly controversial, the effect of vertical ownership arrangements on pricing and foreclosure is much more so.\(^2\)

Concentrating for the moment on full vertical mergers: By the classic Chicago challenge (Bork, 1978; Posner, 1976) these are competitively neutral at worst. Several arguments are around, however, of how vertical mergers can yield higher consumer prices, or even total foreclosure. The arguments rely on particular assumptions, such as additional commitment power of the integrated firm (Ordover et al., 1990), secret contract offers (Hart and Tirole, 1990), or costs of switching suppliers (Chen, 2001).\(^3\)

Throughout, these authors compare complete separation between the raider and the target firm to full joint ownership and control of the two. They do not consider partial ownership. Yet already hindsight suggests that empirically, partial vertical ownership between economically related firms is quite common relative to full ownership.\(^4\)

In this paper, we wish to analyze the incentives to backward integration and their effects on upstream and downstream prices. At the outset, it is important to appreciate that in contrast to under full integration, the direction of acquisition matters here. In addition, partial interests may have substantively different effects when passive vs. controlling. Concentrating on passive interests that we concentrate on here, passive forward ownership of an upstream supplier in one of its customers tends to induce vertical coordination, by reducing double marginalization and thus downstream prices.\(^5\) Obviously, this effect is consumer surplus increasing and pro-competitive. By sharp contrast, the results of this paper tell us that passive backward ownership tends to induce exactly the opposite effect, namely horizontal coordination, by exacerbating double marginalization and increasing downstream prices, which obviously are consumer surplus-reducing and anti-competitive. This is our answer to one of the questions addressed in this article: Is passive partial backward integration really as innocent as believed heretofore, with respect to anti-competitive effects such as increasing

\(^1\)In policy regimes scrutinizing minority ownership, the focus is usually on whether influence on the target’s strategy is feasible. For instance, the German competition law requires decisive influence for merger control to apply. The US has a safe harbor for acquisitions of 10% or less of the company’s share capital solely for purpose of investment. More recently, however, passive partial ownership – in particular in vertically related firms – figures more prominently in the recent European Commission Staff Working Document towards more effective EU merger control. See Commission (2013), Annex 1.

\(^2\)See Flath (1991), or more recently Brito et al. (2010) or Karle et al. (2011) for a theoretical analysis of the profitability of horizontal partial ownership, and Gilo (2000) for examples and an informal discussion of the antitrust effects.

\(^3\)Other specifics include input choice specifications (Choi and Yi, 2000), two-part tariffs (Sandonis and Fauli-Oller, 2006), exclusive dealing contracts (Chen and Riordan, 2007), only integrated upstream firms (Bourreau et al., 2011) and information leakages (Allain et al., 2010).

\(^4\)Allen and Phillips (2000), for instance, show that in the USA 53 percent of corporate block ownership involves firms in related industries.

\(^5\)This is shown in Flath (1989).
Towards that answer, we focus on ownership interests that firms may hold in their suppliers, and distinguish between passive and controlling ownership, where passive ownership involves pure cash flow rights, i.e. claims only on the target’s profits without controlling its decisions. Fixing first that distribution of ownership, we look at the unrestricted pricing decisions of firms in a horizontally differentiated downstream market, and of suppliers in an upstream product market homogeneous just for simplicity, where firms produce at differing levels of marginal costs. We concentrate on upstream competition that is effective in the sense that the difference in the marginal costs between the efficient supplier and its competitors restrict that supplier in its price setting. After discussing the pricing decisions of downstream and upstream firms, we look at the incentives to backward integration.

We borrow this interesting and, we feel, empirically very relevant set up from Chen (2001), with the difference that turns out to be essential, namely that we look at the incentives to, and the effects of passive partial, rather than, as Chen, controlling full backward integration. As we will see, this substantially changes the economics of vertical interaction between the firms. Most importantly, we show passive partial backward integration to be profitable when controlling full backward integration is not. All of this has rather clear policy consequences not considered heretofore.

At any rate, in our model, relative to vertical separation, any downstream firm’s passive participation in the profits of the efficient upstream supplier softens its reaction to a price increase by that supplier, by not as much increasing its downstream price. The reason is that it is reimbursed parts of the so increased upstream profits by its very participation in these profits. In turn, that supplier, by acting independently because backward integration is passive, profitably incorporates the acquirer’s softened reaction, by increasing the nominal price to that acquiring downstream firm beyond the price charged by the second efficient competitor. This way, upstream competition is relaxed by passive backward integration.

At the same time, that price increase is constrained by the second efficient upstream competitor’s minimal price offer: in order to continue serving the acquirer, the efficient supplier must charge an effective price to the acquirer that does not exceed that competitor’s marginal cost. Interestingly enough, that constraint on the efficient supplier’s pricing activity yields that the softened reaction to the supplier’s price (due to the downstream firm’s participation in the upstream firm’s profits), and in reaction the increase in that supplier’s price to the acquirer, perfectly compensate each other, so that with increasing participation in the upstream profits the downstream acquirer continues to procure at effectively the same competitive price.

As the downstream competitors are naturally served by the same efficient supplier, however, the acquirer participates in that supplier’s profits generated from selling to the downstream competitors. This generates a quasi-collusive effect, by which any acquirer incorporates the effect of its own actions on the downstream competitors’ sales, and this increasingly so with increasing passive participation in the efficient upstream supplier’s profits. That acquirer’s incentive to steal business from the downstream competitors thus diminishes, leading
to a price above that under vertical separation. Strategic complementarity in turn induces all downstream competitors to increase theirs.

We also show that as long as competition in both markets is intense but imperfect, the possibility to profitably raise downstream prices incentivizes downstream firms to acquire passive interests in the efficient upstream supplier. Yet, in contrast to what one might expect, passive partial backward acquisition by a downstream firm does not invite the input foreclosure of downstream competitors. Indeed, with equilibrium prices between the downstream sellers of substitutes increasing towards monopoly prices, the competitors tend to benefit from the acquiring firm’s decision inasmuch as the supplier does not absorb the rents so generated.

Towards a comparison of these effects with those arising under full backward integration where the acquirer controls the upstream pricing decisions, we then go on to show that backward integration does not lead to higher, but to lower downstream prices, nor is profitable. The latter result contrasts Chen’s central claim. The essential reason for this drastic difference in outcomes is that in contrast to passive backward integration, the efficient supplier loses under full integration the possibility to credibly commit to a high internal transfer price. That loss in commitment power would benefit consumers, but not the firms.

In all, partial backward integration without the transfer of control rights is effective in raising consumer prices when full integration is not, i.e. when the Chicago argument about the efficiency increasing effect of vertical mergers does hold. Furthermore, backward acquisition incentives are limited to below the level at which the downstream firm takes control over the upstream target’s pricing decisions. By contrast, if it did, the upstream firm would lose its power to commit to high transfer prices, which, as indicated, leads downstream prices to decrease. Hence, in the setting analyzed here, backward acquisitions have an anti-competitive effect only if they are passive.

One could discount these results, as others involving double marginalization effects, by the standard argument that these effects vanish when two part tariffs are allowed upstream. In a section devoted to the discussion and extension of our results, we therefore show all the effects to hold even when the upstream suppliers are allowed to charge two-part tariffs, that in concentrated markets tend to alleviate the double marginalization problem. This motivates our claim that the pricing consequences of passive backward integration should indeed be of concern to competition authorities.

Beyond a contribution to the policy debate on passive interests between related firms, we can generate a number of empirical predictions. A key first one is that even in competitive situations, passive backward acquisitions generically lead to increasing upstream and downstream prices; in particular increases in the acquirers’ input and output prices. The empirical literature that could relate to our results must necessarily be sparse. The reason is

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6At any rate, this tends to remain a purely theoretical argument. Within a very involved case study followed by a questionnaire survey on upstream relationships in the German automotive industry, one of the authors was unable to detect a single nonlinear pricing arrangement that would absorb that effect. There is simply no payment from downstream to upstream, as required in the two-part tariff literature to obtain the efficient outcome. By contrast, many contracts involve fixed payments from upstream to downstream that are akin to slotting allowances and require a very different explanation.
that upstream prices are typically not visible to the general public nor to the econometrician – much more so, however, to the firms in the industry.

There is one very interesting exception, however. Gans and Wolak (2012) report on the effects of passive backward integration of a large Australian electricity retailer into a baseload electricity generation plant. Employing very elaborate alternative methodologies for estimating the pricing effects of that acquisition, they identify a significant increase in wholesale electricity prices. This is exactly in line with our prediction.

A second prediction generated from our model is that with backward acquisition, the target’s valuation tends to decrease, and the valuation of the acquirer tends to increase (all net of the acquisition cost), whence the valuation of its competitors never decreases. Changes in the targets’ and the acquirer’s valuations are commonly observed in the empirical corporate finance literature. TBC

As to a brief review of the theoretical literature pertinent to our subject matter: Flath (1989) shows that with successive Cournot oligopolies, constant elasticity demand and symmetric passive ownership, the effects cancel out, so in his model, pure passive backward integration has no effect. Greenlee and Raskovich (2006) confirm this invariance result for equilibria involving an upstream monopoly and symmetric downstream firms under competition in quantity, and in price – yet under the assumption that downstream demands are linear, and the upstream monopolist is restricted to charge a uniform price to all customers. These invariance results would first suggest that there is no backward integration incentive; and second that there is no need for competition policy to address passive vertical ownership. By contrast, we show that the invariance property of downstream prices does not apply within a more general industry structure involving upstream Bertrand competition with asymmetric costs, with the corresponding prices set by these competitors.

In very interesting papers, Baumol and Ordover (1994), Spiegel (2013) and Gilo et al. (2014) look mainly at the effects of obtaining control over a bottleneck upstream monopolist via partial, as compared to full acquisition. By contrast, our emphasis is on the effects of passive partial acquisition into an efficient upstream competitor. More specifically, Baumol and Ordover (1994) and Gilo et al. (2014) discuss that incentives are naturally distorted when control is exercised over an economically related target with claims only to parts of its profits, as opposed to (implicitly assumed) full claims to those of the raider. In addition to considering controlling acquisitions, Spiegel (2013) also studies partial passive integration. His model differs in many respects. In particular, he excludes double marginalization effects that are in the focus of our arguments. Also, the downstream competitors are served by an upstream bottleneck monopolist, and may vertically differentiate their supply to an undifferentiated final custom by a probabilistic investment function. At any rate, within this very different model with, to some extent, complementary features, he shows that passive backward integration leads to less foreclosure than controlling integration. As in our model, controlling backward integration turns out to be unprofitable, we cannot directly compare this result to ours. But we also show foreclosure not to arise at all with passive backward

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7The demand system is specified, though, such that it does not satisfy standard assumptions.
integration.

Separating control from ownership in order to relax competition is the general theme in the literature on strategic delegation. While that term was coined by Fershtman et al. (1991), our result is most closely related to the earlier example provided by Bonanno and Vickers (1988), where manufacturers maintain profit claims in their retailers through two-part tariffs, but delegate the control over retail prices leading to softened downstream price competition. In the present case, strategic delegation involves backward oriented activities. The particular twist we add to that literature is that the very instrument firms use to acquire control is used here short of implementing it.

The competition dampening effect identified in the present paper relies on internalizing rivals’ sales through a common efficient supplier. This relates to Bernheim and Whinston (1985)’s common agency argument. Strategic complementarity is essential in the sense that rivals need to respond with price increases to the raider’s incentive to increase price. Indeed, acquiring passive vertical ownership is a fat cat strategy, in the terms coined by Fudenberg and Tirole (1984).

A different kind of explanation for backward integration without control is that transferring residual profit rights can mitigate agency problems, for example when firm specific investment or financing decisions are taken under incomplete information (Riordan, 1991; Dasgupta and Tao, 2000). Güth et al. (2007) analyze a model of vertical cross share holding to reduce informational asymmetries, and provide experimental evidence.8 Whereas such potentially desirable effects of partial vertical ownership should be taken into account within competition policy considerations, we abstract from them for expositional clarity.

The empirical literature that could relate to our results is necessarily sparse. The reason is that upstream prices are typically not visible to the general public nor to the econometrician – much more so, however, to the firms involved. For our case, there is one very interesting exception, however. Gans and Wolak (2012) report on the effects of passive backward integration of a large Australian electricity retailer into a baseload electricity generation plant. Employing very elaborate alternative methodologies, they identify a significant increase in wholesale electricity prices associated with this acquisition. This is exactly in line with our main hypotheses, namely that passive backward acquisition is profitable (by revealed preference), and that prices increase as a result (by observation).

The remainder of this article is structured as follows: We introduce the model in Section 2. In Section 3, we solve and characterize the 3rd stage downstream pricing subgame, for passive as well as controlling backward integration. In Section 4, we solve for, and characterize the equilibrium upstream prices arising in Stage 2. There we also derive the essential comparative statics with respect to the downstream firms’ backward interests. In Section 5, we analyze the profitability of partial acquisitions. Section 6 we show first unlike Chen’s claim, full vertical integration is generically unprofitable in the situation discussed here, and compare the

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8Höffler and Kranz (2011a,b) investigate how to restructure former integrated network monopolists. They find that passive ownership of the upstream bottleneck (legal unbundling) may be optimal in terms of downstream prices, upstream investment incentives and prevention of foreclosure. A key difference to our setting is that they keep upstream prices exogenous.
underlying economics with that involved in passive backward integration. In the Discussion
and Extension Section 7, we first look at the effects of bans on upstream price discrimination
common to many competition policy prescriptions. Second and third, we consider the effects
of relaxing structural assumptions: We replace sequential by simultaneous pricing decisions,
and then allow the upstream firms to charge observable two-part, rather than linear tariffs.
The results related to passive backward integration remain unchanged. Fourth, we touch at
the case in which upstream competition is ineffective, so that the efficient firm can exercise
complete monopoly power.\footnote{Last, we briefly compare the effects of passive partial backward
integration with those of passive partial horizontal integration. We conclude with Section 8.
All proofs are removed to an appendix.}

2 Model

Two symmetric downstream firms $i, i \in \{A, B\}$, competing in prices $p_i$, produce and sell
imperfect substitutes demanded in quantities $q_i(p_i, p_{-i})$, that satisfy

**Assumption 1.** $\infty > -\frac{\partial q_i(p_i, p_{-i})}{\partial p_i} > \frac{\partial q_i(p_i, p_{-i})}{\partial p_{-i}} > 0$ (product substitutability).

The production of one unit of downstream output requires one unit of a homogenous
input produced by two suppliers $j \in \{U, V\}$ with marginal costs $c^j$, who also compete in
prices. Assume that $c^U \equiv 0$ and $c^V \equiv c > 0$, so that firm $U$ is more efficient than firm $V$, and $c$ quantifies the difference in marginal costs between $U$ and its less efficient competitor.\footnote{The symmetry assumption downstream, and the restriction to two firms downstream and upstream,
respectively, are without loss of generality. One should be able to order the upstream firms by degree of
efficiency, however. Rather than from $V$, the downstream firms could procure from the world market at
marginal cost $c$.}

All other production costs are normalized to zero. Upstream suppliers are free to price
discriminate between the downstream firms.

Let $x^j_i$ denote the quantities firm $i$ buys from supplier $j$, and $w^j_i$ the associated linear
unit price charged to $i$ by supplier $j$.\footnote{We show in Subsection 7.3 that the results extend to observable two part tariffs.} Finally, let $\delta^j_i \in [0, 1)$ denote the ownership share
downstream firm $i$ acquires in upstream firm $j$. Information is assumed to be perfect.

The game has three stages:

1. Downstream firms $A$ and $B$ simultaneously acquire ownership shares $\delta^j_i$ of suppliers.

2. Suppliers simultaneously set sales prices $w^j_i$.

3. Downstream firms simultaneously buy input quantities $x^j_i$ from suppliers, produce quanti-
ties $q^j_i$, and sell them at prices $p_i$.

Underlying the sequencing is the natural assumption that ownership is less flexible than
prices are, and also observable by industry insiders. This is crucial, as in the following we

\footnote{In a companion paper (Hunold et al. (2014)), we consider ineffective competition and compare the effects
of passive and controlling partial backward and forward integration.}
employ subgame perfection to analyze how ownership affects prices. Yet the assumption that suppliers can commit to upstream prices before downstream prices are set is inessential here.

We use the term **partial ownership** for an ownership share strictly between zero and one. We call **passive** an ownership share that does not involve control over the target firm’s pricing strategy, and **controlling** one that does. The possibility to control the target’s instruments is treated as independent of the ownership share in the target. With this we want to avoid the discussion of at which level of shareholdings control arises. That depends on institutional detail and the distribution of ownership share holdings in the target firm. Although a restriction of ownership shares to below \( \frac{1}{2} \) appears highly plausible for ownership to be passive, our results on passive ownership hold for any partial ownership share. In particular, passive acquisition could involve non-voting shares, and control could be exercised with a few voting shares. Unless indicated otherwise, we assume that acquisitions are passive.

Upstream supplier \( j \)’s profit is given by

\[
\pi^j = \sum_{i \in \{A,B\}} \left( w^j_i - c^j \right) x^j_i. \tag{1}
\]

Downstream firm \( i \)’s profit, including the return from the shares held in upstream firms,

\[
\Pi_i = p_i q_i(p_i, p_{-i}) - \sum_{j \in \{U,V\}} w^j_i x^j_i + \sum_{j \in \{U,V\}} \delta^j_i \pi^j, \tag{2}
\]

is to be maximized with respect to its own price \( p_i \), subject to the constraint \( \sum_j x^j_i \geq q_i \), so that input purchases are sufficient to satisfy quantity demanded.

We define an allocation to involve effective (upstream) competition, if the efficient upstream firm is constrained in its pricing decision by its upstream competitor, i.e. can charge effective unit input prices, as perceived by the typical downstream firm, no higher than \( c \), if it wants to serve that firm’s input demand. Unless indicated otherwise, we consider upstream competition to be effective.

An equilibrium in the third, downstream pricing stage is defined by downstream prices \( p^*_A \) and \( p^*_B \) as functions of the upstream prices \( w^j_i \) and ownership shares \( \delta^j_i \), \( i \in \{A, B\}; j \in \{U, V\} \) held by the downstream in the upstream firms, subject to the condition that upstream supply satisfies downstream equilibrium quantities demanded. In order to characterize that equilibrium, it is helpful to impose the following conditions on the profit functions:

**Assumption 2.**

\[
\frac{\partial^2 \Pi_i(p_i, p_{-i})}{\partial p_i^2} < 0 \quad \text{(concavity)}
\]

**Assumption 3.**

\[
\frac{\partial^2 \Pi_i(p_i, p_{-i})}{\partial p_i \partial p_{-i}} > 0 \quad \text{(strategic complementarity)}
\]

**Assumption 4.**

\[
\frac{\partial^2 \Pi_i(p_i, p_{-i})}{\partial p_i \partial p_{-i}} \Big/ \frac{\partial^2 \Pi_{i-1}(p_{-i}, p_i)}{\partial p_{-i} \partial p_i} > \frac{\partial^2 \Pi_{i-1}(p_{-i}, p_i)}{\partial p_{-i} \partial p_i} \quad \text{(stability)}^{12}
\]

\(^{12}\)The stability assumption implies that the best-reply function of \( i \) plotted in a \((p_i, p_{-i})\) diagram is flatter than the best-reply function of \(-i\) for any \( p_{-i} \), implying that an intersection of the best reply functions is unique.
An equilibrium in the second, upstream pricing stage specifies prices $w^*_j, i \in \{A, B\}; j \in \{U, V\}$ conditional on ownership shares $\delta^j_i, i \in \{A, B\}; j \in \{U, V\}$.

We sometimes compute closed form solutions for the complete game by using the linear demand specification

$$q_i(p_i, p_{-i}) = \frac{1}{1 + \gamma} \left( \frac{1}{1 - \gamma} p_i + \frac{\gamma}{(1 - \gamma)} p_{-i} \right), \quad 0 < \gamma < 1,$$

with $\gamma$ quantifying the degree of substitutability between the downstream products. The two products are independent at $\gamma = 0$ and become perfect substitutes as $\gamma \to 1$. With this demand specification, Assumptions 1 to 4 are satisfied.

### 3 Stage 3: Supplier choice and downstream prices

Downstream firm $i$’s cost of buying a unit of input from supplier $j$ in which it holds a passive partial ownership of $\delta^j_i < 1$ is obtained by differentiating the downstream profit (2) with respect to the input quantity $x^j_i$, i.e.

$$\frac{\partial \Pi_i}{\partial x^j_i} = -w^j_i \underbrace{\partial q_i}_{\text{input price}} + \delta^j_i \underbrace{(w^j_i - c^j_i)}_{\text{upstream profit increase}}.$$

Thus, the unit input price $w^j_i$ faced by downstream firm $i$ is reduced by the contribution of that purchase to supplier $j$’s profits. Call $-\frac{\partial \Pi_i}{\partial x^j_i}$ the effective input price downstream firm $i$ is confronted with when purchasing from firm $j$. The minimal effective input price for downstream firm $i$ is given by

$$w^e_i \equiv \min \left\{ w^U_i \left( 1 - \delta^U_i \right), w^V_i \left( 1 - \delta^V_i \right) + \delta^V_i c \right\}. \quad (4)$$

As natural in this context, firm $i$ buys from the upstream supplier $j$ offering the minimal effective input price. If both suppliers charge the same effective input price, we assume that $i$ buys the entire input quantity from the efficient supplier $U$, as that supplier could slightly undercut to make its offer strictly preferable. Let $j(-i)$ denote the supplier $j$ from which the other downstream firm $-i$ buys its input. Differentiating the two downstream firms’ profits with respect to their own downstream price yields the two first order conditions

$$\frac{\partial \Pi_i}{\partial p_i} = (p_i - w^e_i) \frac{\partial q_i}{\partial p_i} + q_i (p_i, p_{-i}) + \delta^{j(-i)}_i \left( w^{j(-i)}_{-i} - c^{j(-i)}_{-i} \right) \frac{\partial q_{-i}}{\partial p_i} = 0, \quad i \in \{A, B\}. \quad (5)$$

Observe that whenever $\delta^{j(-i)}_i > 0$, downstream firm $i$ takes into account that changing its sales price affects the upstream profits earned via sales quantities $q_{-i}$ to its competitor.$^{13}$

$^{13}$This effect is not present with quantity competition, as then $q_{-i}$ is not a function of the strategic variable $q_i$. 

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By Assumptions 1–4, the equilibrium of the downstream pricing game is unique, stable and fully characterized by the two first order conditions for given input prices and ownership shares. Note that strategic complementarity holds under the assumption of product substitutability if margins are non-negative and \( \frac{\partial^2 q_i}{\partial p_i \partial p_j} \) is not too negative (cf. Equation (5)). Also observe that if prices are strategic complements at \( \delta_A = \delta_B = 0 \), then strategic complementarity continues to hold for small partial ownership shares.

4 Stage 2: Upstream prices under passive partial ownership

\( V \) cannot profitably sell at a (linear) price below its marginal production cost \( c \). \( U \) as the more efficient supplier can profitably undercut \( V \) at any positive upstream price. This implies that, in equilibrium, \( U \) supplies both downstream firms, and this at effective prices at most as high as \( c \).\(^{14}\) To simplify notation, let henceforth \( \delta_i \equiv \delta_i^U \) and \( w_i \equiv w_i^U \). Let \( p_i^U(w_i, w_{-i}| \delta_A, \delta_B) \) denote the equilibrium prices of the downstream subgame as a function of the input prices. Formally, \( U \)'s problem is

\[
\max_{w_A, w_B} \pi^U = \sum_{i=A,B} w_i q_i \left( p_i^U(w_i, w_{-i}| \delta_A, \delta_B), p_{-i}^U(w_{-i}, w_i| \delta_A, \delta_B) \right)
\]

subject to the constraints \( w_i(1-\delta_i) \leq c, i \in \{A, B\} \) such that downstream firms prefer to source from \( U \). Recall that partial ownership restricts \( \delta_i \) to be strictly below 1. Differentiating the reduced-form profit in (6) with respect to \( w_i \) yields

\[
\frac{d\pi^U}{dw_i} = q_i(p_i^*, p_{-i}^*) + w_i \frac{dq_i(p_i^*, p_{-i}^*)}{dw_i} + w_{-i} \frac{dq_{-i}(p_{-i}^*, p_i^*)}{dw_i}.
\]

Starting at \( w_i = w_{-i} = 0 \), it must be profit increasing for \( U \) to marginally increase upstream prices, because both \( q_i > 0 \) and \( q_{-i} > 0 \). By continuity and boundedness of the derivatives, this remains true for not too large positive upstream prices. Hence if \( c \) is sufficiently small, then the constraints are strictly binding for any partial ownership structure, so there is effective upstream competition. In this case, the nominal upstream equilibrium prices are given by

\[
w_i^* = c/(1-\delta_i),
\]

and the effective upstream prices both equal \( c \). Clearly, if \( \pi^U(w_A, w_B) \) is concave, one, or both of the constraints do not bind for \( c \) or \( \delta_i \) sufficiently large, in which case \( U \) can charge the unconstrained monopoly price below \( c \). In this regime, \( U \)'s profits are uniquely given by

\[
\pi^U = \frac{c}{(1-\delta_A)} q_A(p_A^*, p_B^*) + \frac{c}{(1-\delta_B)} q_B(p_B^*, p_A^*),
\]

\(^{14}\)This also implies that none of the downstream firms has an interest in obtaining passive shares from the unprofitable upstream firm \( V \).
and V’s profits are zero. We summarize in

**Lemma 1.** The efficient upstream firm U supplies both downstream firms at any given passive partial backward ownership shares \((\delta_A, \delta_B)\). Under effective upstream competition, i.e. for sufficiently small \(c\), U charges prices \(w^*_i = c/(1 - \delta_i)\), \(i \in \{A, B\}\), so that the effective input prices are equal to the marginal cost \(c\) of the less efficient supplier V.

For example, with the linear demand function introduced in (3) and \(\delta_B = 0\), effective competition is implied by
\[
c < \frac{1}{2} \left( \frac{\gamma + \gamma^2}{2 - (\gamma + \gamma^2)} + \frac{1}{2} \delta_A \gamma (3 - \gamma^2) \right)
\]
Intuitively, the incentive of U to sell more to A than to B increases in the difference of the nominal price \(w_A\) to \(w_B = c\). Shifting demand without losing sales is easier when the downstream firms are closer substitutes. Moreover, the difference between the nominal and effective price increases in \(\delta_A\) and thus the incentive of U to reduce the price to A to increase the sales on which it earns a higher margin.\footnote{We elaborate on this issue within the context of backward acquisition incentives, in Subsection 7.5.}

At any rate, with the upstream prices specified in Lemma 1, downstream profits can be condensed to
\[
\Pi_i = (p_i - c) q_i + \delta_i \frac{c}{1 - \delta_i} q_{-i}.
\]

Observe that if firm \(i\) holds shares in firm U so that \(\delta_i > 0\), its profit \(\Pi_i\), via its upstream holding, increases in the quantity demanded of its rival’s product \(q_{-i}\). All else given, this provides for an incentive to raise the price for its own product. Formally, firm \(i\)’s marginal profit
\[
\frac{\partial \Pi_i}{\partial p_i} = q_i + (p_i - c) \frac{\partial q_i}{\partial p_i} + \delta_i \frac{c}{1 - \delta_i} \frac{\partial q_{-i}}{\partial p_i}
\]

increases in \(\delta_i\). Also, if \(\delta_i > 0\), then the marginal profit of \(i\) increases in \(\delta_{-i}\), as this increases the upstream margin earned on the product of \(-i\). If the downstream products were not substitutable, i.e. \(\frac{\partial q_{-i}}{\partial p_i} = 0\), the marginal profit and thus the downstream pricing would not be affected by backward ownership. As the products \((i, -i)\) become closer substitutes, \(\frac{\partial q_{-i}}{\partial p_i}\) increases and the external effect internalized via the cash flow right \(\delta_i\) becomes stronger, and with it the effect on equilibrium prices.

In all, this yields the following central result:

**Proposition 1.** Let Assumptions 1-4 hold and upstream competition be effective. Then
(i) both equilibrium downstream prices \(p^*_i\) and \(p^*_{-i}\) increase in both \(\delta_i\) and \(\delta_{-i}\) for any non-controlling vertical ownership structure,
(ii) the increase is stronger when the downstream products are closer substitutes.

The following corollary is immediate:
Figure 1: Best-reply functions of downstream firms A, B and the vertically integrated unit UA for linear demand as in (3), with $\gamma = 0.5$ and $c = 0.5$.

**Corollary 1.** Any increase in passive ownership in $U$ by one or both downstream firms is strictly anti-competitive.

Proposition 1 is illustrated in Figure 1 for the case $\delta_A > \delta_B = 0$. The solid line is the inverted best-reply function $p_B'(p_A)^{-1}$ of B at a given $\delta_A > 0$. The dashed line is A’s best reply $p'_A(p_B)$ for $\delta_A = 0$, and the dashed-dotted line above this is A’s best reply for $\delta_A \to 1$. Hence, choosing $\delta_A$ amounts to choosing the best-reply function $p'_A(p_B)$ in the subsequent pricing game. This becomes central when analyzing the profitability of acquisitions in the next section.

Before going on, we should emphasize that the nominal transfer prices charged here are higher for the firm with the larger interest in the efficient upstream supplier. This is interesting because, in view of its potential impact on foreclosure, preferentially low transfer prices between vertically related firms may be more likely to raise concerns of the competition policy analyst.

## 5 Stage 1: Acquisition of passive shares by downstream firms

Here we assess the profitability of downstream firms’ backward acquisitions of passive stakes in upstream firms. We restrict our attention to the acquisition of stakes in firm $U$. This is easily justifiable within the context of our model: As both downstream firms prefer to acquire input from the more efficient firm, the less efficient firm $V$ does not earn positive profits in equilibrium. Hence, there is no scope for downstream firms to acquire passive interests in $V$.

Rather than specifying how bargaining for ownership stakes takes place and conditioning the outcome on the bargaining process, we determine the central incentive condition for backward acquisitions to materialize, namely that there are gains from trading claims to profits in $U$ between that upstream firm and one of the downstream firms.\textsuperscript{16} For the sake of

\textsuperscript{16}From the discussion above, it should have become clear that there is room for simultaneous or, for that matter, sequential passive backward acquisition, given this claim is satisfied.
brevity, we abstain from modeling the ownership acquisition game, that would specify the redistribution of rents to the industry generated from passive backward integration.

In order to enhance the intuition, fix for the moment the stakes held by firm $B$ at $\delta_B = 0$. Gains from trading stakes between $A$ and $U$ arise if the joint profit of $A$ and $U$,

$$\Pi^U_A(\delta_A|\delta_B = 0) \equiv p^*_A q^*_A + c q^*_B,$$

are higher at some $\delta_A \in (0, 1)$ rather than at $\delta_A = 0$, where $p^*_A$, $q^*_A$ and $q^*_B$ all are functions of $\delta_A$.\footnote{Passive backward ownership of $A$ in $U$ benefits $B$ as $A$ prices more softly. Our assessment of the profitability of backward ownership is conservative as this benefit cannot be extracted by $U$ who can at most charge $B$ a unit price of $c$. With commitment to exclusive supply from $U$ or two-part tariffs, $U$ can extract the profit increase of $B$ through a higher marginal price or an up-front fee. See Subsection 7.3 for details.} The drastic simplification of this expression results from the fact that a positive $\delta_A$ just redistributes profits between $A$ and $U$. The gains from trade between $A$ and $U$ can thus arise only via indirect effects on prices and quantities induced by increases in $\delta_A$. Why should there be such gains from trade at all?

The vertical effects of an increase in $\delta_A$ between $A$ and $U$ are exactly compensating as the effective transfer price remains at $c$ (Lemma 1). All that changes are $A$'s marginal profits. They increase in $\delta_A$, because with this $A$ internalizes an increasing share of $U$’s sales to $B$. Again, this leads $A$ to increase $p_A$, which in turn induces $B$ to increase $p_B$. That price increase is not only profitable to $B$, but eventually yields a net benefit to $A$ and $U$. Intuition suggests that this competition softening effect increases the profits of $U$ and $A$ if competition in the industry is fierce. Indeed, evaluating $d\Pi^U_A/d\delta_A$ at small $c$ yields

**Proposition 2.** An increasing partial passive ownership stake of firm $i$ firm in firm $U$ increases the combined profits of $i$ and $U$, if upstream competition is sufficiently intense.

This argument continues to hold if both downstream firms buy shares in the efficient upstream firm, under the obvious restriction that control is not transferred from $U$ to any one of the downstream firms.

**Corollary 2.** Increasing partial passive ownership stakes of firms $i$ and $-i$ in firm $U$ increase the industry profit $\Pi^U_{AB} \equiv p^*_A q^*_A + p^*_B q^*_B$, if upstream competition is sufficiently intense.

Using the linear demand example introduced in (3), we can make explicit how our case assumption that upstream competition is intense enough relates to the intensity of downstream competition. Let $\delta_{-i} = 0$. Then the joint profits of firms $i$ and $U$ are maximized at a positive passive ownership share $\delta_i$ if $c < \gamma^2/4$. For close to perfect downstream competition, i.e. $\gamma$ close to 1, this implies that passive backward ownership is profitable for a range of marginal costs up to $1/2$ of the industry’s downstream monopoly price.\footnote{Recall that a large $\gamma$ corresponds to strong competition downstream, and a small $c$ to strong competition upstream. Hence if overall competition is strong, it is profitable to acquire passive ownership because this increases downstream prices. As the upper bound monotonically increases in $\gamma$, the range of $c$ in which this result holds increases in $\gamma$. At any rate, under this condition, the ownership share maximizing $\Pi^U_i$ is given by}
As a firm’s backward interests confer a positive externality on the second firm’s profits, the industry profits $p_A^*q_A^* + p_B^*q_B^*$ are maximized at strictly positive passive ownership shares by both firms if the less restrictive condition $c < \gamma/2$ holds. The fact that $\gamma^2/4 < \gamma/2$ indicates the internalization of the positive externality on the downstream competitor when interests in the efficient upstream firm are acquired to maximize industry profits.\(^{19}\)

One might worry about the magnitude of the effect derived; also when many inputs are procured to produce a unit of the downstream product. Let us start with the baseline case, in which the downstream products are produced with only one input. Under the assumed close substitutability between the downstream products, the change in own demand induced by a price change is of the same order of magnitude as the change in the competitor’s demand. In equilibrium, the former is weighted by the margin $p_A - c$, whence the latter is weighed by $\frac{\delta_A}{1 - \delta_B} c$. The former can be easily dominated by the latter, even when the shares held by the downstream firms in the upstream efficient supplier are small.

Take now a technology in which the downstream product is produced by two inputs rather than only one. Suppose that input 1 is produced in an industry structured as in the baseline model, and commodity 2 can be procured at a price of $c_2$. The effective downstream margin is now given by $p_A - c - c_2$, which again can be easily dominated by $\frac{\delta_A}{1 - \delta_B} c$. What matters is that the margin of the input on which backward integration takes place is relatively large when compared to the downstream margin. Note also that if a downstream firm integrates backward in the efficient supplier of each input, the overall effect is that of backward integration in case of a one-to-one technology.

In passing, all of these results give rise to interesting hypotheses to be tested empirically. A particularly intricate one is that the externality alluded to here provides incentives to acquire passive shares in suppliers to competitors. While this hypothesis remains to be looked at empirically in detail, it could provide an explanation for the empirical puzzle demonstrated by Atalay et al. (2013) that a majority of backward acquisitions is not accompanied by physical product flows.

One also might want to speculate about the consequences of the effect derived here for the entry of firms downstream and upstream. Due to the externality generated on the outsiders by increasing prices, downstream entry may be invited rather than deterred. By contrast, upstream, the externality results from the fact that all downstream firms are supplied by the efficient firm. This tends to constitute an entry barrier.

\[ \delta_i^* | \delta_i = 0 = \min \left( \frac{4c\gamma(1 + \gamma) + \gamma^2(2 - \gamma - \gamma^2) - 8c}{4c\gamma(2 - \gamma^2)}, \delta \right). \]

\(^{19}\)Under this condition, the industry profit is maximized at

\[ \delta_A^* = \delta_B^* = \min \left( \frac{\gamma - 2c}{\gamma - 2c + 2c\gamma}, \delta \right) \]

with the natural restriction that $\delta \leq 1/2$. 

\[ (13) \]
6 Controlling backward integration, and comparison

In this section, we characterize downstream and upstream equilibrium prices and profits when one of the downstream firm, say $A$, fully integrates backward into the efficient supplier $U$, and compare them with those arising under vertical separation, vs. under passive partial backward integration. Here we also relate to the key claims in Chen (2001). Towards that, observe that the set of assumptions used here corresponds to that used by him.

Let the ownership structure under vertical integration be described by $\{\delta_A = 1, \delta_B = 0\}$, and let $A$ control $U$’s pricing decisions. As $U$ is more efficient than $V$, the fully integrated firm continues, as heretofore, to profitably meet any positive price $w^V_B$ charged by $V$. Under effective upstream competition, it is again optimal to set $w^U_B = c$. Yet, by virtue of being merged with $U$, $A$ takes account of the true input cost normalized to zero.\(^{20}\)

Consider first the effect of full integration of $A$ and $U$ on downstream prices. Still faced with marginal input cost $c$, $B$’s best response remains unchanged. Yet full integration has two countervailing effects on the determination of $p_A$. The first is the one we have studied under passive backward integration: upward price pressure arises because the integrated unit fully internalizes the upstream profit from selling to firm $B$, that is $cq_B(p_B, p_A)$. We call this the integration effect. The other effect not arising under passive backward integration is downward price pressure, following from the elimination of double marginalization on product $A$: the downstream costs $cq_A(p_A, p_B)$ arising under vertical separation are decreased to zero. We call this the efficiency effect. It turns out that under our standard assumptions the latter effect is generically stronger than the former, yielding

**Proposition 3.** Under Assumptions 1 to 4, a vertical merger between one downstream firm and $U$ decreases both downstream prices, as compared to complete separation.

Returning to Figure 1, note that for any $\delta_A > 0$, the best response of the merged entity, $p^*_UA(p_B)$, represented by the dotted line in Figure 1, is located below the one arising under separation.\(^{21}\) We summarize our comparison of downstream equilibrium prices under the two acquisition regimes, in

**Corollary 3.** Under Assumptions 1 to 4 and effective upstream competition, a vertical merger between one of the downstream firms and the efficient upstream firm leads to a decrease of all downstream prices when compared to those arising under vertical separation, whence any passive partial backward ownership of one or both downstream firms in the efficient supplier $U$ leads to an increase in all downstream prices.

We now turn to a comparison of the combined profits of $A$ and $U$ under full vertical separation and full integration. By Proposition 3, vertical integration decreases both downstream prices. This is not desirable for the integrated firm, since by conceptualization of

\(^{20}\)In line with the literature – examples are Bonanno and Vickers (1988), Hart and Tirole (1990), and Chen (2001) – the integrated firm is considered unable to commit to an internal transfer price higher than its true marginal cost.

\(^{21}\)A variant of Proposition 3 is also contained in Chen (2001). See his Lemma 7.
the model the overall margins earned under vertical separation are below the industry profit maximizing level. We now ask whether, at vertical separation, it is profitable nevertheless to move towards integration. It turns out that this is not the case as long as \( c \), i.e. as in Chen, the cost difference between the efficient and the next efficient supplier, is sufficiently small. By continuity, there exists an interval \( (0, \bar{c}) \) such that for any \( c \) in this interval, vertical separation is more profitable than integration. This is shown in

**Proposition 4.** Under Assumptions 1 to 4, a vertical merger between \( A \) and \( U \) is less profitable than complete vertical separation, if upstream competition is sufficiently intense.

This result strictly contradicts Chen (2001)'s central result. His analysis differs from ours, however, in that he assumes that under upstream competition of the type considered here, the integration of \( A \) and \( U \) is profitable to the outsider \( B \), and that the profits so generated can be absorbed by the integrated firm via a higher upstream price. The rent from integration to the outsider is negative, however, as shown in

**Lemma 2.** Under Assumptions 1 to 4, profits to the outsider firm \( B \) are reduced, when \( A \) and \( U \) merge.

In view of Lemma 2, it is natural for the outsider firm \( B \) to switch its sourcing from the integrated firm to the upstream outsider \( V \), as long as there is a low, or a zero switching cost. But then, integration could not arise at all, by Proposition 4. Leaving aside the (complicating) effect of downstream competition on the integration decision: for integration, and with it, an anti-competitive increase in downstream prices to arise in his model, Chen would need to assume somewhat forcedly that \( B \), since naturally supplied by \( U \) before integration, incurs a high cost of switching its sourcing to the upstream outsider \( V \). Thus, the switching cost cannot be (arbitrarily) low or zero, as argued by him. It must not only be high enough to force \( B \) to continue sourcing from \( U \). It must also be high enough to allow the integrated firm to set a transfer price \( w_B^U > c \) sufficiently high to at least cover its loss from integration. On the other hand, it must be low enough to not let \( B \)'s outside option become negative. It is not clear whether such a switching cost interval exists at all. At any rate, the raising-rival’s-cost argument proposed by Chen can only hold if it does.

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22See his Theorem 1.

23These statements do also not conform to Chen’s claims.

24To clarify, let \( \pi_i(y, z) \) denote firm \( i \)'s profit under separation, and \( \Pi_A^U(0, c) \) and \( \pi_B^U(c, 0) \) those under integration, respectively, evaluated at equilibrium prices, that in turn are determined as functions of the equilibrium input costs \( y \) charged to \( i \), and \( z \) charged to \(-i\), respectively. From Lemma 2, we know that \( \pi_B^U(c, 0) < \pi_B(c, c) \). Let \( \underline{s}_B \) be defined so that \( \pi_B^U(c, 0) = \pi_B(c, c) - s \). The right hand side denotes the value of \( B \)'s outside option, that obviously decreases in \( s \), and \( \underline{s}_B \) is the level of the switching cost below which \( B \) will deviate to \( V \) for sure. Let \( w_B^U(s) \) be an increasing function, defined by \( \pi_B^U(w, 0) = \pi_B(c, c) - s \), denoting the transfer price \( B \) is willing to accept from the integrated firm at a given switching cost \( s \) and still not deviate to being supplied by \( V \). The switching cost \( s \) is feasible for \( B \) in the interval \( [\underline{s}_B, \pi_B(c, c)] \). From Proposition 4 we know that \( \Pi_A^U(0, c) < \pi_A(c, c) + \pi_U(c, c) \). Let \( \underline{s}_A \), defined by \( \Pi_A^U(0, u(s)) = \pi_A(c, c) + \pi_U(c, c) \), denote the minimal price the integrated firm must charge to the outsider, so that it covers the loss from vertical integration and does not deviate to vertical separation. It follows that the interval for \( s \) allowing profitable integration à la Chen is given by \( [\max\{\underline{s}_A, \underline{s}_B\}, \pi_B(c, c)] \). Assuming that this interval is non-empty is not innocuous.

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In all, vertical integration does not, as claimed by Chen, change the incentives of the rival in selecting its input supplier. By contrast, since the efficient upstream firm is the natural supplier under vertical separation, backward integration can possibly become profitable only if high switching costs force that rival to maintain its procurement relationship with the integrated firm.

Returning to our main theme, we should emphasize that in contrast to full vertical integration, the outsider’s profits increase with passive backward integration. Note also that our results, namely increased prices from, and incentives to, passive partial backward integration would be strengthened if we would allow the efficient supplier to absorb the rents generated to the outsider firm via an increased transfer price.

At any rate, combining Propositions 2 and 4 yields

**Corollary 4.** Passive partial backward integration of firm $i$ into firm $U$ is more profitable than vertical integration, if upstream competition is sufficiently intense. Then, downstream firms have the incentive to acquire maximal backward interests, short of controlling the upstream firm $U$.

As mentioned before, this result is nicely related to the literature on strategic delegation. The particular twist here is that the very instrument intended to acquire control, namely the acquisition of equity in the target firm, is employed short of controlling the target. This benefits the industry, but it harms consumer welfare.

**A remark on control with partial ownership.** As we have demonstrated, the key driver behind Corrollary 4 is that passive ownership preserves or, with the absorption of the outsider’s profits from passive integration by the efficient supplier, even enhances double marginalization, whereas a vertical merger eliminates it.

This argument is based on an argument commonly used in the literature on vertical relations, that the merged entity cannot commit to internal transfer prices above marginal costs. It is arguably less straightforward with controlling partial ownership, say when $A$ has a controlling block of voting shares of $U$ less than $\delta_A = 1$. If downstream competition is sufficiently strong, then the shareholders of $A$ and $U$ collectively have an incentive to commit to a high transfer price $w_A$. However, $A$ has an individual incentive to be charged a low transfer price, or at least wants to be compensated with a fixed payment. If $A$ cannot be compensated or commitment to a high price is not feasible as renegotiations remain possible, $A$ will use its control to decrease $w_A$, its own input costs. In a standard bargaining framework, the price $w_A$ decreases the more, the stronger the controlling influence of $A$ over the industry.

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25To substantiate our claim, replace, in equation (12), $c$ by $w_B(\delta_A)$, defined by $\pi_B(w_B,c) = \pi_B(c,c)$ at a given $\delta_A$. It is easy to show that $\pi_B(w,c)$ increases in $\delta_A$, so $w_B(\delta_A)$ is an increasing function, with $w_B(0) = c$. The derivative (19) then is augmented by $\frac{d\pi_B}{d\delta_A}q_B^*$, which is strictly positive at any $\delta_A \geq 0$. Whether one would want to classify this effect under raising-rival’s-cost is a matter of taste. At any rate, we maintain that it cannot be classified under foreclosure as claimed by Chen, because it is based on a voluntary profit increasing reaction by the outsider to a price increase initiated by the insider. In fact, in our model involving no switching cost, the outsider’s profits are not reduced as a result of passive backward integration.

26We referred to this in footnote 20.
$U$, whereas the price for $B$ remains unchanged as there is no conflict of interest over it among the shareholders of $U$.

7 Discussion and Extensions

7.1 Non-discriminatory upstream prices

Many competition laws require a firm to charge non-discriminatory prices. While by the U.S. Robinson-Patman Act, non-discrimination is a widely applied rule, Article 102 of the Treaty on the Functioning of the European Union restricts the application of the rule to dominant firms.

Clearly, under effective competition, symmetric passive ownership with $\delta_A = \delta_B > 0$ may arise as an equilibrium. Supplier $U$ then has no incentive to price discriminate. Yet, as we have shown in Proposition 1, symmetric passive ownership is clearly anti-competitive, so in this case, a non-discrimination rule has no effect at all, and in particular no pro-competitive effect.

Consider instead one of the firms’, say $A$’s, incentive to acquire a backward interest in firm $U$ when non-discrimination is effective and $\delta_B = 0$. Then $U$ must charge a uniform price $c$ if it wants to serve both downstream firms. This yields profits to $A$ of

$$\Pi_A = (p_A - c) \cdot q_A + \delta_A c \cdot (q_A + q_B).$$

Differentiating with respect to $p_A$ and $\delta_A$ yields

$$\frac{\partial^2 \Pi_A}{\partial p_A \partial \delta_A} = c \cdot \left[ \frac{\partial q_A(p_A, p_B)}{\partial p_A} + \frac{\partial q_B(p_B, p_A)}{\partial p_A} \right].$$

By Assumption 1, the own price effect dominates the cross price effect, and therefore the cross derivative in (14) is negative at $\delta_A = 0$. Thus marginally increasing $\delta_A$ decreases the marginal profit of $A$. Hence, the best reply $p_A^*(p_B|\delta_A)$ and, in consequence, both equilibrium downstream prices, decrease in $\delta_A$ at $\delta_A = 0$. By continuity, this holds for small positive $\delta_A$. Yet also, the passive backward integration incentive vanishes.

This result generalizes to all feasible $\delta_A$ as long as $\frac{\partial q_B}{\partial p_A} \leq \frac{\partial q_A}{\partial p_B}$ for $p_A < p_B$, e.g. in case of linear demand. Under this condition, if only one downstream firm had passive ownership in $U$ nevertheless, and $U$ optimally served both downstream firms, then such ownership would not anti-competitive under a non-discrimination rule.\textsuperscript{27}

\textsuperscript{27}$U$ wants to serve both downstream firms for a small $\delta_i$, given $\delta_{-i} = 0$. Once $\delta_i$ becomes large, $U$ may find it profitable to set a high nominal price at which only $i$ wants to purchase. This makes $-i$ dependent on $V$. In turn, $V$ can raise the price charged to $-i$ above $c$, yielding partial foreclosure. However, it is unclear whether partial foreclosure is an equilibrium. We will discuss in detail the effects of non-discrimination rules in the different case situations in a forthcoming paper.
7.2 Simultaneous price setting

So far, we have assumed upstream prices to be set before downstream prices. Suppose now that all prices are set simultaneously. Then upstream firms take downstream prices as given. For \( U \), increasing effective prices up to \( c \) does not affect quantity. Hence, effective equilibrium upstream prices must be equal to \( c \). With simultaneous price setting, however, an equilibrium does only exist as long as the participation constraints of downstream firms are not violated at effective upstream prices of \( c \).

**Lemma 3.** Under effective competition, sequential and simultaneous setting of up- and downstream prices are outcome equivalent.

Note also that as long as the participation constraints of downstream firms do not bind, the simultaneous price setting is equivalent to the case in which downstream prices are set first, followed by upstream prices and, finally, downstream firms choose where to buy inputs.

7.3 Two-part tariffs

The assumption of linear upstream prices is clearly restrictive theoretically, as argued already in Tirole (1988) – yet not necessarily empirically so. As to specific theoretical results, Caprice (2006) as well as Sandonis and Fauli-Oller (2006) pointed out that with effective upstream competition, observable two part tariffs offered by the efficient supplier \( U \) implement downstream prices below the industry profit maximizers. One reason is that \( U \) does not want to offer marginal input prices as high so that they maximize industry profits, because downstream firm \( i \)'s alternative to sourcing from \( U \), given its rival \(-i\) sources from \( U \), is more valuable when \( U \) charges \(-i\) a higher marginal price. This induces \( U \) to lower the marginal prices below the industry profit maximizing level, in order to obtain more rents through the fixed fees.

Moreover, if \( U \) cannot offer exclusive contracts, a downstream firm will source inputs alternatively once the marginal input price charged by \( U \) exceeds the alternative input price. In our setting, this implies that without backward interests by a downstream firm, \( U \) cannot implement a marginal price above \( c \) to that firm. We show that in the case discussed heretofore, \( U \) indeed would like to offer marginal prices above \( c \). Thus marginal input prices in equilibrium equal \( c \) and the fixed fee \( F \) equals zero, i.e. the transfer prices \( U \) charges are endogenously linear.

In what follows we formally characterize the two-part contracting problem and show that passive backward ownership can increase downstream prices. At the outset, observe that unlike in the case of full backward integration, deviating to source from \( V \) is less attractive for \( B \) once there is partial ownership of \( A \) in \( U \), because \( B \)'s profits are increased thereby. This generically relaxes the contracting problem of \( U \).

We start our analysis from complete vertical separation so that \( \delta_A = \delta_B = 0 \), and maintain the assumptions that all contract offers are observable to all downstream firms upon acceptance; in particular that acceptance decisions are observed when downstream prices are
set. A tariff offered by supplier \( j \) to downstream firm \( i \) is summarized by \( \{ F_j^i, w_j^i \} \), where \( F_j^i \) is the fixed fee downstream firm \( i \) has to pay the upstream firm \( j \) upon acceptance of the contract, and \( w_j^i \) continues to be the marginal input price. Denote by \( \pi_i^*(w_j^i, w_{k-i}^i), j, k \in \{ U, V \} \), firm \( i \)’s reduced form downstream profits at downstream equilibrium prices as a function of the marginal input price relevant for each downstream firm, but gross of any fixed payment. With the model constructed as in the main part of the paper, the Bertrand logic still holds upstream: \( U \) can always profitably undercut any (undominated) offer by \( V \), so in equilibrium \( U \) exclusively supplies both downstream firms. Yet if upstream competition is effective as assumed throughout, \( U \) is restricted by \( V \) in its price setting. We require that \( V \)’s offers, if accepted, yield it non-negative profits.

More formally, for given contract offers of \( V \) to firm \( A \) and \( B \), \( U \)’s problem is

\[
\max_{F_A^U, F_B^U, w_A^U, w_B^U} \pi^U = \sum_{i \in \{A,B\}} \left[ w_i^U q_i + F_i^U \right] \quad \text{s.t.} \quad \pi_i^* \left( w_i^U, w_{-i}^U \right) - F_i^U \geq \pi_i^* \left( w_i^V, w_{-i}^V \right) - F_i^V. \tag{15}
\]

\( U \) has to ensure that each downstream firm’s deviation to source from \( V \) is not profitable. In equilibrium, the profit constraints of both downstream firms must be binding, as otherwise \( U \) could profitably raise the respective fixed fee \( F_i^U \), until downstream firm \( i \) is indifferent between its and \( V \)’s contract offer.

Let the contracts offered by upstream firms first be non-exclusive, so that an upstream firm cannot contractually require a downstream firm to exclusively procure from it. Then, setting a marginal input price \( w_i^U > c \) with \( F_i^U < 0 \) cannot be an equilibrium, as \( V \) could profitably offer \( \{ F_i^V = 0, w_i^V \in [c, w_i^U) \} \), which would provide incentives to downstream firm \( i \) to accept \( U \)’s contract offer in order to cash in \( F_i^U \), but to source its entire input at the marginal cost \( w_i^V \) offered by \( V \).

The equilibrium contract offers made by \( V \) must be best replies to \( U \)’s equilibrium contract offers. Hence

**Lemma 4.** If \( U \) offers two-part tariffs with \( w_i^U \leq c, i \in \{A, B\} \), then \( \{0, c\} \) is \( V \)’s unique non-exclusive counteroffer that maximizes the downstream firms’ profits and yields \( V \) a non-negative profit.

Using this insight and letting \( w_i \equiv w_i^U \) and \( F_i \equiv F_i^U \) to simplify notation, \( U \)’s problem reduces to

\[
\max_{w_A, w_B} \pi^U = \sum_{i \in \{A,B\}} p_i^*(w_i, w_{-i}) q_i^* - \sum_{i \in \{A,B\}} \pi_i^*(c, w_{-i}) \quad \text{subject to} \quad w_i \leq c, i \in \{A, B\}. \tag{16}
\]

For \( c = \infty \), the outside options equal 0, and \( U \) simply maximizes the industry profit by choosing appropriate marginal input prices. As \( c \) decreases, sourcing from \( V \) eventually yields downstream firms positive profits. Moreover, firm \( i \)’s outside option, the profit \( \pi_i^*(c, w_{-i}) \) it
would obtain when sourcing from $V$, increases in the rival’s cost $w_{-i}$. Hence the marginal profit $\partial \pi^U / \partial w_i$ is below the marginal industry profit. For $c$ sufficiently small, the marginal industry profit is still positive when the arbitrage constraints are binding, i.e. at $w_A = w_B = c$. Hence the motive of devaluing the contract partners’ outside options is dominated by the incentive to increase double marginalization, yielding the result that upstream tariffs are endogenously linear. We summarize in

**Proposition 5.** Let upstream competition be sufficiently intense. Then under vertical separation, \{c, 0\} is the unique symmetric equilibrium non-exclusive two-part tariff offered by both upstream to both downstream firms.

As before, sufficient intensity of upstream competition is to be seen relative to the intensity of downstream competition. In our linear demand example, it suffices to have $c < \gamma^2 / 4$. In passing, this is also the condition ensuring the profitability of an initial increase of passive backward ownership $\delta_i$ to $i$ and $U$.

What does change if we allow for passive partial backward integration? As \{0, c\} is a corner solution, (at least some) passive backward integration does not change the efficient upstream firm’s incentive to charge maximal marginal prices.

Moreover, recall that passive backward ownership of $i$ in $U$ exerts a positive externality on $-i$ as $i$ prices more softly – but only if $-i$ sources from $U$. With two-part tariffs, $U$ can extract the upward jump in $-i$’s payoffs by charging a positive fixed fee.\(^{28}\) Assuming that commitment to only buy from $U$ is not feasible, we obtain

**Lemma 5.** Let upstream competition be sufficiently intense and $\delta_i > \delta_{-i} = 0$. The non-exclusive two-part tariff offered by $U$ to $i$ has $w_i = c / (1 - \delta_i)$ and $F_i = 0$, and the tariff to $-i$ has $w_{-i} = c$ and $F_{-i} > 0$.

Thus, when firm $i$ has acquired a positive share, the effective input price $U$ charges it remains at $c$ as under linear tariffs. With non-exclusivity, a higher marginal input price is not feasible, as then firm $i$ would buy the inputs from $V$, that continues to charge \{0, c\}. Hence Proposition 2 still applies and we obtain

**Corollary 5.** Let upstream competition be sufficiently intense. Then partial passive ownership of downstream firm $i$ in supplier $U$ increases bilateral profits $\Pi^U_i$ as well as industry profits $\Pi^U_{AB}$ compared to complete separation, even if non-exclusive two-part tariffs are allowed for.

Hence the results derived in the main part of the paper for linear tariffs are upheld with non-exclusive observable two-part tariffs, if competition is sufficiently intense. When upstream competition is less intense, it is optimal for $U$ to charge effective marginal prices below $c$ to reduce the downstream firms’ outside options. Thus the no-arbitrage constraint $w_i \leq c / (1 - \delta_i)$ is no longer binding, which is also the case when $U$ offers exclusive two-part tariffs. Yet passive backward integration still relaxes downstream competition for given

\(^{28}\)U could also charge $B$ a marginal price above $c$, but only if commitment to exclusive dealing of $B$ with $U$ is possible. To remain consistent with the main part, we rule this out here, as does Chen (2001).
effective input prices. Moreover, $U$ can still extract the positive externality of backward ownership on downstream competitors by raising either the fixed fee or the marginal price. Assuming that demand is linear and $V$ offers $\{0, c\}$, one can show that passive backward ownership is indeed both profitable and increases downstream prices for large parameter ranges of $c$ and $\gamma$ where contracts with effective marginal input prices above or below $c$ result.\(^{29}\)

### 7.4 Ineffective competition

In the baseline model, we have analyzed the effects of passive partial backward integration when there is effective upstream competition as generated by a difference $c$ in marginal costs between the efficient firm $U$ and the less efficient firm $V$ small enough so that $U$ was constrained in its pricing decision. We now sketch the case in which that cost difference $c$ is so large that $U$ can behave as an unconstrained upstream monopolist.

Consider first complete vertical separation. With linear upstream prices, double marginalization arises so that the equilibrium downstream prices are above the level that maximizes industry profits, and approach the industry profit maximizing prices from above only as downstream competition tends to become perfect. For the industry, it is not desirable to further relax competition. Instead, it is desirable to reduce margins with, for example, resale price maintenance, passive forward integration, or observable two-part tariffs. With observable two-part tariffs, $U$ can maximize the industry profits by choosing the marginal price in accordance to downstream competition and extracting all downstream profits through fixed fees. In this situation the owners of $U$ have no interest in backward ownership, because the profits they can extract are already maximized.

The case with linear tariffs is less straightforward. As before, for given marginal input prices $w_A$ and $w_B$ set by the monopolist, an increase in the passive backward ownership share $\delta_A$ in the supplier reduces $A$’s effective input price, so that $A$ has an incentive to lower its sales price. Yet a positive $\delta_A$ also induces $A$ to internalize its rivals’ sales, so that $A$ wants to increase its sales price. The first effect tends to dominate, so that downstream prices decrease in $\delta_A$ for given (nominal) input prices. As $U$ is unconstrained in its price setting, it can adjust $w_A$ and $w_B$ in response to any ownership change until its marginal profits are zero again. Hence, both effects of an increase in $\delta_A$ on downstream prices are internalized by the unconstrained upstream monopolist. This gives rise to invariant downstream prices in case of symmetric backward ownership.\(^{30}\)

By contrast, with effective upstream competition as in our model, only the first, marginal cost decreasing effect of an increase in $\delta_A$ is counterbalanced by the efficient upstream firm $U$, and that perfectly. Hence the overall effect equals the second effect of internalizing the

\(^{29}\)If $V$ can also offer exclusive contracts, the analysis is more complicated. We simplify here to increase expositional clarity.

\(^{30}\)For linear downstream demands and linear non-discriminatory upstream tariffs, Greenlee and Raskovich (2006) show that upstream and downstream price adjustments exactly compensate when passive backward ownership in the monopoly supplier is symmetric, so downstream prices stay the same independently of the magnitude of partial ownership and the intensity of downstream competition.
rivals’ sales, and thus both downstream prices increase in $\delta_A$.

7.5 Passive backward integration and foreclosure incentives

Suppose again that only firm $A$ hold passive partial ownership rights in firm $U$, so that $\delta_A > 0$, $\delta_B = 0$. We know that when selling to $A$, the upstream supplier obtains a higher unit price than when selling to $B$. It is thus in its interest to manipulate the final demand to the downstream firms to the favor of $A$. The constraint on the efficient supplier’s price setting schedule does not allow to foreclose downstream firm $B$ by setting a high price. $U$ could, however, lower the nominal price quoted to $A$, in order to allow $A$ to foreclose $B$ in the ensuing downstream pricing subgame.

While total foreclosure would be a profitable operation at any given asymmetric backward interests of the downstream firms in particular when their products are very close substitutes, it would violate the incentive to any downstream firm, here $A$, to acquire an interest in the upstream supplier, for passive backward acquisitions would then be allocatively neutral and therefore unprofitable in the ensuing successive monopoly.\footnote{See Greenlee and Raskovich (2006) for a proof.} It follows that this branch of an extended game tree would not be reachable.

7.6 Comparing passive backward with passive horizontal integration

We have shown that passive backward integration of downstream firms, rather than inviting foreclosure, induces downstream horizontal coordination that leads to increasing downstream prices. One is tempted to ask how this price change compares to that induced by direct passive horizontal integration. Let us compare the profits of the integrating downstream firm, say $A$, under the two forms of integration, with the same block share $\delta_A > 0$, and let $\delta_B = 0$. Under backward integration as heretofore, they are, at competitive upstream prices, given by

$$\Pi_A = (p_A - c) q_A + \delta_A c q_B,$$

whence under horizontal integration, they are given by

$$\Pi_A = (p_A - c) q_A + \delta_A (p_B - c) q_B.$$  \hfill (17)

By a first order argument, $A$ internalizes the sales of $B$ more under backward integration if $c > p_B - c$, i.e. if the upstream margin of product $B$ is larger than its downstream margin. With linear demand and effective upstream competition, passive backward integration yields a higher price level than passive horizontal integration if $c > g(\gamma)$, where $g$ is a decreasing function.\footnote{In fact, $g(\gamma) = \frac{2 - \gamma - \gamma^2}{6 - 2\gamma - 2\gamma^2 (2 + \delta_A)}$.} For a given upstream margin $c$, passive backward integration is more anti-competitive if downstream products are sufficiently close substitutes ($g \to 0$ as $\gamma \to 1$).
8 Conclusion

In this article, we consider vertically related markets with differentiated, price setting downstream firms that produce with inputs from upstream firms supplying a homogeneous product at differing marginal costs. We analyze the impact on equilibrium prices of one or more downstream firms holding passive, that is non-controlling ownership shares in the efficient, and therefore common, supplier. In sharp contrast to earlier studies who focused either on Cournot competition or upstream monopoly, we find that if competition is sufficiently intense, passive ownership leads to increased downstream prices and thus is strictly anti-competitive. Most importantly, passive ownership is anti-competitive when a full vertical merger would be pro-competitive. In passing, it also relaxes upstream competition.

We also show that incentives to passive backward acquisitions exist when full controlling integration is not profitable relative to vertical separation. Thus, the firms strictly prefer the former. They voluntarily abstain from controlling the upstream firm, because this would do away with its power to commit to an industry profit increasing high transfer price. The additional feature brought with this to the strategic delegation literature is that the very instrument – here: share acquisition – typically employed to obtain control is used up to the point where control is not attained.

Our result is driven primarily by a realistic assumption on the upstream market structure, in which an efficient supplier faces less efficient competitors. This allows that efficient supplier to soften upstream competition by increasing upstream prices only when the price increasing effect is absorbed by the typical downstream firm, via its claims on upstream cash flows. We show the result to be robust to changes in other assumptions such as linear upstream prices, and sequential price setting upstream and then downstream. Indeed, once allowing upstream firms to offer observable two-part tariffs, we find that the equilibrium contracts are endogenously linear if competition is sufficiently intense. Interestingly enough, under effective upstream competition, passive ownership in suppliers, while anti-competitive when price discrimination is allowed for, tends not to be anti-competitive under a non-discrimination clause.

The theory provides for a number of empirically testable hypotheses. A strong test is already provided in the contribution by Gans and Wolak (2012). For competition policy, it is important to recognize that in contrast to full backward integration, anti-competitive passive ownership in common suppliers is profitable when there is both up- and downstream competition and thus foreclosure potentially not the main concern. Most importantly, proposing passive backward ownership in a supplier as a remedy to a proposed vertical merger tends not to benefit, but rather to harm competition, as long as upstream competition is effective and the upstream supplier serves competitors of the acquirer. The reason is that full vertical integration removes double marginalization via joint control, whilst partial backward integration enhances that.

In the present setting, we abstract from other, potentially socially desirable motives for partial backward ownership. A particularly important effect is the mitigation of agency problems in case of firm-specific investments (Riordan, 1991; Dasgupta and Tao, 2000) such
as investment in specific R&D. Indeed, Allen and Phillips (2000) show for a sample of US companies that vertical partial ownership is positively correlated with a high R&D intensity. Yet such potentially pro-competitive effects need to be weighed against the anti-competitive effects of passive backward integration presented here.
Appendix: Proofs

Proof of Proposition 1. Suppose for the moment that only downstream firm $i$ holds shares in $U$, i.e. $\delta_i > \delta_{-i} = 0$. The first order condition $\frac{\partial \Pi_i}{\partial p_i} = 0$ implied by (11) and, hence, the best-reply $p_i^*(p_i)$ of $-i$ is independent of $\delta_i$. In contrast, the marginal profit $\frac{\partial \Pi_i}{\partial p_i}$ increases in $i$'s ownership share $\delta_i$ for $\delta_{-i} \in [0, 1)$. This implies a higher best reply $p_i^*(p_{-i}|\delta_i)$ for any given $p_{-i}$. By continuity, $\frac{\partial p_i^*(p_{-i}|\delta_i)}{\partial \delta_i} > 0$. Strategic complementarity of downstream prices implies that an increase in $\delta_i$ increases both equilibrium prices. This argument straightforwardly extends to the case where both firms hold shares in $U$ because then $\frac{\partial^2 \Pi_i}{\partial p_i \partial \delta_{-i}} > 0$. \hfill \square

Proof of Proposition 2. Differentiating the combined profits of $A$ and $U$ with respect to $\delta_A$ and using that $\delta_B = 0$ yields

$$\frac{d \Pi^U_A}{d \delta_A} = \left( p^*_A \frac{\partial q_A}{\partial p_A} + q_A + c \frac{\partial q_B}{\partial p_A} \right) \frac{dp^*_A}{d \delta_A} + \left( p^*_A \frac{\partial q_A}{\partial p_B} + c \frac{\partial q_B}{\partial p_B} \right) \frac{dp^*_B}{d \delta_A}. \quad (19)$$

Clearly, at $c = 0$, the derivative is equal to zero as $\frac{dp^*_i}{d \delta_A} = 0$ when the upstream margin is zero. To assess the derivative for small, but positive $c$, further differentiate with respect to $c$ to obtain

$$\frac{d^2 \Pi^U_A}{d \delta_A dc} = \frac{d}{dc} \left( p^*_A \frac{\partial q_A}{\partial p_A} + q_A + c \frac{\partial q_B}{\partial p_A} \right) \frac{dp^*_A}{d \delta_A} + \frac{d}{dc} \left( p^*_A \frac{\partial q_A}{\partial p_B} + c \frac{\partial q_B}{\partial p_B} \right) \frac{dp^*_B}{d \delta_A} + \left( p^*_A \frac{\partial q_A}{\partial p_A} + q_A + c \frac{\partial q_B}{\partial p_A} \right) \frac{d^2 p^*_A}{d \delta_A dc} + \left( p^*_A \frac{\partial q_A}{\partial p_B} + c \frac{\partial q_B}{\partial p_B} \right) \frac{d^2 p^*_B}{d \delta_A dc}.$$

Evaluating this derivative at $c = 0$ yields

$$\frac{d^2 \Pi^U_A}{d \delta_A dc} \big|_{c=0} = p^*_A \frac{\partial q_A}{\partial p_B} \frac{d^2 p^*_B}{d \delta_A dc} \big|_{c=0};$$

because $\frac{dp^*_A}{d \delta_A} \big|_{c=0} = \frac{dp^*_B}{d \delta_A} \big|_{c=0} = 0$ and $p^*_A \frac{\partial q_A}{\partial p_A} + q_A = 0$ (this is the FOC of $\pi_A$ with respect to $p_A$ at $c = 0$). Recall that $\frac{dp^*_B}{d \delta_A} > 0$ for $c > 0$ (Proposition 1) whereas $\frac{dp^*_A}{d \delta_A} = 0$ at $c = 0$. By continuity, this implies $\frac{d^2 p^*_B}{d \delta_A dc} \big|_{c=0} > 0$. It follows that $\frac{d^2 \Pi^U_A}{d \delta_A dc} \big|_{c=0} > 0$ which, by continuity, establishes the result. \hfill \square

Proof of Proposition 3. The best response function of $A$ under complete separation is characterized by

$$\frac{\partial \Pi_A}{\partial p_A} = (p_A - c) \frac{\partial q_A}{\partial p_A} + q_A = 0. \quad (20)$$

When maximizing the integrated profit $p_A q_A + w_B q_B$, it is – as argued before – still optimal to serve $B$ at $w_B \leq c$ and, hence, the corresponding downstream price reaction is characterized by

$$p_A \frac{\partial q_A}{\partial p_A} + q_A + w_B \frac{\partial q_B}{\partial p_A} = 0. \quad (21)$$
Subtract the left hand side (lhs) of (20) from the lhs of (21) to obtain $\Delta \equiv c \frac{\partial q_A}{\partial p_A} + w_B \frac{\partial q_B}{\partial p_A}$. The symmetric fixed point under separation ($\delta_A = \delta_B = 0$ and no shift in price control) has $p_A = p_B$. This implies $\frac{\partial q_B}{\partial p_A} = \frac{\partial q_A}{\partial p_B}$. Hence, at equal prices, $\Delta$ is negative as $\frac{\partial q_A}{\partial p_A} \geq 0$ and $w_B \leq c$. A negative $\Delta$ implies that the marginal profit of $A$ under integration is lower and thus the integrated $A$ wants to set a lower $p_A$. The best-reply function of $B$ is characterized by

$$\frac{\partial \Pi_B}{\partial p_B} = (p_B - y) \frac{\partial q_B}{\partial p_B} + q_B(p_B, p_A) = 0$$

(22)

with $y = c$ under separation and $y = w_B \leq c$ under integration of $A$ and $U$. Hence the best reply function $p^*_B(p_A)$ of $B$ is (weakly) lower under integration. Taken together, strategic complementarity and stability (Assumptions 3 and 4) implies that the unique fixed point of the downstream prices under integration must lie strictly below that under separation.

Proof of Proposition 4. We look at the joint profit $\Pi_A^U$ of $A$ and $U$ when we move from vertical separation to vertical integration. Recall that under effective competition, the upstream firm, integrated or not, will always set the maximal input price $w_B^* = c$ when selling to firm $B$, and this independently of any choice of $w_A$. Also recall that $\Pi_A^U = p_A^* q_A(p_A^*, p_B^*) + c q_B(p_B^*, p_A^*)$. Let the equilibrium downstream prices as a function of input prices be given by $p_A^*(w_A, c) \equiv \arg \max_{p_A} p_A q_A(p_A, p_B^*) + c q_B(p_B^*, p_A^*) \equiv \arg \max_{p_B} (p_B - c) q_B(p_B, p_A^*)$. Note that $w_A = 0$ yields the downstream prices under integration, and $w_A = c$ those under separation.

The effect of an increase of $w_A$ on $\Pi_A^U$ is determined by implicit differentiation. This yields

$$\frac{d \Pi_A^U}{d w_A} = \frac{d \Pi_A^U}{d p_A} \frac{d p_A^*}{d w_A} + \frac{d \Pi_A^U}{d p_B^*} \frac{d p_B^*}{d w_A}.$$  

First, Assumptions 1-4 imply that at $w_A = c$ and hence $p^*_A = p^*_B$, we have both $\frac{d p_A}{d w_A} > 0$ and $\frac{d p_B}{d w_A} > 0$ for $c \geq 0$. Second,

$$\frac{d \Pi_A^U}{d p_A} = q_A(p_A^*, p_B^*) + (p_A^* - c) \frac{\partial q_A}{\partial p_A} + \left[ \frac{\partial q_A}{\partial p_A} + \frac{\partial q_B}{\partial p_A} \right] < 0,$$

but approaches 0 as $c$ goes to zero. Third, $\frac{d \Pi_A^U}{d p_B} = p_A^* \frac{\partial q_A}{\partial p_B} + c \frac{\partial q_B}{\partial p_B}$ is strictly positive for $c$ sufficiently close to zero. In consequence, $\left[ \frac{d \Pi_A^U}{d p_A} \frac{d p_A}{d w_A} \right]_{w_A = c} > 0$ dominates $\left[ \frac{d \Pi_A^U}{d p_B} \frac{d p_B}{d w_A} \right]_{w_A = c} < 0$ as $c$ goes to zero. Summarizing, $\frac{d \Pi_A^U}{d w_A} |_{w_A = c} > 0$ for $c$ sufficiently small. By continuity, decreasing $w_A$ from $c$ to 0 decreases $\Pi_A^U$ for $c$ sufficiently small which implies that moving from separation to integration is strictly unprofitable.

\[\Box\]
Proof of Lemma 2. Let $\pi_B(y, z)$ and $\pi^U_B(y, z)$ denote the rival’s profits at equilibrium prices, determined as functions of the equilibrium input costs $y$ charged to $B$, and $z$ charged to $A$, before and after the integration of $A$ into $U$, respectively; with a corresponding notation on equilibrium prices. Then

$$\pi_B(c, c) = (p_B(c, c) - c) q_B(p_A(c, c), p_B(c, c))$$

$$\geq (p^U_B(c, 0) - c) q_B(p_A(c, c), p^U_B(c, 0))$$

$$> (p^U_B(c, 0) - c) q_B(p^U_A(0, c), p^U_B(c, 0)) = \pi^U_B(c, 0),$$

where the first inequality follows from revealed preference and the second from Proposition 3 as well as Assumption 2.

Proof of Lemma 4. Suppose that firm $-i$ sources only from $U$. The most attractive contract that $V$ can offer $i$ must yield $V$ zero profits, i.e. $F^V_i = x^V_i \cdot (c - w^V_i)$, with $x^V_i$ denoting the quantity $i$ sources from $V$. Given $w^U_i \leq c$, the arbitrage possibility due to multiple sourcing renders contracts with $w^V_i > c$ and thus $F^V_i < 0$ unprofitable as $x^V_i$ would be 0. Recall that $p^*_i(w_i, w_{-i})$ denotes the downstream equilibrium price of $i$ as a function of the marginal input prices. The net profit of $i$ when buying all inputs from $V$ is given by

$$\Pi_i = (p^*_i(w^V_i, w^U_{-i}) - w^V_i) q_i(p^*_i(w^V_i, w^U_{-i}), p^*_{-i}(w^U_{-i}, w^V_i)) - F^V_i.$$

Substituting for $F^V_i$ using the zero profit condition of $V$ with $x^V_i = q_i$ yields

$$\Pi_i = (p^*_i(w^V_i, w^U_{-i}) - c) q_i(p^*_i(w^V_i, w^U_{-i}), p^*_{-i}(w^U_{-i}, w^V_i)).$$

Increasing $w^V_i$ at $w^V_i = c$ is profitable if $d\Pi_i/dw^V_i|_{w^V_i=c} > 0$. Differentiation yields

$$d\Pi_i/dw^V_i = \frac{d\Pi_i}{dp^*_i} \frac{dp^*_i}{dw^V_i} + \frac{d\Pi_i}{dp^*_{-i}} \frac{dp^*_{-i}}{dw^V_i}.$$

Optimality of the downstream prices implies $\frac{d\Pi_i}{dp^*_i} = 0$. Moreover, $\frac{dp^*_i}{dw^V_i} > 0$ follows from the strategic complementarity of downstream prices, and with it, the supermodularity of the downstream pricing subgame. Finally, $\frac{d\Pi_i}{dp^*_{-i}} > 0$ follows directly from $\frac{\partial}{\partial p^*_{-i}} > 0$ (substitutable products). Combining these statements yields

$$d\Pi_i/dw^V_i|_{w^V_i=c} = \frac{d\Pi_i}{dp^*_{-i}} \frac{dp^*_{-i}}{dw^V_i} > 0.$$

This implies that raising $w^V_i$ above $c$ would be profitable for $i$. However, the no arbitrage condition and $w^U_i \leq c$ renders this impossible. Analogously, decreasing $w^V_i$ below $c$ and
adjusting $F_i^V$ to satisfy zero profits of $V$ is not profitable for $i$. In consequence, the contract offer of $V$ most attractive to any downstream firm $i$ is given by $\{0, c\}$.

\[\phantom{\text{Proof of Proposition 5.}}\]

Proof of Proposition 5. Recall that for marginal input prices $w_i$ and $w_{-i}$, $i$’s equilibrium downstream price is given by $p_i^*(w_i, w_{-i})$. Also recall that

$$\pi_i^* (w_i, w_{-i}) \equiv [p_i^*(w_i, w_{-i}) - w_i] q_i \left( p_i^*(w_i, w_{-i}), p_{-i}^*(w_{-i}, w_i) \right)$$

and substitute for $\pi_i^*(c, w_{-i})$ in (16) to obtain

$$\pi^U = \sum_i p_i^*(w_i, w_{-i}) q_i \left( p_i^*(w_i, w_{-i}), p_{-i}^*(w_{-i}, w_i) \right) - \sum_i (p_i^*(c, w_{-i}) - c) q_i \left( p_i^*(c, w_{-i}), p_{-i}^*(w_{-i}, c) \right).$$

The first sum captures the industry profits and the second, as $\{0, c\}$ is $V$’s tariff that maximizes the downstream firms’ profits (Lemma 4), the value of each of the downstream firms’ outside option. An obvious candidate equilibrium tariff of $U$ is $\{F^* = c, w^* = 0\}$ to both downstream firms. This results in $\pi^U = 2c q_i(p^*(c, c), p^*(c, c))$. Let $\{F^*, w^*\}$ denote alternative symmetric equilibrium candidates offered by $U$. Recall that $w^* > c$ with $F^* < 0$ is not feasible, as then the downstream firms would source all quantities from $V$. Towards assessing whether $U$ would benefit from lowering $w$ below $c$ (and increasing $F$), we differentiate $\pi^U$ with respect to $w$ at and evaluate it at $w = c$. If that sign is positive for $w_i, i \in \{A, B\}$ separately and jointly, then $U$ has no incentive to decrease its price below $c$. Differentiation of $\pi^U$ with respect to $w_i$ yields

$$\frac{d\pi^U}{dw_i} = \left( \frac{\partial p_i^*}{\partial w_i} q_i + p_i^* \left( \frac{\partial q_i}{\partial p_i} \frac{\partial p_i^*}{\partial w_i} + \frac{\partial q_i}{\partial w_i} \frac{\partial p_i^*}{\partial p_i} \right) + \frac{\partial p_{-i}^*}{\partial q_i} q_i + p_{-i}^* \left( \frac{\partial q_{-i}}{\partial p_i} \frac{\partial p_i^*}{\partial w_i} + \frac{\partial q_{-i}}{\partial w_i} \frac{\partial p_i^*}{\partial p_i} \right) \right) - \left( \frac{\partial p_i^*}{\partial w_i} q_{-i} + p_i^* \left( \frac{\partial q_{-i}}{\partial p_i} \frac{\partial p_i^*}{\partial w_i} + \frac{\partial q_{-i}}{\partial w_i} \frac{\partial p_i^*}{\partial p_i} \right) \right).$$

Evaluating the derivative at $w_i = w_{-i} = c$, subtracting and adding $c \frac{\partial q_i}{\partial p_i} \left( \frac{\partial p_i^*}{\partial w_i} + \frac{\partial p_{-i}^*}{\partial w_i} \right)$, making use of downstream firm $i$’s FOC $\frac{\partial \pi_i^U}{\partial p_i} = (p_i^* - c) \frac{\partial q_i}{\partial p_i} + q_i = 0$ and simplifying, we obtain

$$\frac{d\pi^U}{dw_i} = c \left( \frac{\partial q_i}{\partial p_i} \frac{\partial p_i^*}{\partial w_i} + \frac{\partial q_{-i}}{\partial p_i} \frac{\partial p_{-i}^*}{\partial w_i} + \frac{\partial q_i}{\partial w_i} \frac{\partial p_i^*}{\partial p_i} + \frac{\partial q_{-i}}{\partial w_i} \frac{\partial p_{-i}^*}{\partial p_i} \right) + (p_i^* - c) \frac{\partial q_i}{\partial p_i} \frac{\partial p_{-i}^*}{\partial w_i}. \tag{24}$$

Substituting for $p_i^* - c = -q_i / \frac{\partial q_i}{\partial p_i}$ from the FOC $\frac{\partial \pi_i^U}{\partial p_i} = 0$ yields that $\frac{d\pi^U}{dw_i} > 0$ iff

$$c < -\left( \frac{q_i}{\frac{\partial q_i}{\partial p_i} + \frac{\partial q_{-i}}{\partial p_{-i}}} \right) \cdot \frac{\partial q_i}{\partial p_i} \frac{\partial p_{-i}^*}{\partial w_{-i}} \cdot \frac{\partial q_i}{\partial w_i} \frac{\partial p_i^*}{\partial w_i} + \frac{\partial q_{-i}}{\partial w_i} \frac{\partial p_{-i}^*}{\partial w_{-i}}. \tag{25}$$

The rhs of (25) remains positive as $c$ goes to zero. Hence (25) holds for $c$ sufficiently small.
This establishes the result. 

Proof of Lemma 5. With passive backward ownership $\delta_A > \delta_B = 0$, the important distinction is that when $B$ buys from $V$, $A$ does not internalize the sales of $B$. Again, given that $V$ charges $\{0, c\}$, $U$ sets the downstream firms indifferent with fees of

$$
F_A = \Pi_{A(U)}(w_A, w_B) - \Pi_{A(V)}(c, w_B), \\
F_B = \Pi_{B(U)}(w_B, w_A) - \Pi_{B(V)}(c, w_A),
$$

where $\Pi^\delta_{i(j)}$, $\Pi_{i(j)}$ are the reduced form total downstream profits of $i$ when sourcing from $j$ as a function of nominal marginal input prices. Substituting the fees in the profit function of $U$ yields

$$
\pi^U = \sum_{i \in \{A, B\}} \left[ p_i^* q_i \left( p_i^*, p_{i,-i}^* \right) \right] - \Pi_{A(V)}(c, w_B) - \Pi_{B(V)}(c, w_A). \tag{26}
$$

As before, the profit consists of the industry profit $\pi^I \equiv \sum_i p_i^* q_i$ less the off-equilibrium outside options. The optimal marginal input prices are characterized by

$$
\frac{\partial \pi^U}{\partial w_A} = \frac{\partial \pi^I}{\partial w_A} - \frac{\partial \Pi_B(c, w_A)}{\partial w_A}, \\
\frac{\partial \pi^U}{\partial w_B} = \frac{\partial \pi^I}{\partial w_B} - \frac{\partial \Pi_A(c, w_A)}{\partial w_B}.
$$

For $w_B = c$ and $w_A = c/(1 - \delta_A)$, the derivatives converge to (24), used in the Proof of Proposition 5, when $\delta_A \rightarrow 0$. Thus the derivatives are still positive when $\delta_A$ increases marginally at 0. By continuity, the corner solutions are sustained for small backward integration shares and $c$ sufficiently small. Moreover, $F_A = \Pi_{A(U)}(c/(1 - \delta_A), c) - \Pi_{A(V)}(c, c) = 0$ and $F_B = \Pi_{B(U)}(c, c/(1 - \delta_A)) - \Pi_{B(V)}(c, c/(1 - \delta_A)) > 0$ as $A$ prices more aggressively when $B$ sources from $V$, because then $A$ does internalize sales via the profit part $\delta_A w_B q_B$. This logic extends to the case that also $\delta_B$ increases at 0. 

References


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